DESIGN OF A ROBUST OPEN SPHERICAL MICROPHONE ARRAY

Gilles Chardon, Wolfgang Kreuzer*†

Acoustics Research Institute
Austrian Academy of Sciences
Wohllebengasse 12–14, 1040 Vienna, Austria

Markus Noisternig‡

UMR STMS IRCAM-CNRS-UPMC
Acoustic and Cognitive Spaces Research Group
1, place Igor Stravinsky, 75004 Paris, France

ABSTRACT

This paper presents a method for designing a robust open spherical microphone array that overcomes the typical problems of open sphere geometries at frequencies related to the zeros of the spherical Bessel functions. The proposed array structure uses only a few additional sampling points inside the spherical volume whose optimal positions are determined by the eigenmodes of the sphere for a given wave number interval. This novel approach minimizes the interpolation error inside the sphere. With illustrate this approach with the design of a 10-th order array using 130 microphones and discuss the simulation results with regard to commonly used error measures (white noise gain, condition number, and interpolation error), and show that the proposed array design compares favorably to previously suggested array designs.

Index Terms— Robust spherical microphone array, spherical harmonics, optimization, sound field interpolation.

1. INTRODUCTION

In 3-D audio applications the auditory scene is fully described by the sound field $p(x, \omega)$, where $x \in \mathbb{R}^3$ is the position vector and ω denotes the angular frequency of the sound wave. A number of formats are available to represent the 3-D sound field (e.g., generalized Fourier series, single layer potential, Herglotz wave function etc.) and different microphone array geometries have been proposed to measure it. In this paper we will focus on spherical microphone arrays (SMA), which allow for expanding the sound field into series of spherical harmonics (see e.g. [1, 2, 3]).

It is known from earlier studies on (open) SMA [4, 5] that numerical instabilities appear at wave numbers which are related to the roots of the spherical Bessel functions. Meyer and Elko [6] proposed to overcome this problem by placing the microphones on a rigid (i.e. a sound hard) sphere. Since this approach is not very well suited for arrays with large radii several authors have proposed alternative array geometries. One way to overcome the numerical ill-conditioning of an open array configuration is to use an array consisting of two concentric spheres with different radii (cf. [7, 8, 9]). However, such arrays require two times the number of microphones of single-sphere arrays. To reduce the number of microphones non-spherical array geometries have been proposed, such as the spherical shell array [10], hybrid array geometries [11], the double-sided cone array [12], and the spindle torus array [13].

In this paper, we present a SMA geometry that uses only a few additional microphones inside the sphere and which provides very small interpolation errors for acoustic fields inside the sphere. We show a method to determine the optimal number and the positions of the additional microphones using the eigenmodes for a sphere with Dirichlet boundary conditions in a given wave number interval.

2. THEORETICAL BACKGROUND

At a given wave number k, the sound field at position (r, θ, ϕ) within a source free sphere can be described as (cf. [3])

$$p(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} p_{nm} j_n(kr) Y_{nm}(\phi, \theta), \qquad (1)$$

where j_n denotes the spherical Bessel functions, k is the wave number, and Y_{nm} are the spherical harmonics. The expansion coefficients p_{nm} can be estimated using the orthogonality of the spherical harmonics

$$p_{nm} = \frac{1}{j_n(kr)} \int_{\mathbb{S}^2} Y_{nm}^*(\theta, \phi) p(r, \theta, \phi, k) dS(\theta, \phi). \qquad (2)$$

The integral is typically evaluated on a finite set of quadrature points on the sphere $\mathbb{S}^2$. The numerical calculation of p_{nm} fails for the eigenfrequencies of the ball with Dirichlet boundary conditions, i.e. for $j_n(kr) = 0$.

