WAVENUMBER TRACKING IN A LOW RESOLUTION FREQUENCY-WAVENUMBER REPRESENTATION USING PARTICLE FILTERING

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ABSTRACT

In underwater acoustics, shallow water environments ($d <$ 200 m) act as dispersive waveguide when considering low-frequency sources ($f < 250$ Hz), and propagation is described by modal theory. Propagated signals are usually multicomponent, and the group delay of each mode (each component) is dispersive and varies with mode number. The waveguide dispersion is characterized by modal wavenumbers, which are widely used as inputs of inversion algorithms to estimate environmental properties. Considering a horizontal array and a source along the axis of the array, wavenumber estimation is equivalent to spectral analysis in the range dimension. A large number of hydrophones (i.e. range samples) is thus required to perform an accurate wavenumber (i.e. spectral) estimation. This paper proposes an original approach for estimating the wavenumbers using a short array and a broadband low-frequency source. The wavenumbers are tracked in the frequency-wavenumber ($f - k$) domain using particle filtering. The waveguide physics provides generic system and state equations to model the $f - k$ diagram. In particular, it is possible to define an iterative relationship for wavenumber at two consecutive frequencies using the dispersion relation, which holds true in every waveguide. The proposed method provides interesting results on simulated data using 10 hydrophones. It is validated on experimental data recorded in the North Sea.

Index Terms— Particle filtering, dispersive waveguide, wavenumber estimation, underwater acoustics

1. INTRODUCTION

Low frequency ($f < 250$ Hz) acoustic propagation in shallow water (depth $d < 200$ m) is dispersive: each frequency propagating with its own speed. Considering a source $S(f)$ located at $r_s = 0$ and at the depth $z = z_s$, the acoustic pressure at $[r, z]$ for the frequency f consists of M dispersive components called modes:

$$P(r, z, f) = S(f) \sum_{m=1}^{M} A_m(f) e^{-j\phi_m(f)}. \quad (1)$$

The phase function of each mode $\phi_m(f) = rk_{rm}(f)$ depends on the source/receiver range r and on the waveguide properties through the horizontal wavenumber $k_{rm}(f)$. The modal amplitude $A_m(f)$ depends both on the waveguide properties and on the source/receiver configuration (range and depth). As the waveguide is dispersive the modal wavenumbers $k_{rm}(f)$ depend non-linearly on frequency. They are widely used as input for inversion algorithms to estimate environmental parameters.

Using a Horizontal Line Array (HLA) and a broadband source in the HLA axis (end-fire position), k_{rm} identification becomes a well known spectral estimation problem (in the range dimension). For wideband sources, one can obtain a frequency-wavenumber ($f - k$) diagram by computing the modulus of the spatial and temporal Fourier Transform (FT) of the array signal, which can be reduced to the spatial FT of equation (1) along the r axis [1]. This representation is suitable to recover the multimodal and dispersive nature of the shallow water waveguide, and thus estimate the environmental properties [2] or localize source depth [3]. However, the spatial FT presents limitations; in particular, an accurate separation of the wavenumbers requires a large amount of sensors. As an example, simulated $f - k$ diagrams computed using 10 and 240 sensors are presented in figures 1(a) and 1(b).

Because deploying long HLA underwater is both tedious and expensive, it is interesting to propose methods that allow wavenumber estimation using a relatively small number of hydrophones. This can be done using high-resolution methods, which classically allows for better spectral analysis [4]. However, these methods are limited to narrowband sources (wavenumbers are excited at a single frequency) [5, 6]. At best, they can be applied in a broadband context for each frequency independently. As an example, $f - k$ diagram can be obtained by replacing the spatial FT with an autoregressive spectrum [7, 8].

