ON RECOVERY OF BLOCK SPARSE SIGNALS FROM MULTIPLE MEASUREMENTS

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ABSTRACT

We consider the problem of recovering block sparse signals which share the same sparsity pattern given multiple measurements. We consider two different noisy measurement models. In the first model, the sensing matrix remains the same for all the measurements. In the second model, we employ different sensing matrices for different measurements. For both these models, we present greedy algorithms for block sparse signal recovery and theoretically establish the recovery guarantees of the proposed algorithms. Using numerical simulations, we study the performance of the proposed algorithms and some existing algorithms. Our results present insights on how the correlation between block sparse signals plays a role on the recovery performance.

Index Terms— Block sparse signal, multiple measurement vectors, subspace matching pursuit, generalized multiple measurement vectors

1. INTRODUCTION AND SYSTEM MODEL

A vector is referred as sparse if the number of non-zero entries in the vector is small compared to its size/dimension. Sparse signals/vectors arise in a wide range of applications and the topic of compressive sensing (CS) deals with the problem of recovering an unknown sparse vector from an under-determined system of linear equations [1]. In many applications (for instance, see [2]), the non-zero entries in a sparse vector appear in blocks (or groups). Block sparse signal recovery has been previously considered in [3–6]. Multiple measurement vector problem in CS literature deals with the joint recovery of several sparse signals which share the same support from multiple measurements [7–10]. In this paper, we are interested in recovering unknown block sparse signals which share the same support from multiple measurements.

Let $\boldsymbol{x}$ be a $PD \times 1$ vector [1] composed of blocks $\boldsymbol{x}_i \in \mathbb{R}^{D\times 1}$ such that $\boldsymbol{x} = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \cdots, \boldsymbol{x}_P^T]^T$. We define ordered set $\mathcal{S} = \{i \in \{1, \cdots, P\} : \|\boldsymbol{x}_i\| > 0\}$ which is the set of indices of non-zero blocks of $\boldsymbol{x}$. We refer $\boldsymbol{x}$ as K-block sparse signal if $|\mathcal{S}| \leq K$ and refer $\mathcal{S}$ as the *sparsity pattern* of $\boldsymbol{x}$. In this paper, our focus is on the block sparse signals which share the same sparsity pattern. Towards this, let $\boldsymbol{x}^{(j)}$ for $j = 1, \cdots, L$ denote K-block sparse signals, composed of blocks $\boldsymbol{x}_i^{(j)}$ for $i = 1, \cdots, P$ with the same sparsity pattern $\mathcal{S}^{(j)} = \mathcal{S}, \forall j = 1, \cdots, L$.

We consider two different noisy observation models. In the first one, the measurement (or sensing) matrix is the same for all the measurements. The sensing matrix $\boldsymbol{A} \in \mathbb{R}^{M\times PD}$ is horizontal concatenation of blocks $\boldsymbol{A}_i \in \mathbb{R}^{M\times D}$ such that $\boldsymbol{A} = [\boldsymbol{A}_1 \cdots \boldsymbol{A}_P]$. The noisy measurement vectors are given by

$$\boldsymbol{y}^{(j)} = \boldsymbol{A}\boldsymbol{x}^{(j)} + \boldsymbol{v}^{(j)},\ j = 1, \cdots, L \tag{1}$$

where $\boldsymbol{v}^{(j)}$ denotes the additive noise. We are interested in scenarios where $KD \leq M \ll PD$. Using the observations $\boldsymbol{y}^{(j)}$, we wish to recover the joint sparsity pattern $\mathcal{S}$ (which is same for all j) and provide the estimates of the block sparse signals $\boldsymbol{x}^{(j)}$ for $j = 1, \cdots, L$. We refer this problem as block sparse signal recovery using multiple measurement vectors (MMV). Special cases of this problem have been addressed previously in the literature. For instance, $L = 1, D = 1$ is the conventional sparse signal recovery problem with single measurement vector (SMV) [1]. For $D = 1, L > 1$, this is the MMV problem for conventional sparse signals addressed in [7, 8, 10]. For $D > 1, L = 1$, the block sparse signal recovery for SMV have been addressed in [3–6].

