A STATISTICAL APPROACH TO INTERFERENCE REDUCTION IN DISTRIBUTED LARGE-SCALE ANTENNA SYSTEMS

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ABSTRACT

This paper considers the problem of interference control in networks where base stations signals are coherently combined (aka network MIMO). Building on an analogy with so-called *massive MIMO*, we show how second-order statistical properties of channels can be exploited when the massive MIMO array corresponds in fact to many antennas randomly spread over a two-dimensional network. Based on the classical one-ring model, we characterize the low-rankness of channel covariance matrices and show the rank is related to the scattering radius. The application of the low-rankness property to channel estimation's denoising and low complexity interference filtering is highlighted.

***Index Terms*—massive MIMO, distributed antennas, channel estimation, interference mitigation, covariance matrix.**

I. INTRODUCTION

Massive vs. Network MIMO. Interference control lies at the core of modern physical-layer design in wireless networks. The aggressive reuse of spectral resources is made necessary by growing capacity demands. However this gives rise to severe signal-to-interference ratio (SIR) limitations especially at cell-edge. Interference impacts channel estimation due to pilots' lack of orthogonality [1], [2], as well as data reception. Recently, approaches relying on the use of additional spatial degrees of freedom (DoF) were proposed to handle interference. Enhanced spatial DoFs are essentially linked to the use of many coherently combined antennas at the network side. The way such antennas are placed in the network determines the nature of the algorithm and gain. In a first approach, referred to as *network-MIMO*, the many antenna elements are those deployed across many (possibly single-antenna) base stations. In this case, some inter-BS cooperation is enabled in the form of coherent combining, allowing all BS antenna signals to be jointly decoded/precoded at one virtual BS node [3], [4]. In the second approach, commonly referred to as *massive* MIMO, by contrast, many antenna elements are deployed at each of (possibly fewer) BS [5], [6]. In this case, the law of large numbers allows for orthogonality build-up between channels linked to different users/cells, as the number of antennas M grows to infinity. This property can in turn be exploited for simple (matched- or MMSE-) filtering out of the interference without much need for inter-BS cooperation [5]–[8].

Second-order statistics of massive arrays. Note that in the case the cluster of jointly processed BSs spans the entire service area, a network-MIMO system can be viewed as a particular massive MIMO setup with antenna elements scattered randomly across the network. Hence an interesting question is whether some of the features of massive linear arrays extend to non-collocated antenna topologies. Notably, a low rank property for channel covariance matrices in uniform linear arrays (ULA) was recently unveiled. In [9], [10] the finite covariance rank is predicted from the angular spread for arriving paths at massive ULAs. This property may lead to powerful yet simple methods for pilot decontamination [9] and multi-user interference filtering [10].

Contributions. In this paper we investigate the behavior of channel covariance in network-MIMO based distributed arrays. We study the regime of a large number of base stations in a fixed network area (so-called *dense* network), under the classical one-ring channel model. Our analysis reveals a low-dimensional signal subspace behavior for distributed arrays in the large number of base station antenna regime, *even discounting path loss effects*. We show the richness of the covariance's signal subspace is primarily governed by the scattering radius around the user terminal. We provide a closed form expression for an upper-bound of the covariance rank and show by simulation how this bound closely matches reality. Finally a simple MRC beamformer is proposed, which exploits low-rank properties of channel covariance and shows significant performance gain.

II. SIGNAL AND CHANNEL MODELS

We consider a large-scale antenna regime, often referred to in the literature as distributed antenna systems. In such a setting, a single virtual base station is deployed, having its M antennas scattered throughout the network. We study the uplink in which joint combining across all BS antennas is assumed possible. Applications scenarios for this case

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include the ideal Cloud Radio Access Network (C-RAN) with remote radio heads, or network MIMO/CoMP systems with a large cluster of cells. The M base station antennas are assumed uniformly and randomly located in a disk-shaped network of fixed radius L, serving single-antenna users.