An alternative method estimates the expansion coefficients by a least-squares approach. While for a 'purely' spherical array the condition number becomes infinite near

*This work was supported in part by the Austrian Science Fund (FWF) START- project FLAME (Frames and Linear Operators for Acoustical Modeling and Parameter Estimation; Y 551-N13),
†the "Research in Paris 2012" program of the City of Paris,
‡and the French ANR CONTINT project Sample Orchestrator 2 (SOR2).

the eigenfrequencies, this method allows for the use of more general array geometries that can be used to stabilize the estimation at all frequencies.

Rafaely [10] proposes a spherical-shell array where some microphones are moved from the boundary of the sphere to the inside of the sphere. The positions of these microphones are determined by a constraint nonlinear optimization procedure minimizing the maximum of the condition number of the $(N+1)^2 \times L$ matrix $\mathbf{B}(k)$ containing the products of the spherical Bessel functions and the spherical harmonics

$$[\mathbf{B}(k)]_{n(n+1)+m+1,i} = 4\pi \mathrm{i}^n j_n(kr_i) Y_{nm}(\theta_i, \phi_i), \quad (3)$$

where N is the order of the microphone and L is the number of different microphone positions (r_i, θ_i, ϕ_i). However, it is not straightforward to determine the number of inside-the-sphere microphones for a given order that ensures stability and convergence of the optimization routine and results in a robust microphone array.

In [14], Chardon et al. theoretically analyzed the sampling of acoustic fields based on least-squares estimation in disks and balls. They show that ensuring stability with the minimal number of measurements requires to place most of the microphones on the sphere and only a few of them inside the sphere. The number of microphones for a stable estimation of N spherical harmonic coefficients is proportional to (at least) $N^{3/2}$ for a uniform distribution of samples in the sphere, whereas it is proportional to N/α for a distribution with α samples on the sphere.

These results clearly show the importance of samples inside the sphere (which also has been recognized in practice), but they do neither indicate the number nor the locations of these inside-the-sphere measurement points. In the following sections we develop an algorithm to determine these points and compare the simulation results with previously suggested array geometries.

3. ARRAY DESIGN

As has been pointed out in the previous section, spherical arrays become unstable at the eigenfrequencies of the ball and can be stabilized by using few microphones inside the sphere. We propose here a method to determine the set of sampling points that stabilizes the estimation of the acoustical field for a given frequency range.

The proposed sampling scheme is composed of two sets of sampling points. The first set of microphones is placed on the sphere in order to estimate the spherical harmonic coefficients of the field when possible. For these points we choose a grid that minimizes the interpolation norm, as has been proposed by Sloan and Womersley [15, 16]. Note that any other set of points may be used, as long as it provides stable interpolation on the sphere. The second set of points is used to stabilize the estimation at the eigenfrequencies of the ball with Dirichlet boundary conditions. Some non-zero fields exist at these frequencies, which are zero on the sphere. It thus is impossible to estimate the spherical harmonic coefficients on the sphere and interpolate the field. Interior points are used for a stable estimation of these coefficients.

Let us first assume that the eigenfrequencies are simple. We now estimate the coefficients for the eigenmodes p_n at a finite number of frequencies ω_n, which requires at least one sample point inside the spherical domain. If carefully chosen, a single point can be used for estimating the coefficients for multiple modes at different frequencies. The eigenmodes p_n are measured at points with maximum amplitude and we thus have to solve the following optimization problem to determine the optimal microphone position:

$$x = \underset{x \in B}{\operatorname{argmax}} \min_n |p_n(x)| \quad (4)$$

where B is the domain inside the array.

In the case of the sphere, the eigenspaces E_n are degenerated. We therefore need m measurements, where m is the maximum multiplicity of the eigenspaces in the frequency band of interest. These points are obtained iteratively by solving the optimization problems

$$x_i = \underset{x \in B}{\operatorname{argmax}} \min_n \max_{\substack{p \in E_n \\ \|p\|=1 \\ (p(x_j)=0)_{0<j<i}}} |p(x)|. \quad (5)$$

These non-convex problems are simply solved by computing the value of the criterion on a sampling grid of the ball.