In the second observation model, the observation vectors are given by

$$\boldsymbol{y}^{(j)} = \boldsymbol{A}^{(j)}\boldsymbol{x}^{(j)} + \boldsymbol{v}^{(j)},\ j = 1, \cdots, L \tag{2}$$

where the sensing matrices $\boldsymbol{A}^{(j)} \in \mathbb{R}^{M\times PD}$ are composed of blocks $\boldsymbol{A}_i^{(j)} \in \mathbb{R}^{M\times D}$ such that $\boldsymbol{A}^{(j)} = [\boldsymbol{A}_1^{(j)} \cdots \boldsymbol{A}_P^{(j)}]$. In the second model (2), the sensing matrices vary with j as opposed to the same sensing matrix in the first model (1). We refer this problem as block sparse signal recovery using generalized multiple measurement vectors (GMMV). For $D = 1, L > 1$ this GMMV problem have been addressed in [9].

We consider bounded noise case such that $\|\boldsymbol{v}^{(j)}\| \leq \epsilon, \forall j$. We assume that the sensing matrices $\boldsymbol{A}$, $\boldsymbol{A}^{(j)}$ are of full rank. We provide greedy algorithms and their recovery guarantees for MMV and GMMV problems in Section 2 and Section 3 respectively. We give numerical results in Section 4 and present insights on the recovery performance.

2. MMV RECOVERY

2.1. Recovery Algorithm

In this section, we address the MMV problem represented in (1) by providing a greedy recovery algorithm and establishing its recovery guarantees. Towards that, horizontally concatenating the observation vectors, unknown vectors, and noise vectors as $\boldsymbol{Y} = [\boldsymbol{y}^{(1)}, \ldots, \boldsymbol{y}^{(L)}]$ and $\boldsymbol{X} = [\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(L)}]$ and $\boldsymbol{V} = [\boldsymbol{v}^{(1)}, \ldots, \boldsymbol{v}^{(L)}]$, we rewrite the equation (1) as

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{V}. \tag{3}$$

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[1]*Notation:* Vectors/matrices are denoted by bold lower/upper case letters. $(\cdot)^T$ denotes transpose, $\|\cdot\|$ denotes ℓ_2 norm of vectors, $\|\cdot\|_F$ denotes Frobenius norm of matrices and $\mathrm{col}(\cdot)$ denotes the column space of matrices. Ordered sets are denoted by mathcal font $\mathcal{A}$, $|\mathcal{A}|$ denotes set size.

We refer $\boldsymbol{X}$ as K-block sparse matrix. We can view sensing matrix $\boldsymbol{A}$ as a concatenation of P subspaces where the blocks $\boldsymbol{A}_i$'s form basis for each of the P subspaces. We extend the subspace matching pursuit (SMP) [6] algorithm for block sparse recovery to our MMV problem. We refer this algorithm as SMP-MMV which is described below. We use $\Pi_i(\cdot)$ to denote the projection operator onto subspace $\mathrm{col}(\boldsymbol{A}_i),\ i = 1, \ldots, P$.

Step 1. Initialize: $\boldsymbol{R}_0 = \boldsymbol{Y}$, $k = 1$, $\boldsymbol{\Phi} = [\,]$, $\Gamma_0 = \emptyset$.

Step 2. Find the index γ_k such that

$$\gamma_k = \underset{i=1,\cdots,P}{\arg\min} \|\boldsymbol{R}_{k-1} - \Pi_i(\boldsymbol{R}_{k-1})\|_F.$$

Step 3. $\Gamma_k \leftarrow \Gamma_{k-1} \cup \{\gamma_k\}$ and $\boldsymbol{\Phi}_k \leftarrow [\boldsymbol{\Phi}_{k-1}\ \boldsymbol{A}_{\gamma_k}]$.

Step 4. Calculate new estimate of the signal:

$$\hat{\boldsymbol{X}} = \underset{\underline{\boldsymbol{X}}}{\arg\min} \|\boldsymbol{Y} - \boldsymbol{\Phi}_k \underline{\boldsymbol{X}}\|_F.$$

Step 5. $\boldsymbol{R}_k = \boldsymbol{Y} - \boldsymbol{\Phi}_k \hat{\boldsymbol{X}}$ and $k \leftarrow k + 1$.

Step 6. If $k \le K$ go to Step 2.