II-A. Distributed Array Channel Models

In order to facilitate the analysis, we adopt the classical one-ring model [11] where users are surrounded by a ring of $P \gg 1$ local scatterers (see Fig. 1) located r meters away from the user. In the one-ring model, propagation from user to base is assumed to follow P paths (hereafter referred to as scattering paths), where each path p bounces once on the p-th scatterer before reaching destinations. Hence, the path length from user k to the m-th antenna via the p-th scatterer is $r + d_{kpm}$, where d_{kpm} is the distance between the p-th scatterer of the k-th user and the m-th BS antenna. The path loss of the p-th scattering path is modeled by:

$$\beta_{kpm} = \frac{\alpha}{(d_{kpm} + r)^{\gamma}}, \tag{1}$$

where α is a constant that can be computed based on desired cell-edge SNR, and γ is the path loss exponent. We scale the amplitude of each path by $\sqrt{P}$. The channel between user k and all BS antennas is given by:

$$\mathbf{h}_k \triangleq \frac{1}{\sqrt{P}} \sum_{p=1}^{P} \mathbf{h}_{kp}, \tag{2}$$

where $\mathbf{h}_{kp}$ is the p-th scattering path vector channel between user k and all base stations:

$$\mathbf{h}_{kp} \triangleq \begin{bmatrix} \sqrt{\beta_{kp1}} e^{-j2\pi \frac{d_{kp1}+r}{\lambda}} \\ \vdots \\ \sqrt{\beta_{kpM}} e^{-j2\pi \frac{d_{kpM}+r}{\lambda}} \end{bmatrix} e^{j\varphi_{kp}}, \tag{3}$$

where λ is the wavelength and $e^{j\varphi_{kp}}$ denotes the random common phase of that scattering path vector due to possible random perturbations of the user location around the ring center. φ_{kp} is assumed i.i.d. and uniformly distributed between 0 to 2π.

II-B. A Low Rank Model for Distributed Arrays

We are now interested in characterizing the rank of channel covariance for distributed antenna systems. In attacking this problem it is important to distinguish the rank reduction effect due to path loss from the intrinsic finite rank behavior of the large antenna channel covariance in an equal path loss regime. In fact, in an extended network (i.e. where some base station antennas can be arbitrarily far from some users), any given user will be received over only a limited number of antennas in its vicinity, thereby effectively limiting the channel rank to the size of this neighborhood. To circumvent this problem, we consider below a (dense) network where the path loss terms are set artificially to be all equal (to one) and study finite rankness under such conditions.

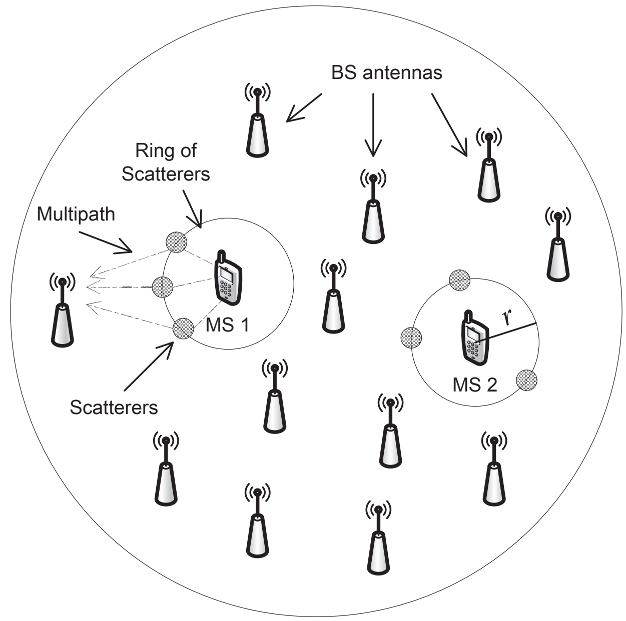

Fig. 1. The distributed large-scale antenna setting.

Theorem 1. *The rank of the channel covariance matrix for a distributed antenna system satisfies:*

$$\textit{rank}(\mathbf{R}) \leq \frac{4\pi r}{\lambda} + o(r). \tag{4}$$

Proof: Can be found in [12]. □

In reality we show below that the right hand side of (4) is a very close approximation of the actual rank. Theorem 1 shows a linear dependency of the rank on the size of the scattering ring. When r increases, the richer scattering environment expands the dimension of signal space. Fig. 2 shows the behavior of the covariance rank with respect to the scattering radius r. We can see the rank scales linearly with the slope $4\pi/\lambda$. However because of the finite number

