ANALYSIS OF RF ENERGY HARVESTING IN LARGE-SCALE NETWORKS USING ABSORPTION FUNCTION

Koji Ishibashi† *and Giuseppe Abreu*∗

† Advanced Wireless Communication Research Center (AWCC)
The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan
∗ School of Engineering and Sciences,
Jacobs University Bremen, Germany Campus Ring 1, 28759 Bremen, Germany

ABSTRACT

This paper reveals an impairment of well-known path-loss model generally assumed in conventional works with radio frequency (RF) energy harvesting. We will prove that, when RF energy harvesting is considered in a large-scale network, the summation of energy transferred to nodes in the network diverges while the radiated energy must be finite. Thus, we propose a new absorption function meeting this law and derive several expressions of average harvested energy based on the model. Furthermore, the effectiveness of RF energy harvesting with Poisson point process (PPP) network with a single transmitter is demonstrated.

Index Terms— Poisson point process (PPP), RF energy harvesting, absorption function

1. INTRODUCTION

From the beginning of wireless communications with mobile devices, their lifetime is strictly limited by a finite-sized battery. Conventional works, hence, focus on reducing energy consumption of mobile devices to maximize their lifetime by using powerful capacity-achieving error correcting coding such as turbo codes and low-density parity check (LDPC) codes, adaptive power control, and so on. Recently, dramatic paradigm shift happened due to the development of energy harvesting technologies which enable devices to harness energy from ambient sources such as solar, vibration, themoelectric effects, and so on. Since this might be an ultimate solution of the crucial energy constraint, it has gained much attention from researchers. Especially, radio frequency (RF) energy harvesting does not depend on availability of ambient energy sources where ambient RF radiation is captured by the receiver antennas and converted into a direct current (DC) voltage through appropriate circuits such as rectennas [1, 2]. Therefore, this energy transfer is considered as one of the most attractive candidate technologies to realize self-sustaining networks.

Information and energy transfer has been studied in [3, 4] for the point-to-point single antenna transmission. In [5], RF transfer-based beamforming schemes has been proposed for a three-node multiple-input multiple-output (MIMO) setup and practical solutions for simultaneous energy and information transfer have been investigated. Three-node cooperative network with RF energy harvesting has been analyzed in [6, 7] where the relay exploit the additional energy radiated by the source to charge its own battery, which results in superior outage performance. More recently, RF energy harvesting in large-scale networks has been analyzed in [8, 9]. In [8], opportunistic energy harvesting in cognitive radio network has been studied where the secondary users harness the energy radiated by nearby active primary transmitters. Also, in [9], simultaneous information and energy transfer with feasible strategy has been studied in large-scale networks with/without relaying.

These conventional works presented the great possibility of RF energy harvesting and provide some guidelines and insights to design the systems with harvesting ability. Although these works generally assume well-known path-loss model, when the large-scale network with RF energy harvesting is considered, the summation of energy transferred from the transmitter to harvesters in the network diverges while the radiated energy must be finite [10]. This inconsistency of the path-loss model has been ignored in the literature. Therefore, in this paper, we mathematically clarify the essential impairment of conventional works with well-known path-loss model in terms of energy conservation law and propose new *absorption function* which meets the law and explains the effect of RF energy harvesting. Moreover we derive the upper-bound of harvested energy based on this model and further analyze the stationary Poisson point process (PPP) network with RF energy harvesting ability to demonstrate its effectiveness.