2.2. Recovery Guarantees

To obtain the recovery guarantees of SMP-MMV, we use a quantity called *mutual subspace incoherence* (denoted by μ) of the matrix $\boldsymbol{A}$ which is defined as [6],

$$\mu(\boldsymbol{A}) \quad = \quad \max_{\substack{i,j \in \{1,\ldots,P\} \\ i \neq j}} \left\{ \max_{\substack{\boldsymbol{a} \in \mathrm{col}(\boldsymbol{A}_i) \\ \boldsymbol{b} \in \mathrm{col}(\boldsymbol{A}_j)}} \frac{|\langle \boldsymbol{a}, \boldsymbol{b} \rangle|}{\|\boldsymbol{a}\|\ \|\boldsymbol{b}\|} \right\}.$$

Geometrically, $\mu(\boldsymbol{A})$ measures the smallest angle between any two subspaces from the set $\{\mathrm{col}(A_i), i \in \{1, \cdots, P\}\}$. Mutual subspace incoherence of randomly generated matrices with some structure has been characterized in [2].

Consider the noiseless observations, $\tilde{\boldsymbol{Y}} = \boldsymbol{A}\boldsymbol{X}$ and note that each column $\tilde{\boldsymbol{y}}^{(j)},\ j = 1, \cdots, L$ of matrix $\tilde{\boldsymbol{Y}}$ is a linear combination of vectors from K subspaces chosen from the collection $\{\mathrm{col}(\boldsymbol{A}_i), i \in \{1, \cdots, P\}\}$. For convenience in notations, without loss of generality, we assume that the first K blocks in $\boldsymbol{x}^{(j)}$ are non-zero. So we can write,

$$\tilde{\boldsymbol{y}}^{(j)} = \sum_{i=1}^{K} w_i^{(j)} \boldsymbol{z}_i^{(j)},\ j = 1, \ldots, L \tag{4}$$

where $\boldsymbol{z}_i^{(j)} \in \mathrm{col}(\boldsymbol{A}_i)$ and $\|\boldsymbol{z}_i^{(j)}\| = 1$. For convenience, without loss of generality, we assume that the scalars are ordered such that $|w_1^{(j)}| \ge |w_2^{(j)}| \ge \cdots \ge |w_K^{(j)}| > 0$.

Theorem 1. *SMP-MMV can perfectly identify the sparsity pattern of K-block sparse matrix $\boldsymbol{X}$ from noisy observations $\boldsymbol{Y}$ in (3) if*

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\boldsymbol{A})}\right) - \frac{1}{\mu(\boldsymbol{A})} \frac{\epsilon}{w_{\min}}, \tag{5}$$

where $w_{\min} = \min\{|w_K^{(1)}|, \cdots, |w_K^{(L)}|\}$.

Proof. The following proof is an extension of the proof for the single measurement case given in [6]. SMP-MMV algorithm will pick one of the first K subspaces in the first step if

$$\|\Pi_1(\boldsymbol{Y})\|_F > \|\Pi_k(\boldsymbol{Y})\|_F \ \text{ for all } k > K. \tag{6}$$

To lower bound the left hand side of the above equation,

$$\left\|\Pi_1(\tilde{\boldsymbol{Y}} + \boldsymbol{V})\right\|_F \ge \left\|\Pi_1(\tilde{\boldsymbol{Y}})\right\|_F - \|\Pi_1(\boldsymbol{V})\|_F. \tag{7}$$

First part in (7) can be written as,

$$\left\|\Pi_1(\tilde{\boldsymbol{Y}})\right\|_F = \sqrt{\sum_{j=1}^{L} \left\|\Pi_1\left(\tilde{\boldsymbol{y}}^{(j)}\right)\right\|^2}. \tag{8}$$

Lower bound to the each term in (8) is found as below,

$$\begin{aligned} \left\|\Pi_1(\tilde{\boldsymbol{y}}^{(j)})\right\| &= \left\| w_1^{(j)} \boldsymbol{z}_1^{(j)} + \sum_{i=2}^{K} w_i^{(j)} \Pi_1(\boldsymbol{z}_i^{(j)}) \right\| \\ &\ge \|w_1^{(j)} \boldsymbol{z}_1^{(j)}\| - \sum_{i=2}^{K} |w_i^{(j)}| \left\|\Pi_1(\boldsymbol{z}_i^{(j)})\right\| \\ &\ge |w_1^{(j)}| - |w_1^{(j)}| \sum_{i=2}^{K} \mu(\boldsymbol{A}). \end{aligned} \tag{9}$$