Secondly, in the case of the sphere, the number of microphones needed is at most equivalent to the product $k_M R$, where k_M is the upper bound of the considered frequency band. This is small compared to the necessary number of microphones on the sphere, which is proportional to $(k_M R)^2$. As a result, the number of microphones needed for the proposed array geometry is, for high frequencies, almost half the number than the number of sensors needed for e.g. a double sphere structure. It is also lower than the number of microphones for Rafaely's shell array structure, i.e. the sum of the multiplicities of the eigenfrequencies in the considered band. By using Weyl's law [17], this quantity can be shown to be proportional to k_M^3.

4. RESULTS AND DISCUSSION

The proposed array, as well as other arrays obtained by existing methods, are compared in terms of interpolation error for a plane wave, condition number of the least-squares approximation, and the white noise gain. The white noise gain is a commonly applied measure for the robustness of a microphone array or beamformer against sensor noise under the assumption that the noise is spatially uncorrelated (cf. [18]). It is defined using the signal-to-noise ratios of the input signal and the array output signal. With the assumptions given in [10] the white noise gain calculates to

$$wg(k) = \frac{1}{||\mathbf{B}(k)^{\dagger}\mathbf{Y}||^2}, \quad (6)$$

where $\mathbf{B}(k)^{\dagger}$ is the pseudoinverse of the matrix $\mathbf{B}(k)$ defined in Eq. (3) and the vector $\mathbf{Y} = (Y_{00}(\theta_0, \phi_0), \cdots, Y_{NN}(\theta_0, \phi_0))^T$ contains the spherical harmonics evaluated at the direction of the incoming plane wave.

In the following the error measures are estimated for a spherical array of order 10 in a range of $k \in [2, 9]$. Without loss of generality we assume the radius of the sphere to be $r = 1$. The sample points on the sphere are distributed using the sampling grid suggested by Sloan and Womersley [15, 16], which minimizes the interpolation error. The number of sample points inside the sphere is 9, the maximal multiplicity of the eigenmodes of the sphere in the chosen frequency range. To allow for a fair comparison of the different arrays, random points are added to the arrays with no internal points (i.e. the single sphere, double sphere, and spherical shell arrays). Therefore, all arrays have the same number of sampling points.

In summary, we are comparing the following array geometries and, if not explicitly stated otherwise, use the Sloan-Womersley grids for the angular distribution of sampling points:

Simple sphere: Open sphere array with 121 regularly distributed plus 9 randomly distributed sensors on the sphere.

Double sphere: Double sphere array with 130 sensors distributed over two open spheres. The sphere radii are chosen according to [7]: $r_1 = 18/(18+\pi)$ and $r_2 = 1$. The angular distribution of sample points uses a regular grid of order 7 plus 1 additional sensor at a random position at each sphere.

Proposed array: Open sphere array with 121 regularly distributed sensors on the sphere and 9 additional sensors, which are positioned inside the sphere using the algorithm proposed Eq. (5); typical computation time: 10 s.

Shell (Rafaely): Spherical shell array with 130 sensors using the same angular distribution as for the simple sphere configuration and random radii between $r_1 = 18/(18+\pi)$ and $r_2 = 1$ according to [10].

Sphere + random: Open sphere array with 121 sensors on the sphere and 9 additional sensors that are randomly distributed inside the sphere.

Sphere + interior (Rafaely): Open sphere array with 121 sensors on the sphere plus 9 additional sensors (i.e. a grid of order 2) at radii obtained from the nonlinear optimization method in [10]. We used Octave's sequential quadratic programming (SQP) solver [19] with default parameters and initial radii that were randomly distributed in the interval $[0, 1]$. For an open sphere, the prominent singularities of the condition number of matrix $\mathbf{B}$ in Eq. (3) are in the wave number range of $k \in [5, 9]$ (cf. Fig. 2). Therefore, the additional 9 sampling points inside the sphere were placed with respect to the minimum of the condition number inside this wave number interval and were restricted to be in the interval $[0, 1]$; typical computation time: 3 min.