2. SYSTEM MODEL

2.1. Network Model

Throughout this article, we consider a random network in two-dimensional Euclidean space, modeled by a stationary PPP of intensity λ in $\mathbb{R}^2$. Let us select an arbitrary reference point defining the origin of the space, and order the remaining points $i \in \mathbb{N}$ according to their Euclidean distances r_i to such reference. It is well-known [11] that the distances r_i are such that $0 < r_1 < r_2 < \cdots$ almost surely, such that this property is implicitly used henceforth to support the assumption that each node in the network can be unequivocally identified by its distance to the origin (source). In a two-dimensional PPP with intensity λ, the distance between an arbitrary point and its ith neighbor follows a generalized Gamma distribution [12]

$$p(r; i, \lambda) = r^{2i-1}\frac{2(\lambda\pi)^i}{\Gamma(i)}\exp(-\lambda\pi r^2). \tag{1}$$

Assume that every node in the network is an RF-energy harvesting node which absorbs all the RF-energy impinging on its antenna and a single node randomly becomes a transmitter at every time slot.

This work was supported in part by Heiwa Nakajima Foundation and in part by JSPS KAKENHI 24760295.

Under such assumptions, it has been shown in [13] that the amount of power reaching a node at distance r of a source is given by

$$f(r;\alpha) = \frac{\exp(-\alpha r)}{2\pi r}, \qquad (2)$$

where $\alpha > 0$. In conventional context of wireless communications $f(r;\alpha)$ describes a path loss law, and α would be typically referred to as a path loss exponent. In the context of this article, however, the function $f(r;\alpha)$ can be interpreted as a *absorption function*, such that the parameter α can be understood as the *absorption factor*, which will be discussed in detail in the next section.

2.2. Battery and RF Energy Harvesting Model

In our scenario, every node is equipped with an energy battery of size $E_B = \beta P$ with $\beta > 0$. RF transmitter circuit including a power amplifier (PA) is most power consuming at the node and that of receiver circuit is negligible compared with the transmitter. We, hence, assume that each transmission consumes the energy P from the battery but each reception does not. Although, if the remaining battery is less than P but not equal to zero, the node cannot be the transmitter anymore, it still can be the receiver. Once the node completely drains the battery, i.e., $E_B = 0$, it can be neither the transmitter nor receiver but the energy harvester. In the rest of paper, P is assumed to be unity and β is an integer without loss of generality.

Consider that the energy harvester is r apart from the transmitter with the unit transmit power. Then, we define the energy that can be harvested from the received signal to be equal to

$$E_H(r) = \epsilon f(r;\alpha), \qquad (3)$$

where $\epsilon \in (0,1]$ denotes the conversion efficiency[1]. Upon energy harvesting, the absorbed energy will be stored in the battery.

3. ANALYSIS OF ABSORPTION MODEL

Under the interpretation here provided an infinite network should be able to absorb all the energy radiated by a given source. In other words, the following *energy conservation law* must be satisfied

$$\sum_{i=1}^{\infty}\left[\int_0^{\infty} p(r;i,\lambda) f(r;\alpha)\,\mathrm{d}r\right] \le 1, \qquad (4)$$

where the energy radiated by the source was normalized to the unity, without lack of generality, and where equality is to be achieved if and only if no absorption by air molecules or any other physical obstacles rather than the nodes themselves occurs (perfect propagation).

3.1. Path-loss Exponent Model

Before discussing our absorption model given by (2), we investigate the well-known path-loss model and clarify the mathematical impairment of that model when the RF-energy harvesting is considered. The absorption function with path-loss exponent model is simply written by $f_e(r;\alpha) = (1/r)^{\alpha}$. From Campbell's theorem [16], we have

$$\sum_{i=1}^{\infty}\left[\int_0^{\infty} p(r;i,\lambda) f_e(r;\alpha)\,\mathrm{d}r\right] = \left.\frac{2\pi\lambda}{2-\alpha} r^{2-\alpha}\right|_{r=0}^{\infty}. \qquad (5)$$

[1] In [14], practical cellular energy harvesting was investigated of which conversion efficiency ϵ was 0.25. Latest results confirm that this efficiency can be around 0.37 [15].

Practically, the path-loss exponent ranges from 2 to 4 and thus the above equation diverges due to the lower bound, which is a consequence of the path-loss law and the property of the PPP that nodes can be arbitrarily close. This observation clearly declares that the well-known path-loss exponent model is inconsistent with the energy conservation law and the model cannot be used in any scenarios with RF energy harvesting.