Substituting in (8) we get,

$$\left\|\Pi_1(\tilde{\boldsymbol{Y}})\right\|_F \ge \sqrt{\sum_{j=1}^{L} \left(w_1^{(j)}\right)^2}\ [1 - (K-1)\mu(\boldsymbol{A})]. \tag{10}$$

Let $\boldsymbol{w}_1 = [w_1^{(1)} w_1^{(2)} \ldots w_1^{(L)}]^T$, then

$$\|\Pi_1(\tilde{\boldsymbol{Y}})\|_F \ge \|\boldsymbol{w}_1\|[1 - (K-1)\mu(\boldsymbol{A})].$$

Similarly, second part in (7) can be written as,

$$\|\Pi_1(\boldsymbol{V})\|_F = \sqrt{\sum_{j=1}^{L} \|\Pi_1\left(\boldsymbol{v}^{(j)}\right)\|^2} \ \le\ \epsilon\sqrt{L}, \tag{11}$$

since $\|\Pi_1(\boldsymbol{v})\| \le \|\boldsymbol{v}\|$ for any $\boldsymbol{v}$. Similarly right hand side of (6) can be written as,

$$\|\Pi_k(\boldsymbol{Y})\|_F \le \sqrt{\sum_{j=1}^{L} \left\|\Pi_k\left(\tilde{\boldsymbol{y}}^{(j)}\right)\right\|^2} + \epsilon\sqrt{L}. \tag{12}$$

Upper bound to terms in square root in (12) can be found as,

$$\left\|\Pi_k(\tilde{\boldsymbol{y}}^{(j)})\right\| = \left\| \sum_{i=1}^{K} w_i^{(j)} \Pi_k(\boldsymbol{z}_i^{(j)}) \right\| \ \le\ |w_1^{(j)}| \mu(\boldsymbol{A}) K. \tag{13}$$

Substituting in (12), we have

$$\|\Pi_k(\boldsymbol{Y})\|_F \le \|\boldsymbol{w}_1\| \mu(\boldsymbol{A}) K \ + \epsilon\sqrt{L}. \tag{14}$$

From (6),(7),(10), (11) and (14) we have the required condition,

$$\|\boldsymbol{w}_1\|[1-(K-1)\mu(\boldsymbol{A})] - \epsilon\sqrt{L} \ >\ \|\boldsymbol{w}_1\|\mu(\boldsymbol{A})K + \epsilon\sqrt{L}, \text{ or}$$

$$K < \frac{1 + \mu(\boldsymbol{A})}{2\mu(\boldsymbol{A})} - \frac{1}{\mu(\boldsymbol{A})} \frac{\epsilon\sqrt{L}}{\|\boldsymbol{w}_1\|}. \tag{15}$$

Repeating the above argument for each iteration, defining $\boldsymbol{w}_k = [w_k^{(1)}, \cdots, w_k^{(L)}]^T$, we have the following conditions after K iterations,

$$K \ <\ \frac{1 + \mu(\boldsymbol{A})}{2\mu(\boldsymbol{A})} - \frac{1}{\mu(\boldsymbol{A})} \frac{\epsilon\sqrt{L}}{\|\boldsymbol{w}_k\|}, \text{ for } k = 1, \cdots, K. \tag{16}$$

Due to the ordering of $w_i^{(j)}$'s, with $w_{min} = \min\{w_K^{(1)}, w_K^{(2)}, \ldots, w_K^{(L)}\}$, we have $\|\boldsymbol{w}_k\| \ge \sqrt{L}\ w_{min}$ for $k \in \{1, \cdots, K\}$. Thus the sufficient condition in (16) can be written as in (5). □

Corollary 1. *A sufficient condition for SMP-MMV algorithm to perfectly recover K-block sparse matrix $\boldsymbol{X}$ from noiseless observations $\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X}$, is*

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\boldsymbol{A})}\right) \tag{17}$$

2.3. Remarks on Recovery

SMP-MMV noiseless recovery condition in (17) is the same as that of SMP for single measurement case [6]. Note that when the unknown vectors are identical $\boldsymbol{x}^{(1)} = \cdots = \boldsymbol{x}^{(L)}$, the noiseless observations are identical $\boldsymbol{y}^{(1)} = \cdots = \boldsymbol{y}^{(L)}$. Hence, having more observations does not provide "new" information for $L > 1$ and hence MMV recovery becomes identical to SMV. Since our recovery guarantee in Corollary 1 holds true for any K-block sparse vectors, it has to hold for the identical unknowns as well. Hence Corollary 1 does not show improvement over SMV. For the conventional sparse signal recovery, such a conclusion has been drawn for the extension of greedy orthogonal matching pursuit (OMP) algorithm [11] to the MMV problem in [7, 9]. On the other hand, we will see from our simulation results (Section 4) that SMP-MMV performs better than SMP for SMV when the unknown vectors are not identical.

