PERFORMANCE OF PARTIAL ZERO-FORCING BEAMFORMING IN LARGE RANDOM SPECTRUM SHARING NETWORKS

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ABSTRACT

Mutual interference is the main bottleneck on the throughput of large random spectrum sharing networks. This work examines the extent to which the performance of such networks can be improved by employing multiple transmitting antennas, without degrading the average performance of individual users. By extending partial zero-forcing beamforming to spectrum sharing networks, the aggregate interference towards primary receivers is reduced, and the desired signals at both primary and secondary receivers are boosted. Considering randomly distributed users and spatially independent Rayleigh fading channels, this work provides upper and lower bounds on the maximum permissible density of secondary transmitters with respect to the numbers of primary and secondary transmitting antennas. The simulation results show that substantial increase in the density of secondary transmitters can be obtained while meeting the outage requirements of the spectrum sharing users.

Index Terms— Spectrum sharing, multiple transmitting antennas, interference nulling, stochastic geometry.

1. INTRODUCTION

Spectrum sharing improves the spectrum usage efficiency and increases the spatial throughput by allowing different networks to access the same spectrum. In an underlay spectrum sharing scenario, the access of licensed primary users (PUs) to the spectrum is to be guaranteed, and unlicensed secondary users (SUs) can access the band so long as the interference imposed on the PUs and SUs lies below a tolerable threshold [1,2]. Previous studies have explored underlay spectrum sharing for small deterministic networks with given channel state information and user location information; e.g., [3–5]. However, to satisfy increasing demands, spectrum sharing networks must grow in size. In large networks, acquiring substantial channel state information and location information used in the previous approaches would consume an unreasonably large fraction of the resources provided by the channel. Therefore, the development of techniques to manage mutual interference in large random networks has become essential [6].

In this work, we study spectrum sharing between two networks where the transmitters are randomly located on a two-dimensional plane according to homogeneous Poisson point processes (HPPPs), and each transmitter communicates with its corresponding receiver located at a fixed distance. We study the spectrum sharing networks based on a spatial throughput framework. In this framework, each transmitter attempts communication at a fixed rate, and the performance of spectrum sharing is measured through network densities and network-wide successful transmission probabilities that are computed over random transmitter locations and random channels. A similar stochastic geometry model [7,8] has been adopted recently in some analyses of underlay spectrum sharing networks with single-antenna nodes; e.g., [9–16].

To explore spectrum sharing opportunities in dense networks, exploiting spatial dynamics with the aid of multiple transmitting antennas is a promising approach. Multi-antenna communication is a key enabling technique in LTE and WiMax standards [17]. The benefit of partial zero-forcing beamforming (ZFBF) has been analyzed for a single network in [18–20].

This work extends partial ZFBF to large random spectrum sharing networks. In particular, each primary transmitter (PT) and secondary transmitter (ST) nulls its interference towards adjacent primary receivers (PRs) and boosts the signal to its target receiver. The analytical difficulty arises from two facts: (1) due to the priority difference, SUs null their interference towards PUs, but PUs do not null their interference to SUs; (2) a transmitter nulling its interference towards its n^{th} closest receivers does not imply that the interference from the n^{th} closest transmitters to a receiver is nulled. By applying stochastic geometry, this work quantifies the maximum density of STs that preserves specified successful transmission probabilities for nodes in each network, as a function of the number of antennas at the primary and secondary transmitters.

This study is related to prior works [21, 22] that have addressed underlay spectrum sharing problems of randomly distributed nodes with multiple antennas. In [21], multiple antennas were employed for maximum ratio transmission but not for interference reduction. In [22], multiple antennas were used only in SUs to mitigate inter-interference to PUs and to boost secondary signals. However, the effects of intra-interference reduction and signal boosting within PUs have not been considered in the existing works despite their importance in a dense spectrum sharing network. To address this gap, the present study quantifies the impacts of the numbers of both the primary and the secondary transmitting antennas on spectrum sharing.

2. SYSTEM MODEL

We consider an underlay spectrum sharing scenario with two types of users: PUs and SUs. These users coexist in the same region and share the same spectrum. The spectrum sharing network comprises transmitter-receiver pairs. Each transmitter communicates to its corresponding receiver in a point-to-point manner, and each receiver treats all other transmissions as interference.