In this paper, we show that it is possible to track wavenumbers in a badly resolved $f - k$ diagramm obtained by spatial FT, thanks to physical knowledge of the waveguide propagation. In particular, wavenumbers are tracked using particle

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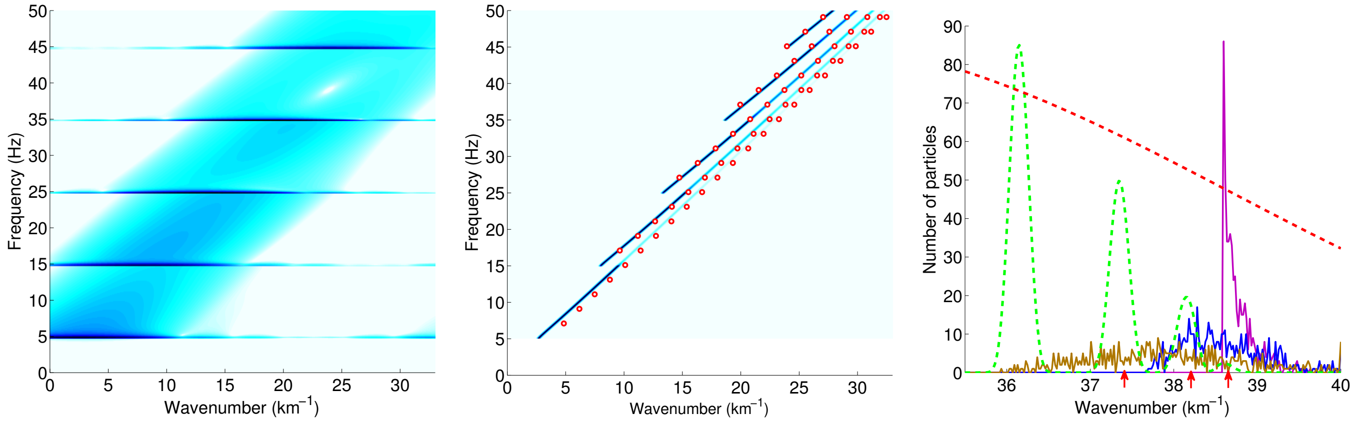


(a) 10 sensors $f-k$ diagram (b) Tracking result (c) Particle probability density

Fig. 1. Simulated data: (a) $f-k$ diagram obtained with spatial FT on 10 sensors; (b) mean trajectories of 500 particles per mode obtained with 10 sensors (red dots) over a $f-k$ diagram obtained with 240 sensors; (c) particle distributions for the first, second and third modes at 54 Hz (respectively purple, blue and brown curves), along with the wavenumber spectra at 54 Hz computed with 10 sensors (dashed red curve) and 240 sensors (dashed green curve). Red arrows show the theoretical wavenumber of modes 1, 2, and 3 estimated on the 240 sensor $f-k$ diagram. The spectrum amplitudes are arbitrarily normalized.

filtering (PF) method [9] and a state-space relation based on the generic dispersion relationship. Note that PF algorithms have been applied in several domains of acoustics [10] and underwater acoustics [11]: time of arrival estimation, detection of variations in the waveguide, mammal tracking, etc. However, none of these methods take advantage of propagation knowledge to define physics-based state-space relation.
The remainder of the paper is organized as follows. In section 2, the waveguide physics will be introduced to obtain a state equation, which links k_{rm} values from one frequency to the other; an observation equation will also be introduced to generate the spectrum associated to a particular wavenumber set. Then, in section 3, the bayesian formulation of the problem will be presented to apply a tracking algorithm for the wavenumber estimation. Finally, section 4 presents simulated and experimental results. The proposed method allows to properly estimate wavenumbers using 10 sensors, both on simulated data and experimental marine data recorded in the North Sea.

2. DISCRETE AND DYNAMIC MODEL OF THE $f-k$ DIAGRAM

Let us consider a discrete $f-k$ diagram $\mathbf{D}[\nu,\kappa]$ with $\nu \in [1, N_f]$ the discrete frequency and $\kappa \in [1, N_k]$ the discrete wavenumber. At discrete frequency ν, the wavenumber spectrum $\mathbf{D}[\nu, 1 : N_k]$ can be modeled as a discrete dynamical system, parametrized by two equations: a system equation and an observation equation. The observation equation is the relation generating the wavenumber spectrum from the wavenumber vector $[k_{r1}[\nu], ..., k_{rM}[\nu]]^T$ (as a reminder, M is the number of modes). The system equation is the iterative relation that links $k_{rm}[\nu+1]$ to $k_{rm}[\nu]$.