3. GMMV RECOVERY

3.1. Recovery Algorithm

For the GMMV problem represented in (2), we propose a greedy recovery algorithm which is referred as SMP-GMMV algorithm. We use $\Pi_i(\boldsymbol{y}^{(j)})$ to denote projection of vector $\boldsymbol{y}^{(j)}$ onto $\text{col}(\boldsymbol{A}_i^{(j)})$.

Step 1a. Initialize: $k = 1, \Gamma_0 = \emptyset$.
Step 1b. Initialize: $\boldsymbol{\Phi}^{(j)} = [\,]$, $\boldsymbol{r}_0^{(j)} = \boldsymbol{y}^{(j)}$ for $j = 1, \ldots, L$.
Step 2. Find the index γ_k such that

$$\gamma_k = \underset{i=1,\cdots,P}{\arg\min} \sum_{j=1}^{L} \|\boldsymbol{r}_{k-1}^{(j)} - \Pi_i(\boldsymbol{r}_{k-1}^{(j)})\|.$$

Step 3. $\Gamma_k \leftarrow \Gamma_{k-1} \cup \{\gamma_k\}$ and $\boldsymbol{\Phi}_k^{(j)} \leftarrow [\boldsymbol{\Phi}_{k-1}^{(j)}\ \boldsymbol{A}_{\gamma_k}^{(j)}]$ for $j = 1, \ldots, L$.
Step 4. Calculate new estimate of the signal:

$$\hat{\boldsymbol{x}}^{(j)} = \underset{\underline{\boldsymbol{x}}^{(j)}}{\arg\min} \|\boldsymbol{y}^{(j)} - \boldsymbol{\Phi}_k^{(j)}\underline{\boldsymbol{x}}^{(j)}\| \text{ for } j = 1, \ldots, L.$$

Step 5. $\boldsymbol{r}_k^{(j)} \leftarrow \boldsymbol{y}^{(j)} - \boldsymbol{\Phi}_k^{(j)}\hat{\boldsymbol{x}}^{(j)}$ for $j = 1, \ldots, L$ and $k \leftarrow k+1$.
Step 6. If $k \leq K$ go to Step 2.

3.2. Recovery Guarantees

In order to derive the recovery guarantees, as before, we note that each observation vector $\boldsymbol{y}^{(j)}$ can be written as a sum of vectors from K-subspaces from the collection $\{\text{col}(\boldsymbol{A}_i^{(j)}), i = 1, \cdots, P\}$. Without loss of generality, we assume that the first K subspaces are nonzero and we write each $\boldsymbol{y}^{(j)}$ as

$$\boldsymbol{y}^{(j)} = \sum_{i=1}^{K} w_i^{(j)} \boldsymbol{z}_i^{(j)},$$

where $\boldsymbol{z}_i^{(j)} \in \text{col}(\boldsymbol{A}_i^{(j)})$, with $\|\boldsymbol{z}_i^{(j)}\| = 1$ and $|w_1^{(j)}| \geq |w_2^{(j)}| \geq \cdots \geq |w_K^{(j)}| > 0$. We denote the mutual subspace incoherence of $\boldsymbol{A}^{(j)}$ by $\mu^{(j)}$.

Theorem 2. *SMP-GMMV can perfectly identify the sparsity pattern of K- block sparse vectors from the noisy measurements in (2) if*

$$K < \min_k \left(\frac{1}{2}\left(\frac{\sum_{j=1}^{L} |w_k^{(j)}|}{\sum_{j=1}^{L} |w_k^{(j)}|\mu^{(j)}} + 1 \right) - \frac{L\epsilon}{\sum_{j=1}^{L} |w_k^{(j)}|\mu^{(j)}} \right). \tag{18}$$

Proof. We proceed in the similar fashion as the proof of Theorem 1. The algorithm will pick a correct subspace in the first step if,

$$\sum_{j=1}^{L} \left\|\Pi_1(\boldsymbol{y}^{(j)})\right\| > \sum_{j=1}^{L} \left\|\Pi_k(\boldsymbol{y}^{(j)})\right\|, \text{ for } k > K. \tag{19}$$