The PTs are modeled as being distributed according to a two-dimensional HPPP. Let $\Phi_p = \{T_{pi}\}$ denote the set of coordinates of PTs, where $T_{pi} \in \mathbb{R}^2$ is the coordinate of the i^{th} PT. Let λ_p denote the spatial density of PTs (i.e., the expected number of PTs in a

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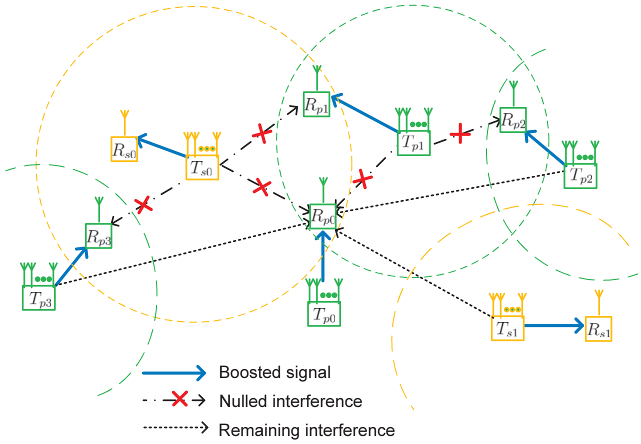

Fig. 1: An illustration of spectrum sharing between PUs (in green) and SUs (in yellow) with multiple transmitting antennas. Partial ZFBF is used at each transmitter to boost signals and avoid causing interference to adjacent PRs. In this example, PT T_{p1} and ST T_{s0} both avoid transmitting in the direction of the primary reference receiver R_{p0}, but PTs T_{p2} and T_{p3} and ST T_{s1} do not try to avoid transmitting in the direction of R_{p0}. In addition, that receiver's desired transmitter, T_{p0}, boosts the received signal with constrained spatial matched filtering. Note that the cardinalities and the locations of the transmitters that avoid causing interference to R_{p0} and those of the residual interferers are unknown at receiver R_{p0}.

unit area). Due to the stationarity of HPPP[1], the performance of the PUs can be evaluated through a reference transmitter-receiver pair. For simplicity, the reference PR R_{p0} is assumed to be located at the origin, which is a distance d_p away from its associated PT located at T_{p0}. Each PT has N_p antennas, and each PR has a single antenna (i.e., multiple-input single-output).

The locations of the STs are also described by an independent HPPP but of density λ_s and are denoted by $\Phi_s = \{T_{sj}\}$. The reference transmitter-receiver location pair for SUs is denoted by (T_{s0}, R_{s0}), and the associated reference transmission distance is denoted by d_s. Each ST has N_s antennas, and each secondary receiver (SR) has a single antenna.

We consider interference-limited spectrum sharing in the sense that the impact of the thermal noise is negligible compared with that of the interference[2]. The received signal at the reference PR R_{p0} in the presence of intra (network) interference from the other PTs and inter (network) interference from the STs is

$$y_p = \underbrace{d_p^{-\alpha/2}\mathbf{h}_{00}^*\mathbf{u}_0 x_{p0}}_{\text{desired primary signal}} + \underbrace{\sum_{i:T_{pi}\in\Phi_p\setminus\{T_{p0}\}} \left|T_{pi}\right|^{-\alpha/2}\mathbf{h}_{0i}^*\mathbf{u}_i x_{pi}}_{\text{intra-interference from PTs}} + \underbrace{\sum_{j:T_{sj}\in\Phi_s} \left|T_{sj}\right|^{-\alpha/2}\boldsymbol{\ell}_{0j}^*\mathbf{v}_j x_{sj}}_{\text{inter-interference from STs}}, \tag{1}$$