3.2. Valid Absorption Model

To show the validity of (2), we analyze the energy conservation law with the model. Substituting (1) and (2) into (4), the energy conservation law yields

$$\sum_{i=1}^{\infty}\left[\int_0^{\infty} \frac{2}{r}\frac{(\lambda\pi r^2)^i}{\Gamma(i)}\exp(-\lambda\pi r^2)\cdot\frac{\exp(-\alpha r)}{2\pi r}\right] \le 1. \qquad (6)$$

After some mathematical manipulations, the above inequality can be rewritten by

$$\lambda\cdot\int_0^{\infty}\exp(-\alpha r)\,\mathrm{d}r \le 1. \qquad (7)$$

Finally, recognizing that $\int_0^{\infty}\exp(-\alpha r)\,\mathrm{d}r = 1/\alpha$ yields the *energy conservation condition*

$$\alpha \ge \lambda \qquad (8)$$

It is interesting to notice that within the derivation of $f(r;\alpha)$ offered in [13], the parameter α was assumed to indicate an estimate of the density of the obstacles in the environment. Indeed, the latter result indicates that in fact $\alpha = \lambda$ implies that the entire RF energy radiated by the source is absorbed by the nodes in the network. An analysis conducted under such conditions will therefore provide bounds on the energy harvesting capability of random networks. It is worth noting that this energy conservation condition is valid even in the three dimensional space.

This absorption function is yet imperfect especially when RF energy harvesting is studied since the function might be greater than one when r is small while no receiver can obtain more power than was transmitted. In this case, an alternative option is the use of minimum possible path-loss degradation which ensures the accuracy of path-loss model for short distances [10]. For example

$$f_m(r;\alpha) = \min\left\{\frac{\exp(-\alpha r)}{2\pi r}, \frac{\exp(-\alpha)}{2\pi}\right\}. \qquad (9)$$

The minimum possible path-loss degradation $\exp(-\alpha)/2\pi$ never exceeds one for $\alpha > 0$ and the energy conservation law is apparently satisfied.

4. UPPER-BOUND ON AVERAGE HARVESTED RF-ENERGY

Based on the functions described above, the average energy that can be collected by a node that is the ith closest from a source radiating a unit energy can be calculated. Specifically,

$$E_m(i;\lambda,\alpha) \triangleq \int_0^{\infty} p(r;i,\lambda) f_m(r;\alpha)\,\mathrm{d}r. \qquad (10)$$

Unfortunately, this equation does not have the closed-form expression and thus we consider the upper-bound of $E_m(i;\lambda,\alpha)$ by stepping back to the function $f(r;\alpha)$ which obviously overrates the RF energy harvesting due to the short distance signals while it makes mathematical analysis tractable. Then, we get

$$\begin{aligned} E_m(i;\lambda,\alpha) &< E(i;\lambda,\alpha), \\ &\triangleq \int_0^\infty p(r;i,\lambda) f(r;\alpha)\, \mathrm{d}r, \\ &= \frac{(\lambda\pi)^i}{\pi\Gamma(i)} \int_0^\infty r^{2i-2} \exp(-\lambda\pi r^2 - \alpha r)\, \mathrm{d}r. \end{aligned} \quad (11)$$