2.1. System equation

Because of modal propagation properties [12], the horizontal wavenumbers k_{rm} follows

$$\left(\frac{2\pi f}{c}\right)^2 = k_{rm}(f)^2 + k_{zm}(f)^2, \tag{2}$$

where c is the water sound speed and k_{zm} the vertical wavenumber. Equation (2) is called the dispersion relationship, and is valid in every waveguides. Note that in shallow water waveguides, the vertical wavenumber $k_{zm}(f)$ weakly depends on frequency f.
The wavenumber $k_{rm}(f)$ has two distinct behaviors depending on a particular frequency f_m called the cutoff frequency of mode m. If $f > f_m$, the wavenumber $k_{rm}(f)$ is a real number. In this case, using Eq. (2), it is possible to derive the iterative relation

$$\begin{aligned} k_{rm}[\nu+1]^2 = k_{rm}[\nu]^2 + (2\nu+1)\left(\frac{2\pi\Delta_f}{c}\right)^2 \\ + k_{zm}[\nu]^2 - k_{zm}[\nu+1]^2, \end{aligned} \tag{3}$$

where Δ_f is the resolution of the frequency axis. As k_{zm} barely depends on frequency, $k_{zm}[\nu]^2 - k_{zm}[\nu+1]^2$ can be neglected; in the following, this difference is assimilated to a random gaussian uncertainty.
When $f < f_m$, the wavenumber k_{rm} is an imaginary number and does not propagate; it is evanescent and does not impact the $f-k$ diagram. The cutoff frequencies can be relatively well known using minimal *a priori* information about the waveguide, or directly estimated on the $f-k$ diagram.

2.2. Observation equation

The waveguide has a modal behavior, its response presents resonances at the k_{rm} values. A simple observation equation inspired from the receptance response in modal analysis can be used to describe the normalized acoustic pressure $y(f, k)$ [13]:

$$y(f, k) = \sum_{m=1}^{M} \frac{1}{|k^2 - k_{rm}(f)^2 + j\xi|}, \quad (4)$$

which naturally leads to discrete wavenumber spectra $\mathbf{y}_\nu$ of size $[1 \times N_k]$. In (4), ξ is a resolution parameter and is estimated on the measured wavenumber spectrum using peak picking method [13] at the first value after the first cutoff frequency. It can be seen as a damping coefficient multiplied by a wavenumber but it does not depend on the frequency. Thanks to ξ, the estimated $\mathbf{y}_\nu$ is as badly resolved as $\mathbf{D}[\nu, 1 : N_k]$.

The dynamical model of the $f - k$ diagram consists of equations (3) and (4). Bayesian framework offers great opportunity for estimating the k_{rm} and tracking their evolution using classical PF algorithm. Since the system equation is nonlinear and the observation is not gaussian (k_{rm} can not take negative values), PF is an adapted tracking algorithm.

3. PARTICLE FILTERING

The wavenumbers k_{rm} are tracked using a classical bootstrap algorithm [9, 14]. Note that as a preliminary step, each line of the $f - k$ diagram is normalized, which leads to normalized wavenumber spectra $\mathbf{y}_\nu$.

An initial discrete frequency $\nu = \nu_0$ is chosen as first frequency where one mode is present. In the following, mode number is thus $M(\nu) = 1$, and $x_\nu = k_{r1}[\nu]$ is the tracked wavenumber. The measured wavenumber spectrum can be seen as distribution of x_ν. Thanks to Monte-Carlo integration, it can be discretized as a vector $\mathbf{x}_\nu$ of N_s particles x_ν^n (the exponent n denoting the particle number). Using Eq. (3), the evolution of the particles is defined as a Markov Models [9], leading to $x_{\nu+1}^n$. Corresponding wavenumber spectra $\mathbf{y}_{\nu+1}^n$ can then be generated using the observation equation (4).

Importance Sampling (IS) strategy is required to correctly approximate the wavenumber spectrum distribution [9]. It consists in associating weights $w_{\nu+1}^n$ to the particles $x_{\nu+1}^n$. Thanks to the bayesian inference, $w_{\nu+1}^n = \mathcal{L}(x_{\nu+1}^n | \mathbf{y}_{\nu+1})$, where $\mathcal{L}$ is the likelihood function for spectral estimation [15]. The IS is finally applied by updating the particles $x_{\nu+1}^n = \tilde{w}_{\nu+1}^n x_{\nu+1}^n$ where $\tilde{w}_{\nu+1}^n = w_{\nu+1}^n / \left(\sum_{n=1}^{N_s} w_{\nu+1}^n\right)$.