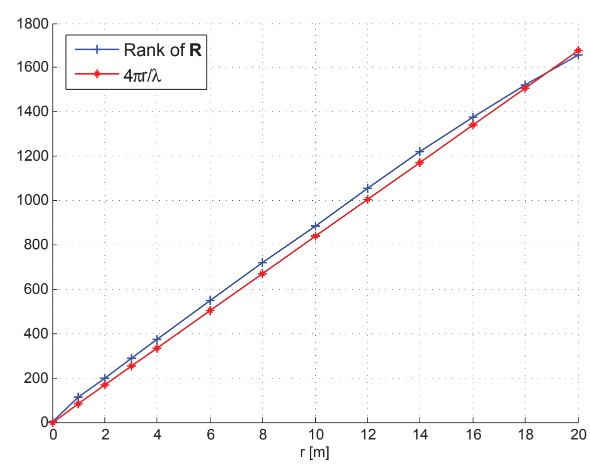

Fig. 2. Rank vs. r, $M = 2000$, λ = 0.15m, $L = 500$m.

of antennas the rank will finally saturate towards M when r keeps increasing.

III. CHANNEL ESTIMATION AND BEAMFORMING

In this section we will exploit the low-dimension property for pilot contamination reduction and interference rejection.

III-A. Channel Estimation

It is well known that channel covariance information can be exploited to improve channel estimation performance [13], [14]. In the presence of pilot contamination, e.g., there are K users sharing the same pilot sequence, a Bayesian (or equivalently MMSE) estimator of the target channel vector $\mathbf{h}_1$ is given by [7], [9]:

$$\widehat{\mathbf{h}}_1 = \mathbf{R}_1 \left(\sigma_n^2 \mathbf{I}_M + \tau \sum_{k=1}^{K} \mathbf{R}_k \right)^{-1} \bar{\mathbf{S}}^H \mathbf{y}, \tag{5}$$

where $\mathbf{R}_k, k = 1, \cdots, K$, is the channel covariance matrix of user k. $\bar{\mathbf{S}}$, $\mathbf{y}$, τ, and σ_n^2 are the training matrix, received signal vector, pilot length, and noise power respectively [9]. The channel estimation performance of (5) is related to how much structural information we can exploit from the covariance matrices [9]. To this end, the low-rankness is beneficial for channel estimation. Imagine an extreme case when the covariances are close to an identity matrix, i.e., they have full rank, there is little information we can obtain from the covariances. Under such condition the performance of (5) is close to an Least Squares (LS) estimator. However if the covariances have low rank as predicted in Theorem 1, the Bayesian estimator is less affected by pilot contamination, as will be shown in section IV.

III-B. Subspace-based Beamforming

In this section, we propose a simple beamforming strategy building on the low dimensionality of the signal subspace, which requires no accurate channel estimation. We consider a K-user network with the first user being a target user and all other users being interference users. Denote the sum of interference covariances as $\mathbf{R}_I = \mathbf{R}_2 + \mathbf{R}_3 + \cdots + \mathbf{R}_K$. Consider the eigenvalue decomposition (EVD) of $\mathbf{R}_I$:

$$\mathbf{R}_I = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H, \tag{6}$$

where $\mathbf{\Sigma}$ is a $M \times M$ diagonal matrix with the eigenvalues of $\mathbf{R}_I$ on its main diagonal. Suppose the eigenvalues are in descending order and the first m eigenvalues are non-negligible while the others can be neglected. We construct the spatial filter at the BS side for user 1 as:

$$\mathbf{W}_1 = \left[\mathbf{u}_{m+1} | \mathbf{u}_{m+2} | \dots | \mathbf{u}_M \right]^H,$$

where $\mathbf{u}_m$ is the m-th column of $\mathbf{U}$. We can assume approximately that:

$$\mathbf{W}_1 \mathbf{h}_k \approx \mathbf{0}, \forall k \neq 1,$$

$$\mathbf{W}_1 \mathbf{Y} \approx \mathbf{W}_1 \mathbf{h}_1 \mathbf{s}^T + \mathbf{W}_1 \mathbf{N}, \tag{7}$$

where $\mathbf{N} \in \mathbb{C}^{M \times \tau}$ is the spatially and temporally white additive Gaussian noise, $\mathbf{Y} \in \mathbb{C}^{M \times \tau}$ is the received training signal at the base station, and $\mathbf{s} \in \mathbb{C}^{\tau \times 1}$ is the shared pilot sequence by the K users.