The latter integral admits the closed-form [17, EQ. 3.462.1] so that with $\nu = 2i-1$, $\beta = \lambda\pi$ and $\gamma = \alpha$, we obtain

$$E(i) = 2^{\frac{1}{2}-i} \sqrt{\frac{\lambda}{\pi}} \frac{\Gamma(2i-1)}{\Gamma(i)} \exp\left(\frac{\alpha^2}{8\lambda\pi}\right) D_{1-2i}\left(\frac{\alpha}{\sqrt{2\lambda\pi}}\right), \quad (12)$$

where $D_n(x)$ is the parabolic cylinder function. An expression for $E(i)$ as a function of generalized Hermite polynomials[2] $H_n(x)$ can be obtained as follows. First, using the relation [18, EQ. 22.5.58]

$$D_{1-2i}\left(\frac{\alpha}{\sqrt{2\lambda\pi}}\right) = 2^{i-\frac{1}{2}} \exp\left(\frac{-\alpha^2}{8\lambda\pi}\right) H_{1-2i}\left(\frac{\alpha}{\sqrt{2\lambda\pi}}\right), \quad (13)$$

into equation (12) readily yields

$$E(i) = \sqrt{\frac{\lambda}{\pi}} \frac{\Gamma(2i-1)}{\Gamma(i)} H_{1-2i}\left(\frac{\alpha}{\sqrt{2\lambda\pi}}\right), \quad (14)$$

which can be slightly simplified using the following property of the Gamma function

$$\Gamma(2z) = (2\pi)^{-1/2} 2^{2z-1/2} \Gamma(z)\Gamma(z+1/2), \quad (15)$$

to expand the term $\Gamma(2i-1)$, with $2z = 2i-1$, into

$$\Gamma(2i-1) = (2\pi)^{-1/2} 2^{2i-3/2} \Gamma(i-1/2)\Gamma(i), \quad (16)$$

such that

$$E(i) = \frac{4^i\sqrt{\lambda}}{4\pi} \Gamma(i-1/2)\, H_{1-2i}\left(\frac{\alpha}{\sqrt{2\lambda\pi}}\right). \quad (17)$$

Since calculating the latter expression requires the evaluation of the Hermite polynomials in integral form, it may be convenient also to note the relation [19, EQ. 34]

$$H_n(x) = 2^n U(-n/2, 1/2; x^2), \quad (18)$$

where $U(a,b;x)$ is the Whittaker's confluent hypergeometric function of the second kind, which yields

$$E(i) = \frac{\sqrt{\lambda}}{2\pi} \Gamma(i-1/2)\, U\left(i-\frac{1}{2}, \frac{1}{2}; \frac{\alpha}{\sqrt{2\lambda\pi}}\right). \quad (19)$$

Finally, we also note that using the relationship between the Hermite polynomial and the Kummer confluent hypergeometric function of the first kind [20], the following expression of $E(i)$ can also be obtained

$$\begin{aligned} E(i) = \frac{1}{2\pi\Gamma(i)} \Bigg[&\sqrt{\lambda\pi}\Gamma(i-1/2)\, {}_1F_1\left(i-\frac{1}{2}, \frac{1}{2}; \frac{\alpha^2}{4\pi\lambda}\right) \\ &-\alpha\,\Gamma(i)\, {}_1F_1\left(i, \frac{3}{2}; \frac{\alpha^2}{4\pi\lambda}\right)\Bigg]. \end{aligned} \quad (20)$$

[2]Here $H_n(x)$ are generalized for negative integer n, as allowed by the integral form [17, EQ. 8.951] $H_n(x) = \frac{2^n}{\sqrt{\pi}} \int_{-\infty}^{\infty} (x+it)^n \exp(-t^2)\, \mathrm{d}t$.

5. SUM CAPACITY WITH RF-ENERGY HARVESTING

To demonstrate the effectiveness of RF-energy harvesting, the sum capacity of the network is investigated. We assume that the network only has a single transmitter at each time slot to avoid the interference. This assumption is valid if the network utilizes *ideal* carrier sense multiple access with collision avoidance (CSMA/CA). When the ith neighbor node becomes the receiver and the others become energy harvesters, the average channel capacity at the ith neighbor node is given by

$$C(i) = \int_0^\infty p(r;i,\lambda) \log\left[1 + \frac{Pf(r;\alpha)}{N_0}\right] \mathrm{d}r, \quad (21)$$

where N_0 is one-sided power spectral density of additive white Gaussian noise (AWGN). The closed form expression of (21) cannot be obtained. Thus, using Jensen's inequality, we get

$$\begin{aligned} C(i) &\geq \log\left[1 + \int_0^\infty p(r;i,\lambda) \frac{P\exp(-\alpha r)}{2\pi r N_0}\, \mathrm{d}r\right] \\ &= \log\left(1 + \frac{PE(i)}{N_0}\right). \end{aligned} \quad (22)$$