As in the proof of Theorem 1, we bound the left and right hand sides as,

$$\sum_{j=1}^{L} \left\|\Pi_1(\boldsymbol{y}^{(j)})\right\| \geq \sum_{j=1}^{L} \left[|w_1^{(j)}|(1 - \mu^{(j)}(K-1))\right] - L\epsilon \tag{20}$$

$$\sum_{j=1}^{L} \left\|\Pi_k(\boldsymbol{y}^{(j)})\right\| \leq \sum_{j=1}^{L} \left[|w_1^{(j)}|\mu^{(j)}K\right] + L\epsilon. \tag{21}$$

From (19), (20) and (21), for the first iteration $k = 1$, we get the following condition,

$$K < \frac{1}{2}\left(\frac{\sum_{j=1}^{L} |w_k^{(j)}|}{\sum_{j=1}^{L} |w_k^{(j)}|\mu^{(j)}} + 1 \right) - \frac{L\epsilon}{\sum_{j=1}^{L} |w_k^{(j)}|\mu^{(j)}}. \tag{22}$$

Repeating for K iterations, we get the above condition (22) for $k = 1, \cdots, K$, which can be combined as (18). ☐

For the noiseless observations, the recovery guarantee of SMP-GMMV is obtained by substituting $\epsilon = 0$ in (18). If the unknown vectors are identical $\boldsymbol{x}^{(1)} = \cdots = \boldsymbol{x}^{(L)}$, we have $w_i^{(1)} = \cdots = w_i^{(L)}$ for all i and the recovery condition in (18) can be further simplified. Towards that, we define the average mutual subspace incoherence $\breve{\mu} = \frac{1}{L}\sum_{j=1}^{L} \mu^{(j)}$.

Corollary 2. *For identical unknown K-block sparse vectors, SMP-GMMV algorithm perfectly recovers all the non-zero blocks from the noiseless GMMV measurements if*

$$K < \frac{1}{2}\left(\frac{1}{\breve{\mu}} + 1\right). \tag{23}$$

3.3. Remarks on Recovery

Note that the recovery guarantee in (23) depends on the *average* value of the mutual subspace incoherence of sensing matrices $\{\boldsymbol{A}^{(j)}\}$. In compressive sensing, it is typical to generate the sensing matrices randomly. In that regard, for MMV, if $\mu(\boldsymbol{A})$ is bad for a given random matrix, then recovery condition in Corollary 1 may fail. On the other hand, for GMMV, as long as the average value $\breve{\mu}$ satisfies the recovery condition in Corollary 2, recovery will be successful even if some $\mu^{(j)}$'s fail to individually satisfy the recovery condition in Corollary 1. Thus our result theoretically establish the superiority of GMMV recovery over MMV recovery when the unknown vectors are identical.