where $\mathbf{h}_{00} \in \mathbb{C}^{N_p\times 1}$ is the channel between the reference PT T_{p0} and its associated PR R_{p0}, $\mathbf{h}_{0i} \in \mathbb{C}^{N_p\times 1}$ is the interfering channel between the i^{th} PT and R_{p0}, and $\boldsymbol{\ell}_{0j} \in \mathbb{C}^{N_s\times 1}$ is the interfering channel between the j^{th} ST and R_{p0}. The channel is modeled as being a rich scattering environment, and hence the elements of the channel vectors are modeled as being i.i.d. and distributed as $\mathcal{CN}(0,1)$. The superscript $*$ denotes the conjugate transpose. The vector $\mathbf{u}_i \in \mathbb{C}^{N_p\times 1}$ is the normalized beamforming vector used by the i^{th} PT, $\mathbf{v}_j \in \mathbb{C}^{N_s\times 1}$ is the normalized beamforming vector used by the j^{th} ST, α is the path loss exponent, $|\cdot|$ denotes the distance from the origin; x_{pi} and x_{sj} are data signals transmitted from the i^{th} PT and the j^{th} ST, respectively, with $x_{pi}, x_{sj} \sim \mathcal{CN}(0,1)$. The received signal y_s at the reference SR R_{s0} is

$$y_s = \underbrace{d_s^{-\alpha/2}\mathbf{g}_{00}^*\mathbf{v}_0 x_{s0}}_{\text{desired secondary signal}} + \underbrace{\sum_{j:T_{sj}\in\Phi_s\setminus\{T_{s0}\}} \left|T_{sj}\right|^{-\alpha/2}\mathbf{g}_{0j}^*\mathbf{v}_j x_{sj}}_{\text{intra-interference from STs}} + \underbrace{\sum_{i:T_{pi}\in\Phi_p} \left|T_{pi}\right|^{-\alpha/2}\mathbf{f}_{0i}^*\mathbf{u}_i x_{pi}}_{\text{inter-interference from PTs}}, \tag{2}$$

where $\mathbf{g}_{00} \in \mathbb{C}^{N_s\times 1}$ is the channel between the reference ST T_{s0} and its associated SR R_{s0}, $\mathbf{g}_{0j} \in \mathbb{C}^{N_s\times 1}$ is the interfering channel between the j^{th} STs and R_{s0}, and $\mathbf{f}_{0i} \in \mathbb{C}^{N_p\times 1}$ is the interfering channel between the i^{th} PTs and R_{s0}. In the next section, we analyze the performance of spectrum sharing based on this system model.

3. MULTI-ANTENNA SPECTRUM SHARING

In this section, we analyze the performance of partial ZFBF in large random spectrum sharing networks. As illustrated in Fig. 1, each transmitter avoids causing interference to adjacent PRs and boosts the signal towards the target receiver. The spectrum sharing performance is evaluated by the maximum permissible density of STs.

3.1. Partial Zero Forcing Beamforming

Each ST uses $n_s \in \{0, 1, 2, \ldots, N_s - 1\}$ degrees of freedom to null the interference to its n_s closest PRs in terms of distance and uses the remaining $N_s - n_s$ degrees of freedom to boost the desired signal to its associated SR. This transmission strategy is called partial ZFBF [19], and can be viewed as constrained transmit spatial matched filtering. As we can see below, when $n_s = N_s - 1$ partial ZFBF is equivalent to full ZFBF, and when $n_s = 0$ it corresponds to transmit beamforming. To illustrate partial ZFBF scheme, let

$$\mathbf{L}_j = [\widetilde{\boldsymbol{\ell}}_{1j} \ \ldots \ \widetilde{\boldsymbol{\ell}}_{nj} \ \ldots \ \widetilde{\boldsymbol{\ell}}_{n_s j}],\ 0 \le n_s \le N_s - 1, \tag{3}$$

be an $N_s \times n_s$ matrix of channels from T_{sj} to its n_s closest PRs, where vectors $\widetilde{\boldsymbol{\ell}}_{1j} \ \ldots \ \widetilde{\boldsymbol{\ell}}_{nj} \ \ldots \ \widetilde{\boldsymbol{\ell}}_{n_s j}$ are in the increasing order of distance between T_{sj} and its n^{th} closest PR. First, the partial ZFBF vector $\mathbf{v}_j$ is chosen such that it is in the null space of $\mathbf{L}_j$, $Null(\mathbf{L}_j)$. Then, to maximize the power of the desired signal, $\mathbf{v}_j$ is the projection of the desired channel $\mathbf{g}_{jj}$ onto $Null(\mathbf{L}_j)$. More precisely, by defining the superscript † as the left pseudo-inverse and $||\cdot||$ as Euclidean norm, normalized beamformer $\mathbf{v}_j$ is computed as [20, 22]