It is known that PF is not suitable for a high number of iterations as it could lead to degeneration of the particles: the weight associated to a majority of particles tends toward zero while ν increases [9]. To prevent this phenomenon, the weight efficiency is evaluated by $w_{\nu+1}^{\text{eff}} = 1/\sum_{n=1}^{N_s} (w_{\nu+1}^n)^2$. When $w_{\nu+1}^{\text{eff}}$ becomes smaller than a threshold w_s, a new particule set $\mathbf{x}_{\nu+1}$ is drawn. This step is called multinomial resampling [9]; wheter it is required or not, frequency ν is then incremented and the whole process repeated.

Note that smoothing [9] could be applied at the end of the bootstrap algorithm. It classicaly increases the PF performances. However, it will not be applied in our modal tracking context because the paper focuses on the physics-based model and system equations.

In our modal tracking context, it is important to slightly modify this classical bootstrap algorithm. Indeed, the number of tracked modes increases with frequency: mode m exists when $f > f_m$. As a consequence, mode number $M(\nu)$ can be greater than one and depends on frequency ν. When ν becomes greater than a discrete cutoff frequency ν_m, the mode number is incremented and a new particle set is drawn. As a result, number of particles required to sample $\mathbf{y}_\nu$ is actually $M(\nu)N_s$; and the particule vector $\mathbf{x}_\nu$ can be decomposed into $M(\nu)$ subsets, each subset corresponding to a given mode. It is thus important to know (or estimate) the cutoff frequency of each mode.

4. APPLICATION

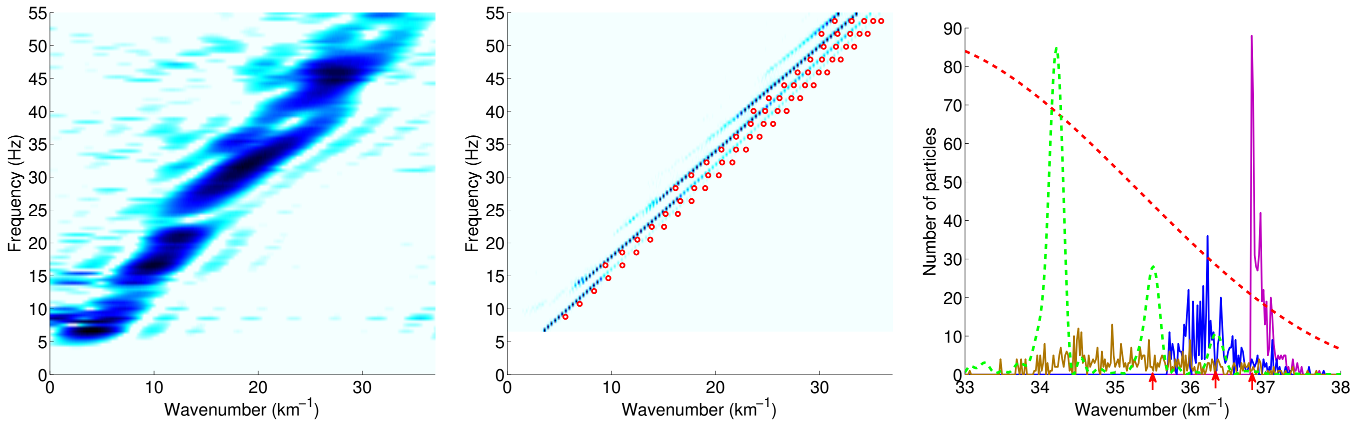
The tracking of the wavenumbers is applied on a simulated waveguide and on measurements realized in North Sea by CGG.

4.1. Pekeris Waveguide

The methodology is first applied on a simulated Pekeris waveguide, a classical shallow water model [12]. The environment consists of an isovelocity water column (depth 130 m, sound speed 1500 m.s^{-1}, density 1 kg.m^{-3}) over a semi-infinite fluid seabed (sound speed 2000 m.s^{-1}, density 2 kg.m^{-3}). The receiving array lies on the bottom, the hydrophone spacing is 25 m which produces a 6 km long array. The source signal is a perfect impulse with white spectrum from 0 to 60 Hz. It is localized at depth $z_s = 130$ m at the same range than the first HLA hydrophone. A white Gaussian noise is added on each hydrophone signal so that signal to noise ratio is 5 dB per sensor.