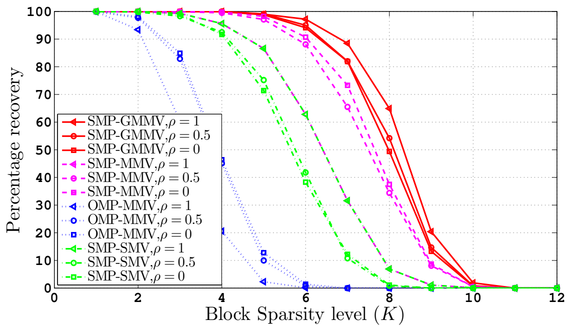

Fig. 1. Performance with K for $M = 40, P = 30, D = 4, L = 2$

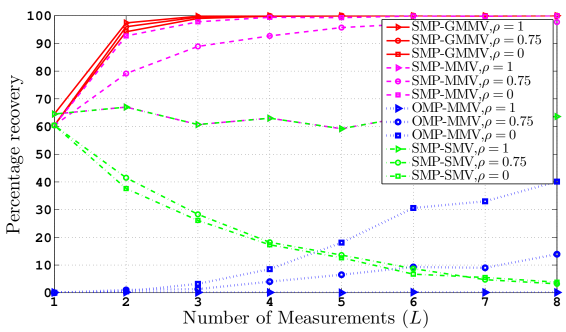

Fig. 2. Performance with L for $M = 40, P = 30, D = 4, K = 6$

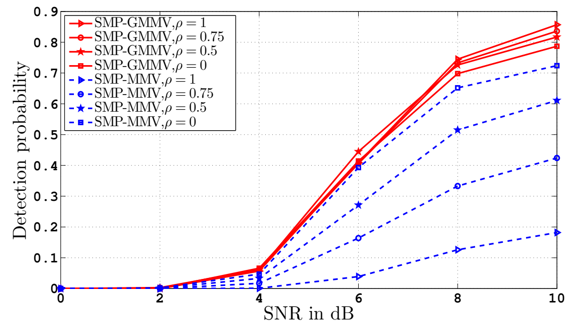

Fig. 3. Noisy recovery:$M = 64, P = 168, D = 8, K = 6, L = 2$

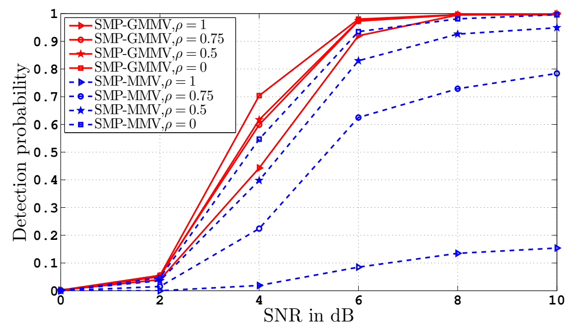

Fig. 4. Noisy recovery:$M = 64, P = 168, D = 8, K = 6, L = 4$

4. SIMULATION RESULTS

The entries of the sensing matrices $\boldsymbol{A}, \boldsymbol{A}^{(j)}$ are generated using i.i.d. normal distribution. For various values of L and K, we compare the recovery performance of SMP-MMV and SMP-GMMV. We say that the recovery is successful only if the support of all the K non-zero blocks in all the L observations are recovered perfectly. As noted in Sections 2.3 and 3.3, the recovery performance is related to the correlation among the observations (see also [8]). Using $\boldsymbol{x}_o^{(j)}$ to denote the vector composed of non-zero entries of $\boldsymbol{x}^{(j)}$, we generate $\boldsymbol{x}_o^{(j)}$ using standard normal distribution with $\mathrm{E}\{\boldsymbol{x}_o^{(j)}\boldsymbol{x}_o^{(j+1)\mathsf{T}}\} = \rho\boldsymbol{I}$ for $j = 1, \cdots, L-1$. Note that ρ captures the correlation among the unknown vectors. For $\rho = 1$, all the unknown vectors are identical and for $\rho = 0$, the unknown vectors are uncorrelated. For completeness, we also show the performance of SMP algorithm applied to the L observations separately and also the OMP-MMV algorithm given in [7]. Note that SMP ignores the fact that all the L observations share the same support and OMP-MMV ignores the fact that the unknown vectors have block sparse structure. For various values of ρ, we compare the noiseless recovery performance in Figures 1, 2 and noisy recovery performance in Figures 3, 4. Following observations can be made from the plots.

- SMP-GMMV performs better than all the other algorithms and its performance does not change much with respect to ρ.
- For $\rho = 1$, performance of SMP-MMV is inferior to SMP-GMMV as established from our recovery guarantees (Section 3.3). As ρ decreases, performance of SMP-MMV improves and gets closer to that of SMP-GMMV.
- SMP-MMV performs better than OMP-MMV for all ρ since OMP ignores the block sparse nature of the unknowns (Figures 1 and 2).
- For $\rho = 1$, performance of SMP-MMV becomes identical to that of SMP as discussed in Section 2.3 (Figures 1 and 2). However, as ρ decreases, SMP-MMV performs better than SMP.
- For small values of L, SMP outperforms OMP-MMV (Fig. 2). In this regime, the value of exploiting block sparsity outweighs the value of exploiting joint sparsity of multiple measurements.
- For $\rho < 1$, as L increases, performance of OMP-MMV improves and that of SMP degrades (See Fig. 2). Reason for the degradation in SMP is because we declare success only if support is recovered perfectly for all the L unknown vectors.

5. CONCLUSIONS

We presented block sparse signal recovery algorithms for MMV and GMMV cases. We theoretically established that SMP-GMMV is superior to SMP-MMV for identical unknown signals. Our numerical results give insights on how the correlation among the unknown vectors plays a role on the recovery performance.

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