$$\mathbf{v}_j = \frac{(\mathbf{I} - \mathbf{L}_j\mathbf{L}_j^\dagger)\mathbf{g}_{jj}}{||(\mathbf{I} - \mathbf{L}_j\mathbf{L}_j^\dagger)\mathbf{g}_{jj}||}. \tag{4}$$

Furthermore, in this study we also consider that each PT uses $n_p \in \{0, 1, 2, \ldots, N_p - 1\}$ degrees of freedom to null the interference to its closest PRs in terms of distance and uses the remaining

[1]All the receivers in an HPPP network have the same statistics for signal reception [6].

[2]The effect of noise can be addressed by including an extra term in a straightforward manner [14].

$N_p - n_p$ degrees of freedom to boost the desired signal to its associated PR. Let

$$\mathbf{H}_i = [\widetilde{\mathbf{h}}_{1i} \ \ldots \ \widetilde{\mathbf{h}}_{ni} \ \ldots \ \widetilde{\mathbf{h}}_{n_p i}], \ 0 \le n_p \le N_p - 1, \tag{5}$$

be an $N_p \times n_p$ matrix of channels from T_{pi} to its n_p closest PRs (except R_{pi}), the beamformer of T_{pi} is the normalized projection of $\mathbf{h}_{ii}$ onto $Null(\mathbf{H}_i)$.

$$\mathbf{u}_i = \frac{(\mathbf{I} - \mathbf{H}_i\mathbf{H}_i^\dagger)\mathbf{h}_{ii}}{||(\mathbf{I} - \mathbf{H}_i\mathbf{H}_i^\dagger)\mathbf{h}_{ii}||} \tag{6}$$

Note that the interferer locations and the vector channels are random, but we assume that each transmitter has knowledge of the channel to its intended receiver and the channels to the adjacent receivers that the transmitter is interfering with. The same assumption has been considered in [18, 19, 22]. Given that, the interference nulling at the transmitter side is perfect[3].

3.2. Successful Transmission Probabilities

Let Ψ_p and Ψ_s be the sets of the PTs and STs, respectively, that null their interference towards R_{p0}. From (1) and (2), the signal-to-interference ratio (SIR) at the reference PR and that at the reference SR can be written as

$$\begin{aligned}\text{SIR}_p =& d_p^{-\alpha}|\mathbf{h}_{00}^*\mathbf{u}_0|^2\Big(\sum_{i:T_{pi}\in\Phi_p\setminus\Psi_p}\big|T_{pi}\big|^{-\alpha}|\mathbf{h}_{0i}^*\mathbf{u}_i|^2 + \\ &\sum_{j:T_{sj}\in\Phi_s\setminus\Psi_s}\big|T_{sj}\big|^{-\alpha}\big|\boldsymbol{\ell}_{0j}^*\mathbf{v}_j\big|^2\Big)^{-1},\end{aligned} \tag{7}$$

$$\begin{aligned}\text{SIR}_s =& d_s^{-\alpha}|\mathbf{g}_{00}^*\mathbf{v}_0|^2\Big(\sum_{j:T_{sj}\in\Phi_s\setminus\{T_{s0}\}}\big|T_{sj}\big|^{-\alpha}|\mathbf{g}_{0j}^*\mathbf{v}_j|^2 + \\ &\sum_{i:T_{pi}\in\Phi_p}\big|T_{pi}\big|^{-\alpha}|\mathbf{f}_{0i}^*\mathbf{u}_i|^2\Big)^{-1}.\end{aligned} \tag{8}$$