When we consider the $W \times H$ area, the expected number of nodes in the area is given by $N_T = \lambda W H$. When the ith node is communicating with the source radiating a unit energy, maximum absorbed energy via all the energy harvesters in the network is given by

$$E_{HN}(i) = \epsilon \sum_{\substack{j=1 \\ j\neq i}}^{N_T} E(j). \quad (23)$$

where infinite size of battery is implicitly assumed here since, if the battery size is limited, the harvested energy is partially stored in their batteries when some energy harvesters are fully charged. Although the assumption of infinite battery capacity is practically unfeasible, ignoring this nonlinearity makes the subsequent analysis tractable. Moreover, even with the assumption, the obtain analysis provides a reasonable upper bound of RF-harvested energy as will be discussed later.

If N_T is infinitely large, the average absorbed energy is approximated by

$$E_{HN}(i) \approx \epsilon\left(\frac{\lambda}{\alpha} - E(i)\right). \quad (24)$$

In our model, nodes in the network transmit by turns and thus each transmission only consumes P of transmitter's battery while $PE_{HN}(i)$ out of the transmit energy is collected by RF energy harvesting. Thus, the maximum number of transmissions of the RF energy harvesting network is *roughly* given by

$$\begin{aligned} T_{EH} &< \frac{N_T E_B}{P}\left\{1 + E_{HN}(i) + E_{HN}(i)^2 + \cdots\right\} \\ &= \frac{N_T E_B}{P}\left\{\frac{1}{1 - E_{HN}(i)}\right\}. \end{aligned} \quad (25)$$

The above inequality is loose since the whole energy in the network cannot be efficiently utilized for transmissions. Then, the sum capacity of the network is given by

$$C_{EH}(i) \approx \frac{N_T E_B}{P}\left\{\frac{1}{1 - E_{HN}(i)}\right\} \log\left(1 + \frac{PE(i)}{N_0}\right). \quad (26)$$

Although the inequality sign of the above equation cannot be determined, the approximations of T_{EH} and $E(i)$ are even looser than

Jensen's inequality. Thus, (26) eventually becomes an upper-bound on $C_{EH}(i)$.

Also, the sum capacity without energy harvesting is analyzed. When the ith neighbor node is chosen as the designated receiver, at least i nodes should remain as candidate receiver. Hence, the maximum number of transmissions is written by $T_M < \frac{(N_T - i)E_B}{P}$. Hence, the sum capacity is given by

$$C_M(i) \approx T_M \log\left(1 + \frac{PE(i)}{N_0}\right). \tag{27}$$

Again, this equation eventually provides an upper-bound on $C_M(i)$ due to T_M and $E(i)$.

Compared with the capacity without energy harvesting, the resulting capacity with energy harvesting increases in proportion to $\frac{N_T}{(1-E_{HN}(i))(N_T-i)}$, which is referred to as *energy harvesting gain* G_{EH} in the rest of the paper.

6. NUMERICAL RESULTS

In this section, some numerical examples are shown to confirm our analysis. In the following, PPP with $\lambda = 0.1$ in 16×16 area is assumed and $\epsilon = 0.25$ is used as the feasible value of conversion efficiency. Also, α is assumed to be $2\lambda = 0.2$ to meet the energy conservation law. Moreover, the signal-to-noise ratio (SNR) at the unit distance from the transmitter is assumed 15 dB. In computer simulations, 100 time slots for one given network are observed and 1.0×10^5 different PPP realizations are averaged to obtain the results. Note that, for simulations, (9) is used as path-loss function and the effect of limited size of rechargeable battery is considered where $\beta = 1.0$.