4.1. Interpolation error

The interpolation error for the reconstruction of a plane wave for different array types is depicted in Fig. 1. The error is averaged on 50 realizations of noise (white Gaussian noise, SNR = 40 dB) and plane wave directions.

We first compare the proposed robust array to conventional array geometries (see Fig. 1, left subplot), such as the simple sphere and the double sphere. For the proposed array two operation modes are clearly visible: for approximately $k < 6$ the interpolation error is dominated by the noise level, while for $k > 6$ the approximation error for a plane wave and spherical harmonics decomposition with order $N = 10$ dominates. It can be seen that the proposed array is stable for all frequencies within the chosen frequency band, whereas the simple sphere becomes unstable at the eigenfrequencies of the ball. The double sphere (with an internal radius as suggested in [7]) is stable at all frequencies; however, only an order $N = 7$ can be used for the decomposition and thus the interpolation error is higher for $k > 4$.

Next, we compare the performance of different array geometries with interior points using the optimization method proposed by Rafaely in [10] (see Fig. 1, right subplot). Here again the proposed array provides the lowest interpolation error. For the shell array the angular sampling grid is randomly distributed on different radii bounded by inner and outer spheres (see [10] for more details). The mediocre performance of the shell array (even in comparison to the simple sphere) can be explained as follows. The convex hull of the points of the shell array is smaller than that of the spherical array and the interpolation is only relevant in this convex hull; the larger resonance peaks are due to the splitting of the eigenvalues of the sphere. Rafaely's spherical array with 9 additional sampling points in the ball at optimized radial distances clearly reduces the instabilities of some, but not of all eigenvalues. It only performs slightly better than an array with randomly chosen interior points. This is very likely linked to the non-convex shape of the function to be optimized (i.e. the maximal condition number on a given interval of frequencies), which is hard to minimize (unknown gradient and 9 variables).

4.2. Condition number and white noise gain

As already mentioned, the condition number of the matrix $\mathbf{B}$ in Eq. (3), is used as a measure for the robustness in [10]. In Fig. 2, the peaks of the condition number for the open

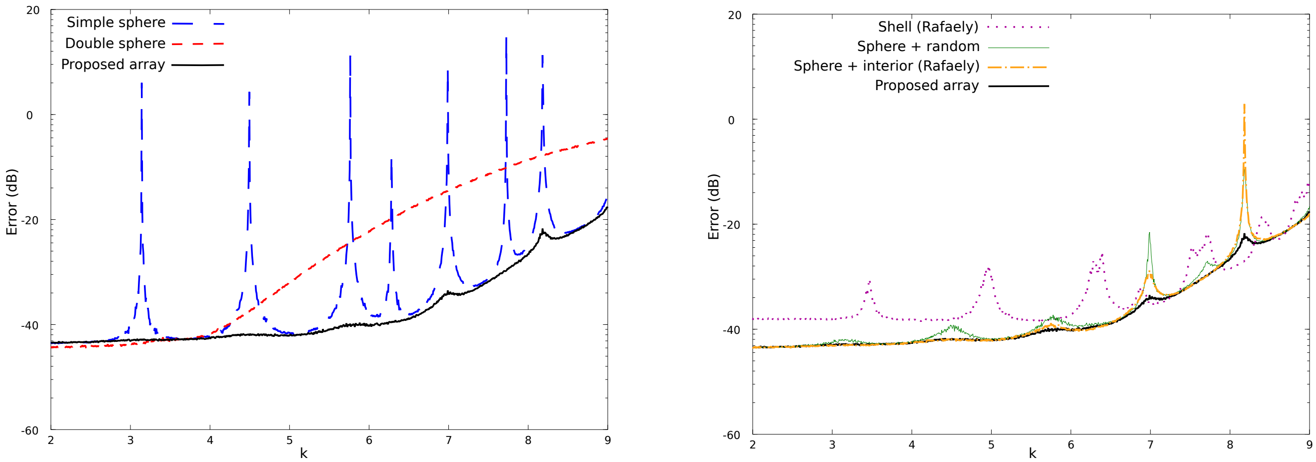

Fig. 1. Interpolation error for a plane wave inside the spherical arrays, for 50 trials at each wave number $k \in [2, 9]$. All the arrays have 130 microphones; the SNR = 40 dB.