A performance metric of interest in this study is the successful transmission probability of PUs p_p with respect to a predefined SIR threshold β_p. This performance metric is given by

$$p_p = \mathbb{P}\big[\text{SIR}_p > \beta_p\big]. \tag{9}$$

Typically, the target SIR β_p is selected so that if this SIR is achieved, then the chosen coding scheme for PUs can communicate successfully with high probability. For that reason, we refer to p_p as the successful transmission probability and to $1 - p_p$ as the outage probability of PUs. Similarly, the successful transmission probability of SUs p_s with respect to a predefined SIR threshold β_s is

$$p_s = \mathbb{P}\big[\text{SIR}_s > \beta_s\big]. \tag{10}$$

The received SIR depends on the design of the beamforming vectors and on the random interferer locations. In the next section, we study the maximum permissible density of STs based on the successful transmission probabilities.

[3]In practice, channel state information could be obtained through the transmission of pilot symbols and feedback in frequency division duplexing (FDD) systems or by exploiting reciprocity in time division duplexing (TDD) systems. For the imperfect cases, the result in this work provides an analytical upper bound.

3.3. Maximum Density

Network throughput is defined as the product of the sum rate per unit area and the successful transmission probability that this target rate is achieved [23]. Based on this definition, spectrum sharing throughput is determined by the density of STs λ_s [10, 22]. Therefore, we are interested in exploring the maximum permissible density of STs $\lambda_s^\star$ subject to outage probabilities ϵ_p of PUs and ϵ_s of SUs; i.e.,

$$\lambda_s^\star = \sup\{\lambda_s : p_p \ge 1 - \epsilon_p, p_s \ge 1 - \epsilon_s\}. \tag{11}$$

To establish the relation of the density $\lambda_s^\star$ to the numbers of transmitting antennas N_p and N_s when partial ZFBF is adopted, we choose the numbers of nulled interferers $n_p = \theta_p N_p$ for a constant $0 < \theta_p < 1$ and $n_s = \theta_s N_s$ for a constant $0 < \theta_s < 1$. In the following theorem, $\lceil\cdot\rceil$ denotes the ceiling function, $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function, and $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Beta function.

Theorem 1. *The maximum permissible density of STs, $\lambda_s^\star$, is bounded by N_p and N_s in the following way,*

$$\min\{\lambda_a, \lambda_b\} \le \lambda_s^\star \le \min\{\lambda_c, \lambda_d\}, \tag{12}$$

where

$$\begin{aligned}\lambda_a =& \Big[\Big(\frac{\epsilon_p(\alpha/2-1)[(1-\theta_p)N_p - 1](\theta_p N_p + \theta_s N_s - \lceil\alpha/2\rceil)^{\alpha/2-1}}{\pi^{\alpha/2}\beta_p d_p^\alpha}\Big)^{\frac{2}{\alpha}} \\ &- \lambda_p\Big],\end{aligned} \tag{13}$$

$$\lambda_b = \Big[\frac{\epsilon_s\alpha[(1-\theta_s)N_s]^{2/\alpha}}{2\pi B(2/\alpha, 1-2/\alpha)\beta_s^{2/\alpha}d_s^2} - \lambda_p\Big], \tag{14}$$

$$\lambda_c = \Big[\frac{\theta_p N_p + \theta_s N_s + \alpha/4 + 1.5}{(1-\epsilon_p)\pi^{\alpha/2}\beta_p d_p^\alpha[(1-\theta_p)N_p - 1]^{2/\alpha}} - \lambda_p\Big], \tag{15}$$

$$\lambda_d = \Big[\frac{\epsilon_s\alpha\Gamma(1-2/\alpha)[(1-\theta_s)N_s]^{2/\alpha}}{2\pi B(2/\alpha, 1-2/\alpha)\beta_s^{2/\alpha}d_s^2} - \lambda_p\Big]. \tag{16}$$

Proof: See the Appendix. ■

This analytical result quantifies the increase in the density of secondary nodes that can be obtained by exploiting partial ZFBF. In particular, for channels with a path loss exponent $\alpha > 2$ and spectrum sharing networks with sufficiently large number of transmitting antennas $N_p = N_s = N$, the maximum permissible density of STs grows as fast as $\min\{N^{1-2/\alpha}, N^{2/\alpha}\}$.