The $f - k$ diagram obtained using 10 hydrophones is represented in figure 1(a). The k dimension spectrum is obtained by FT on 2048 points using zeropadding and a Hann window. Because of the small hydrophone number, wavenumber resolution is really poor and wavenumber values can not be estimated. Wavenumbers are tracked over this 10-sensor $f-k$ diagram using the methodology presented in section 3. The tracking is performed using 500 particles per wavenumber over 500 frequency bins.

The mean tracks of the wavenumbers are plotted over a 240-sensors $f-k$ diagram in figure 1(b). The wavenumber trajec-



(a) 10 sensors normalized $f - k$ diagram (b) Tracking result (c) Particle probability density

Fig. 2. Measured data: (a) $f - k$ diagram obtained with spatial FT on 10 sensors; (b) mean trajectories of 500 particles per mode obtained with 10 sensors (red dots) over a $f - k$ diagram obtained with 240 sensors; (c) particle distributions for the first, second and third modes at 50 Hz (respectively purple, blue and brown curves), along with the wavenumber spectra at 50 Hz computed with 10 sensors (dashed red curve) and 240 sensors (dashed green curve). Red arrows show wavenumber values of modes 1, 2, and 3 estimated on the 240 sensor $f - k$ diagram. The spectrum amplitudes are arbitrarily normalized.

tory coincides really well with the 240-sensor $f - k$ diagram, demonstrating the method ability to track wavenumbers using 10 sensors. The first mode trajectory presents a deviation at its start but it is corrected among the frequencies. The associated particle distributions for $f = 54$ Hz are presented for the first, second and third mode with purple, blue and brown lines in figure 1(c). For comparison, the 10 and 240 sensor spectra are plotted as dashed red and green lines. Vertical red arrows show the theoretical position of the first three modes. The particle distribution associated to the first mode is sharper than the peak obtained with 240 sensors whereas the particle distribution associated to the second mode present the same spreading than the 240-sensor spectrum. The third mode particles present high variation. This phenomenon can be explained by the duration of the tracking: mode 1 and 2 are tracked over a greater frequency band, so that weighting and resampling lead to sharper distributions.

4.2. North Sea measurements

The methodology is also applied on experimental data collected in the North Sea by CGG [2]. The source signal is an airgun, emitting a short impulsion with a relatively white spectrum from 0 to 60 Hz. The array is composed of 240 omnidirectionnal hydrophones, equally spaced every 25 m. The array is then 6 km long and lays on the seabed. Data are recorded at sampling frequency 250 Hz. The environment can be approximated by a Pekeris waveguide with the parameters that have been used in section 4.1 [2].

As for the simulation wavenumber are tracked over a 10-sensor $f - k$ diagram (corresponding array is 225 m long) using 500 particles per wavenumber, and experimental results are presented in figure 2. The 10-sensor $f - k$ diagram is shown in figure 2(a): it is particularly irregular and noisy when compared with the simulated one. The estimated wavenumber trajectories are superimposed on a 240-sensor $f - k$ diagram in figure 2(b). The estimation seems good, except for a small deviation at the start of each modes. This deviation tends to be corrected as frequency increases thanks to the weighting process. The particle distributions at 50 Hz are presented in figure 2(c). These results have a similar behavior than the simulated ones, although the distributions have a more important variance, which is probably due to the initial noisy and irregular $f - k$ diagram.

5. CONCLUSIONS AND PERSPECTIVES

This paper presents an original approach for estimating wavenumbers in shallow water waveguide using a small HLA and a broadband source. Physical information from the waveguide theory is used to construct a model of the $f - k$ diagram. The strength of the proposed method is to inject physical *a priori* through the dispersion relationship. This relationship is at the same time true for every waveguide and robust enough to allow wavenumber tracking using PF. The method is successfully applied on simulated and experimental marine data. The particle distributions give an accurate estimation of the wavenumbers and provide estimation uncertainty.

The estimated wavenumbers and uncertainties can be used as the input of inverse algorithms. Considered applications include geoacoustic inversion, and particularly estimation of the attenuation in the sediment.

6. REFERENCES

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