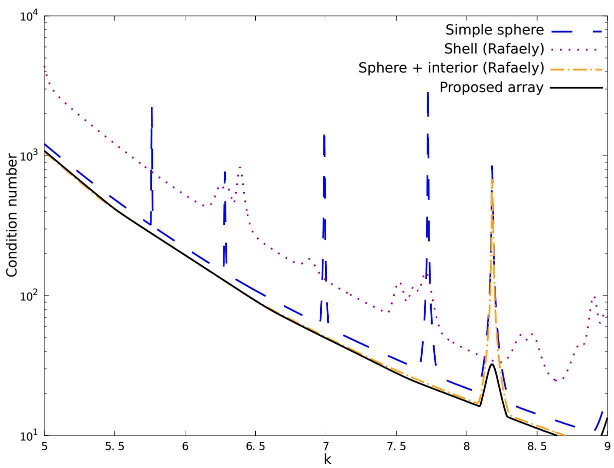

Fig. 2. Condition number of the least-square estimation for 4 different arrays with 130 microphones.

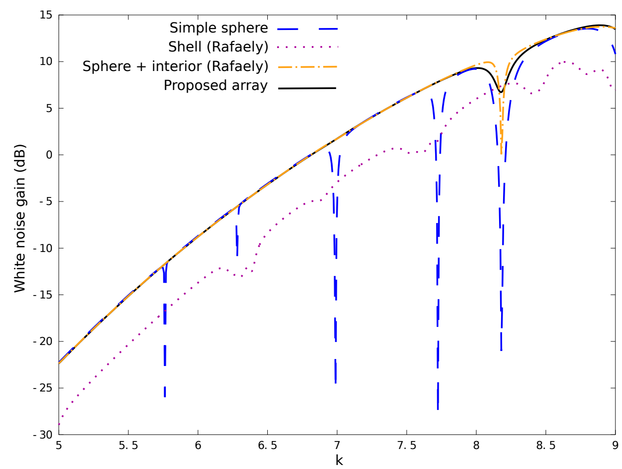

Fig. 3. White noise gain for 4 different arrays with 130 microphones.

sphere at the zeros of the spherical Bessel function can be clearly observed for wave numbers $k \in [5, 9]$. Most of the peaks can be suppressed by adding additional microphones inside the sphere, however, for the peak around $k = 8.18$ our method yields the biggest suppression compared to all other approaches. Because the microphones are inside the ball and not on the sphere, the condition number for the shell array is larger over the whole wave number interval.

The white noise gain depicted in Fig. 3 shows a similar behavior as the condition number. The effect of the roots of the spherical Bessel function again can be clearly seen for the simple open sphere and the improvement by the additional microphones inside the sphere is similar to the improvement in the condition number. Again, the worse performance of the regular shell array is observed.

5. CONCLUSION

In this work we have presented a simple method for generating robust spherical microphone arrays by adding additional sampling points inside the sphere. Our approach is motivated by the results from [14]. The additional microphones are positioned using the eigenmodes for a sphere with Dirichlet boundary conditions in a given wave number interval. Compared with the approach by [10] this approach has the advantages that the number of additional microphones is known once the wavenumber range is set, the computation time is of the order of 10 second on a standard computer, and a better stability is achieved. In our numerical experiments it was shown that this novel approach results in a robust microphone array that, compared with other approaches in literature, has a smaller interpolation error for acoustic fields in the sphere.