4. SIMULATION RESULTS AND DISCUSSION

In this section, we present Monte Carlo simulations to evaluate the performance of spectrum sharing using partial ZFBF. The procedure for simulating a large random spectrum sharing network follows that in [24, 25]. The simulated PUs and SUs lie in a two-dimensional disk and the number of transmitter-receiver pairs in each network is Poisson distributed. The mean number of PTs is 200. Then, the area of the overlaid region is adjusted according to the PT density (i.e., the radius of the disk is $\sqrt{200/\lambda_p/\pi}$). The reference PR is located at the center of the disk. The following parameters are used: the density of PTs is $\lambda_p = 0.05\text{m}^{-2}$, path-loss exponent is $\alpha = 4$, reference transmission distances are $d_p = 3$ m, $d_s = 1$ m, target SIR thresholds are $\beta_p = \beta_s = 1$, outage probabilities are bounded by $\epsilon_p \le 0.1, \epsilon_s \le 0.15$, and interference nulling ratios are $\theta_p = \theta_s = 0.5$.

Fig. 2 shows that both the analytic bounds and the numerical results for the maximum density $\lambda_s^\star$ are increasing functions of the

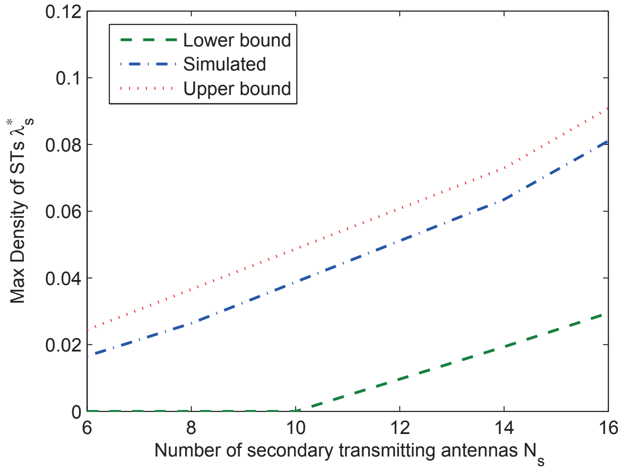

Fig. 2: The maximum permissible density of STs $\lambda_s^\star$ versus the number of secondary transmitting antennas N_s ($N_p = 6$). The simulated density is obtained by Monte Carlo simulations. The analytical bounds on the density are obtained in Theorem 1.

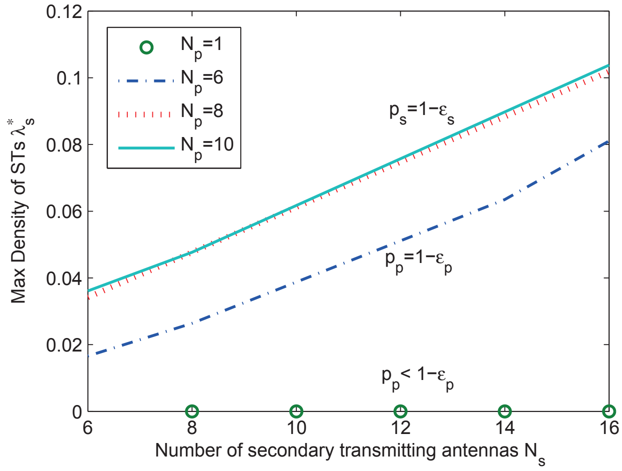

Fig. 3: The maximum permissible density of STs $\lambda_s^\star$ versus the number of secondary transmitting antennas N_s with different numbers of primary transmitting antennas N_p. The simulated density is obtained by Monte Carlo simulations.

number of secondary transmitting antennas N_s. This is because the additional interference generated by increasing $\lambda_s^\star$ is complemented by the increasing N_s through interference nulling and signal boosting.