Figure 1 shows sum capacity of *energy harvesting network* (EHN) and *conventional non-energy-harvesting network* (CN). In order to evaluate the approximations in theoretical analyses, simulations with (2) and *infinitely rechargeable battery* (IRB) are also shown in the figure. From the figure, all the results calculated by the derived equations show good agreement with those obtained by Monte-Carlo simulations. Moreover, the gap between EHN and CN becomes larger as the receiver number increases although the resulting sum capacity monotonically decreases owing to decrease of the capacity of each transmission. When the closest neighbor is chosen as the receiver, the energy harvesting gain becomes about 1.12 and the use of energy harvesting increases the sum capacity by 12%.

Interestingly, the results with (9) is almost same as those with IRB and (2). This observation concludes that introduction of IRB and (2) can be consider as a reasonable assumption and does not affect the performance so much while it makes the analytic calculation even more tractable. However, when there are multiple transmitters in the network, the assumption of (2) would seriously affect the results since the distribution of received signal energy plays important role rather than average [9].

The effect of different λ/α with different conversion efficiency 0.25 and 0.37 is shown in Fig. 2 where the other parameters are same as Fig. 1 and the closest node is chosen as the receiver. From the figure, when λ/α approaches one, energy harvesting gain is maximized. As described earlier, $\lambda/\alpha = 1.0$ means that no absorption by air molecules or any other physical obstacles rather than the nodes themselves occurs. Hence, in order to efficiently obtain the energy harvesting gain, the energy harvesters are densely placed compared with the given path-loss exponent α which is defined by the observation of real environments. Also, the higher energy harvesting gain

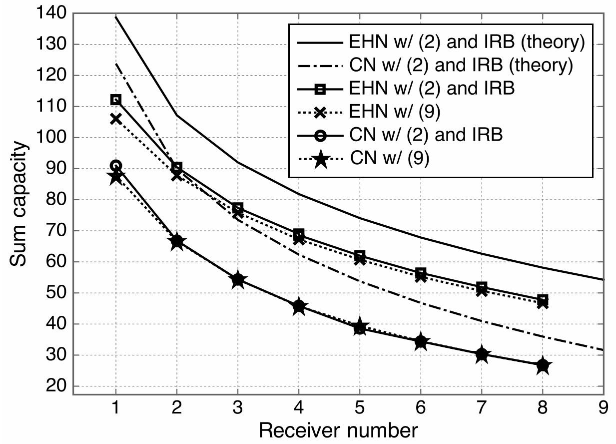

Fig. 1. Sum capacity of energy harvesting network (EHN) and conventional non-energy harvesting network (CN) where $\eta = 0.25$, $\lambda = 0.1$, and $\alpha = 0.2$.

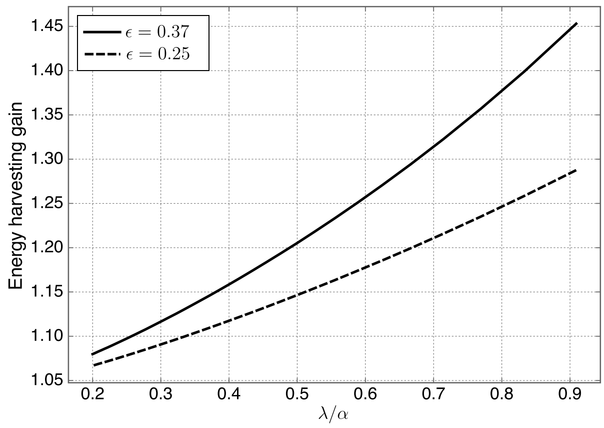

Fig. 2. Energy harvesting gain with different λ/α where $\epsilon = 0.25$ and 0.37.

can be obtained as the conversion efficiency ϵ increases. Obviously the improvement of the conversion efficiency is remarkable in RF energy harvesting.