6. REFERENCES

[1] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics (Part 2)*, McGraw-Hill Publishing, 1953.

[2] G. Weinreich and E. B. Arnold, "Method for measuring acoustic radiation fields," *The Journal of the Acoustical Society of America*, vol. 68, no. 2, pp. 404–411, 1980.

[3] E. G. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography*, Academic Press, 1999.

[4] T. D. Abhayapala and D. B. Ward, "Theory and design of high order sound field microphones using spherical microphone array," in *Acoustics, Speech and Signal Processing (ICASSP), 2002 IEEE International Conference on*, 2002, pp. 1949–1952.

[5] B. N. Gover, J. G. Ryan, and M. R. Stinson, "Measurements of directional properties of reverberant sound fields in rooms using a spherical microphone array," *The Journal of the Acoustical Society of America*, vol. 116, no. 4, pp. 2138–2148, 2004.

[6] J. Meyer and G. W. Elko, "A highly scalable spherical microphone array based on an orthonormal decomposition of the soundfield," in *Acoustics, Speech and Signal Processing (ICASSP), 2002 IEEE International Conference on*, 2002, pp. 1781–1784.

[7] I. Balmages and B. Rafaely, "Open-Sphere Designs for Spherical Microphone Arrays," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 15, no. 2, pp. 727–732, 2007.

[8] A. Parthy, C. T. Jin, and A. van Schaik, "Acoustic holography with a concentric rigid and open spherical microphone array," in *Acoustics, Speech and Signal Processing (ICASSP), 2009 IEEE International Conference on*, 2009, pp. 2173–2176.

[9] C. T. Jin, N. Epain, and A. Parthy, "Design, Optimization and Evaluation of a Dual-Radius Spherical Microphone Array," *Audio, Speech, and Language Processing, IEEE/ACM Transactions on*, , no. 99, pp. 1, 2013.

[10] B. Rafaely, "The Spherical-Shell Microphone Array," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 16, no. 4, pp. 740–747, May 2008.

[11] T. D. Abhayapala and A. Gupta, "Alternatives to spherical microphone arrays: Hybrid geometries," in *Acoustics, Speech and Signal Processing (ICASSP), 2009 IEEE International Conference on*, Apr. 2009, pp. 81–84.

[12] A. Gupta and T. D. Abhayapala, "Double sided cone array for spherical harmonic analysis of wavefields," in *Acoustics, Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on*, 2010, pp. 77–80.

[13] D. L. Alon and B. Rafaely, "Spindle-Torus Sampling for an Efficient-Scanning Spherical Microphone Array," *Acta Acustica United With Acustica*, vol. 98, no. 1, pp. 83–90, Jan. 2012.

[14] G. Chardon, A. Cohen, and L. Daudet, "Reconstruction of solutions to the Helmholtz equation from punctual measurements," in *10th International Conference on Sampling Theory and Applications*, Bremen, Germany, July 2013.

[15] Robert S. Womersley and Ian H. Sloan, "How good can polynomial interpolation on the sphere be?," *Advances in Computational Mathematics*, vol. 14, no. 3, pp. 195–226, 2001.

[16] Rob Womersley, "Interpolation and cubature on the sphere," http://web.maths.unsw.edu.au/˜rsw/Sphere/.

[17] R Seeley, "A sharp asymptotic remainder estimate for the eigenvalues of the laplacian in a domain of $\{R3\}$," *Advances in Mathematics*, vol. 29, no. 2, pp. 244 – 269, 1978.

[18] S. Doclo and M. Moonen, "Superdirective Beamforming Robust Against Microphone Mismatch," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 15, no. 2, pp. 617–631, Feb. 2007.

[19] D. Bateman J. W. Eaton and S. Hauberg, *GNU Octave Manual Version 3*, Network Theory Limited, 2008.