Fig. 3 shows how the maximum density $\lambda_s^\star$ of STs grows with the number of secondary transmitting antennas N_s and that of primary transmitting antennas N_p. Based on the definition in (11), the maximum density of STs is achieved when one of the outage constraints is active (i.e., $p_p = 1 - \epsilon_p$ or $p_s = 1 - \epsilon_s$). When $N_p = 1$ (i.e., single antenna), the intra-interference of PUs is so large that $p_p < 1 - \epsilon_p$ even in the absence of SUs; hence, no spectrum sharing opportunity is provided at all. When N_p grows ($N_p = 6, 8, 10$), intra-interference from PUs is nulled and the primary signal is boosted, thus allowing concurrent transmission from SUs. When $N_p = 6$, PUs are sensitive to inter-interference from SUs, and thus primary outage constraint is active. Now that the increasing inter-interference from increasing $\lambda_s^\star$ is offset by improved interference nulling from increasing N_s, the primary outage constraint is satisfied. Consequently, $\lambda_s^\star$ grows with N_s. When $N_p = 8$ or $N_p = 10$, the primary signal is largely boosted, and intra-interference from PUs is largely nulled, and thus PUs are not as sensitive to inter-interference from SUs. Then, the secondary outage constraint is active rather than the primary outage constraint. Now that the increasing intra-interference from increasing $\lambda_s^\star$ is balanced by largely boosted the secondary signal from increasing N_s, the secondary outage constraint is satisfied. Consequently, $\lambda_s^\star$ grows with N_s. Note that if the major bottleneck of spectrum sharing is the secondary outage constraint, increasing N_p is not very useful since PUs do not reduce interference to SUs. Therefore, the increase of N_p from 8 to 10 does not provide a significant increase in spectrum sharing opportunities for additional SUs.

The simulation results suggest that employing multiple primary transmitting antennas is essential to support successful transmission in dense PUs and to possibly permit spectrum sharing. Nevertheless, excessive use of primary transmitting antennas may not offer substantial additional spectrum sharing opportunities.

5. CONCLUSION

In this study, we have examined the performance gains that can be obtained by extending partial ZFBF to large random spectrum sharing networks. These performance gains are due to the reduction in aggregate interference towards the PRs, and the boosting of both the primary and secondary signals. We have quantified how the maximum density of STs grows with the numbers of both primary and secondary transmitting antennas under outage constraints. Our results indicated that spectrum sharing opportunities can be enhanced in large random networks by exploiting multiple primary and secondary transmitting antennas.

6. APPENDIX

The bounds on p_p can be derived by using the properties of stochastic geometry and the features of partial ZFBF. Given that the signal coefficient $|\mathbf{h}_{00}^*\mathbf{u}_0|^2$ follows Chi-squared distribution $\chi^2_{2(N_p-n_p)}$, the interference terms $|\mathbf{h}_{0i}^*\mathbf{u}_i|^2$ and $|\boldsymbol{\ell}_{0j}^*\mathbf{v}_j|^2$ follow independent χ^2_2 [19,22], and the ordered squared-distances follow a one-dimensional HPPP [6,26], we have

$$p_p \geq 1 - \pi^{\alpha/2}\beta_p d_p^\alpha(\alpha/2-1)^{-1}(\lambda_p+\lambda_s)^{\alpha/2}\times (N_p-n_p-1)^{-1}(n_p+n_s-\lceil\alpha/2\rceil)^{1-\alpha/2}, \tag{17}$$

$$p_p \leq \pi^{-\alpha/2}\beta_p^{-1}d_p^{-\alpha}(\lambda_p+\lambda_s)^{-\alpha/2}\times (N_p-n_p-1)^{-1}(n_p+n_s+\alpha/4+1.5)^{\alpha/2}, \tag{18}$$

From [27], we have the bounds on p_s as follows

$$p_s \geq 1 - 2\pi\alpha^{-1}B\Big(2/\alpha, 1-2/\alpha\Big)\beta_s^{2/\alpha}d_s^2(N_s-n_s)^{-2/\alpha}\times (\lambda_p+\lambda_s), \tag{19}$$

$$p_s \leq 1 - 2\pi\alpha^{-1}B\Big(2/\alpha, 1-2/\alpha\Big)/\Gamma(1-2/\alpha)\beta_s^{2/\alpha}d_s^2\times (N_s-n_s)^{-2/\alpha}(\lambda_p+\lambda_s), \tag{20}$$

The density bounds arise from the definition in (11). Details are omitted due to space limitations.

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