Define the effective channel $\underline{\mathbf{h}}_1 \triangleq \mathbf{W}_1 \mathbf{h}_1$. Note that $\underline{\mathbf{h}}_1$ has a reduced size, which is $(M - m) \times 1$. An LS estimate of $\underline{\mathbf{h}}_1$ is:

$$\widehat{\underline{\mathbf{h}}}_1 = \mathbf{W}_1 \mathbf{Y} \mathbf{s}^* \left(\mathbf{s}^T \mathbf{s}^* \right)^{-1}, \tag{8}$$

The key idea is that channel estimate $\widehat{\underline{\mathbf{h}}}_1$ is coarse, yet can be used as a modified MRC beamformer as it lies in a subspace orthogonal to the interference and is also aligned with the signal subspace of $\mathbf{h}_1$. During uplink data transmission phase:

$$\mathbf{y} = \mathbf{h}_1 \mathbf{s}_1^T + \sum_{k=2}^{K} \mathbf{h}_k \mathbf{s}_k^T + \mathbf{n}, \tag{9}$$

where the lengths of transmitted signal sequence $\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K$ are τ_u. $\mathbf{y}, \mathbf{n} \in \mathbb{C}^{M \times \tau_u}$ are the received signal and noise respectively.

The subspace-based MRC beamformer is $\widehat{\underline{\mathbf{h}}}_1^H \mathbf{W}_1$:

$$\widehat{\underline{\mathbf{h}}}_1^H \mathbf{W}_1 \mathbf{y} = \widehat{\underline{\mathbf{h}}}_1^H \underline{\mathbf{h}}_1 \mathbf{s}_1^T + \underbrace{\widehat{\underline{\mathbf{h}}}_1^H \mathbf{W}_1 \sum_{k=2}^{K} \mathbf{h}_k \mathbf{s}_k^T}_{\approx \mathbf{0}} + \widehat{\underline{\mathbf{h}}}_1^H \mathbf{W}_1 \mathbf{n}. \tag{10}$$

In case there is no null space for $\mathbf{R}_I$, e.g., the number of users is large or the interference users have rich scattering environments, the subspace-based method can still avoid the strong eigen modes of interference and therefore reject a good amount of interference.

IV. NUMERICAL RESULTS

We examine both the channel estimation quality and the sum-rate performance under the subspace-based modified matched filter, based on covariance information. As one performance metric, the Mean Squared Error (MSE) of channel estimation for the k-th user is defined as:

$$\text{MSE}_k \triangleq 10\log_{10} \left(\frac{\left\| \widehat{\mathbf{h}}_k - \mathbf{h}_k \right\|^2}{\left\| \mathbf{h}_k \right\|^2} \right). \tag{11}$$

In the simulation we average the channel estimation MSE over different users and different channel realizations in order to obtain the MSE curve. Another metric is the sum-rate defined as follows:

$$\text{sum-rate} \triangleq \sum_{k=1}^{K} \log_2 (1 + \text{SINR}_k), \tag{12}$$

where SINR_k is the uplink signal-to-noise-plus-interference ratio (SINR) of the k-th user.

We consider a random network with radius $L = 500$m, with cell-edge signal-to-noise ratio (SNR) equal to 3dB. The path loss exponent $\gamma = 2.5$. The number of scatterers is 50. The wavelength is $\lambda = 0.15$m (the carrier frequency is 2GHz [15]). We obtain channel covariance matrix by averaging $\mathbf{hh}^H$ over the random locations of scatterers. We first show the pilot decontamination reduction effect of the MMSE estimator (5). Assume user 1 is located at the origin while an interfering user is moving over the horizontal axis at increasing distances from user 1. As we can observe in Fig. 3, when the two users have identical pilot sequence, an LS estimator is unable to separate the desired channel and interference channel. By contrast, the channel estimation error of MMSE estimator decreases almost linearly with user spacing. One may also notice the constant performance gap between LS and MMSE estimator in interference-free scenario, which indicates that covariance information is still helpful even in a highly distributed antenna system.