7. CONCLUSIONS

This paper pointed out the essential impairment of conventional works with well-known path-loss model in terms of energy conservation law and proposed the absorption function meeting the energy conservation law. Moreover, several expressions of average harvested energy were derived and the effectiveness of RF energy harvesting was discussed. Computer simulations confirmed our theoretical analyses. In this paper, we only studied the case with single transmitter. From practical point of view, the regular networks are composed of multiple transmitters and thus there is interference in the network even with CSMA/CA [10]. In order to analyze the system with the interference effect, its distribution has to be derived. However, this derivation is quite difficult and is still an open issue.

8. REFERENCES

[1] X.-X. Yang, C. Jiang, A. Elsherbeni, F. Yang, and Y. Q. Wang, "A novel compact printed rectenna for data communication systems," *IEEE Trans. Antennas Propag.*, vol. 61, pp. 2532–2539, May 2013.

[2] U. Olgun, C.-C. Chen, and J. Volakis, "Investigation of rectenna array congurations for enhanced RF power harvesting," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 262–265, 2011.

[3] L. Varshney, "Transporting information and energy simultaneously," in *Proc. Int. Symposium on Inform. Theory (ISIT)*, (Toronto, ON), pp. 1612–1616, July.

[4] P. Grover and A. Sahai, "Shannon meets tesla: Wireless information and power transfer," in *Proc. Int. Symposium on Inform. Theory (ISIT)*, (Austin, TX), pp. 2363–2367, June.

[5] R. Zhang and C. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," in *Proc. 2011 IEEE Global Commun. Conf.*, (Houston, TX), pp. 1–5, Dec.

[6] I. Krikidis, S. Timotheou, and S. Sasaki, "RF energy transfer for cooperative networks: Data relaying or energy harvesting?," *IEEE Commun. Lett.*, vol. 16, pp. 1772–1775, Nov. 2012.

[7] K. Ishibashi, H. Ochiai, and V. Tarokh, "Energy harvesting cooperative communications," in *Proc. of IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, (Sydney, NSW), pp. 1819–1823, Sept. 2012.

[8] S. Lee, R. Zhang, and K. Huang, "Opportunistic wireless energy harvesting in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, pp. 4788–4799, Sept. 2013.

[9] I. Krikidis, "Simultaneous information and energy transfer in large-scale networks with/without relaying." ArXiv[Online]. Available: http://http://arxiv.org/abs/1310.6511, Oct. 2013.

[10] M. Haenggi, *Stochastic Geometry For Wireless Networks*. Cambridge University Press, 1 ed., 2013.

[11] M. Haenggi, "A geometric interpretation of fading in wireless networks: Theory and applications," *IEEE Trans. Inf. Theory*, vol. 54, pp. 5500–5510, Dec. 2008.

[12] M. Haenggi, "On distances in uniformly random networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3584–3586, Oct. 2005.

[13] M. Franceschetti, J. Bruck, and L. Schulman, "A random walk model of wave propagation," *IEEE Trans. Antennas Propag.*, vol. 52, pp. 1304–1317, May 2004.

[14] C. Mikeka and H. Arai, "Design of a cellular energy-harvesting radio," in *2009 European Wireless Tech. Conf.*, (Rome, Italy), pp. 73–75, 2009.

[15] A. M. Hawkes, A. R. Katko, and S. A. Cummer, "Applied physics letters," *A microwave metamaterial with integrated power harvesting functionality*, vol. 103, pp. 163901 – 163901–3, Oct. 2013.

[16] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127–248, 2009.

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. San Diego: Academic Press, sixth ed., 2000.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Dover Publications, 10 ed., 1965.

[19] `mathworld.wolfram.com/HermitePolynomial.html`.

[20] `functions.wolfram.com/HypergeometricFunctions/Hypergeometric1F1/03/01/02/0013/`.