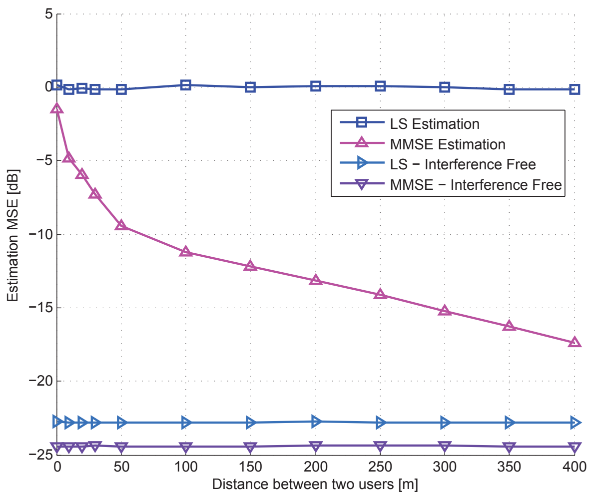

Fig. 3. Estimation MSE vs. user spacing, $M = 2000$, $r = 15$m.

In the following we will show in Fig. 4 and Fig. 5 the performance of subspace-based MRC beamforming in a network where two users share the same pilot. We compare the subspace-based MRC beamformer and two traditional MRC methods. In the figures "LS + MRC" denotes the sum-rate performance of MRC beamforming using the LS channel estimate; while "MMSE + MRC" is the performance of MRC beamforming using the MMSE estimate (5). The total number of distributed antennas is 500. The simulation in Fig. 4 indicates the simple subspace-based method has a very good performance. Due to pilot contamination, the MRC beamformer using MMSE channel estimate is not as good as subspace-based method. The reason is that $\mathbf{R}_1$ and $\mathbf{R}_2$ generally have overlapping signal subspaces in a distributed antenna system.

In Fig. 5, we show the uplink sum-rate performance

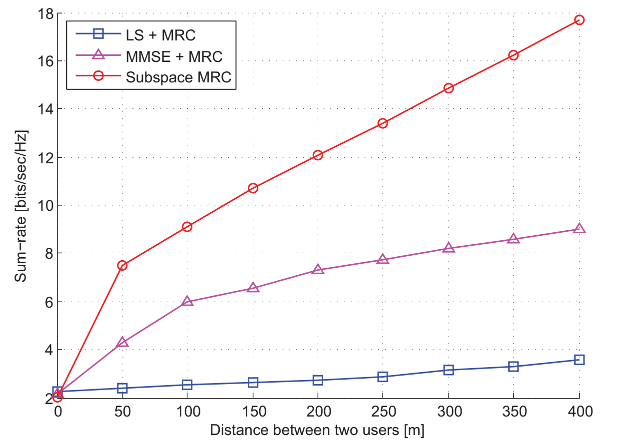

Fig. 4. Uplink sum-rate vs. user spacing, $M = 500$, $r = 15$m.

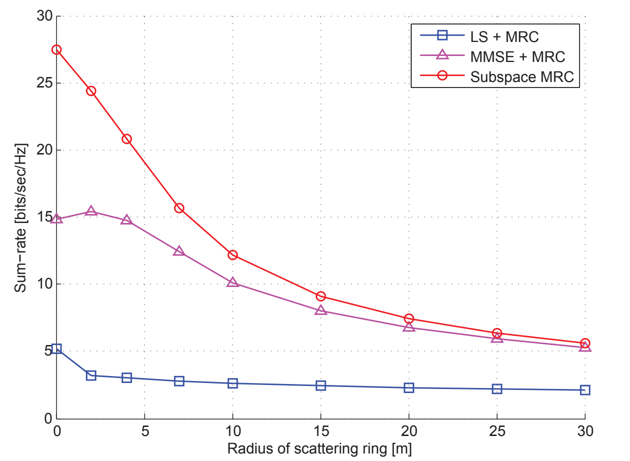

Fig. 5. Uplink sum-rate vs. r, $M = 500$.

of subspace-based MRC beamforming as a function of scattering radius r. The inter-user distance is 100m. The subspace-based beamforming shows performance gains over traditional methods especially when the radius of scattering ring is smaller.

V. CONCLUSIONS

We investigate the low dimensional properties of covariance signal spaces and extend previous results known in the uniform linear array case to the case of arrays with randomly scattered antennas over a 2D dense network. A correlation model is derived which is exploited to gain insight on the interference rejection capability of low complexity matched filter-based receivers in distributed antenna settings.

ACKNOWLEDGMENT

This work was supported by the Seventh Framework Programme for Research of the European Commission under grant number HARP-318489.

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