IMPROVED COMPRESSED SENSING RADAR BY FUSION WITH MATCHED FILTERING

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ABSTRACT

Compressed Sensing (CS) provides a rich mathematical framework to efficiently acquire a sparse signal from few non-adaptive measurements. In radar imaging, most scenes are sparse and CS can be successfully applied for efficiently acquiring the target scene. Although the use of CS in radar is advantageous in many aspects, a higher noise in the received signal makes the output of CS unreliable. We propose a framework based on CS and matched filtering to improve the performance of CS particularly in high noise scenarios. We realize this framework by CS on chirp signal and discuss some limitations associated with it. Numerical experiments confirm a substantial performance improvement using the proposed framework compared to conventional CS reconstruction.

Index Terms— compressed sensing, sparse reconstruction, matched filtering, data fusion.

1. INTRODUCTION

Range and velocity estimation is the first step in moving target indication in Radar. The range-velocity plane is often viewed as a linear time-varying system $\mathbf{H}$. A probing signal $\mathbf{f}$ is used to determine the target characteristics from the reflected observation $\mathbf{y} = \mathbf{Hf}$. For efficient and accurate estimation of time delays and Doppler shifts from $\mathbf{y}$, one needs to overcome the fundamental time-frequency uncertainty, and mitigate noise & clutter.

We consider target detection in 1-dimensional, far-field, narrowband monostatic radar. The target is modeled as a non fluctuating point target in the range-velocity (delay-Doppler) plane, moving with a constant radial velocity. In real-life scenarios, the number of targets in the imaging plane of the radar is unknown. Matched filtering (MF) typically search for all possible time delays and Doppler shift combinations in the reflected signal $\mathbf{y}$, and its output (time-frequency plane) suffers from the inherent time-frequency limitations of the probing signal [1].

Consider the situation of only a few airplanes against the wide sky; hence the final results delivered by the radar imaging contains very few significant detections compared to the data collected in the front end. The theory of compressed sensing can be employed in different constructs [2, 3] to address various problems arising in the radar community. Particularly, most of the works focus on applying CS towards reducing data without compromising performance [4–6].

Although applications of CS in radar signal processing are very promising, one needs to address its satisfactory operating regime for the optimal usage. Compressed sensing, in specific cases, can improve the resolution [1, 6] thereby increase the accuracy of target detection. However, the reliability of CS drops heavily when the noise in the reflected signal is high, a situation that often occurs in the case of single-pulse radar returns. Secondly, a slight violation in sparsity may lead to failure in CS reconstruction algorithms.

In this paper, we introduce an abstract framework to improve the reliability of the CS reconstruction by utilizing the information from matched filtering, particularly in the low SNR regime. We propose an improved CS reconstruction algorithm based on compressive sampling matching pursuit (CoSaMP), followed by fusing the output of CS and MF to improve the detection accuracy. We emphasize that the proposed approach utilizes all Nyquist samples. In this paper, we explore ways to use best of both worlds, i.e., CS and MF, with the aim of improving the accuracy of the conventional radar systems [7].

In Section 2, we explain the proposed compressed sensing-matched filtering framework and its components. Numerical results were presented in Section 3. We offer concluding remarks in Section 4.

2. COMPRESSED SENSING-MATCHED FILTERING FRAMEWORK

A schematic of the proposed framework is shown in Fig. 1. We refer to this system as compressed sensing-matched filtering (CS-MF) framework, where we consider CS as an augmentation to the conventional system rather considering it as a replacement. In Fig. 1, the matched filtering line is shown in the bottom, referred as standard line, with auxiliary CS line on top of it. The CS reconstruction of the target scene is achieved by a random subset of samples acquired via Nyquist sampling; more importantly, we consider matched filtering output to guide CS reconstruction. Optionally, the received baseband signal can be subjected to coherent integration be-

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fore CS-MF framework to improve the SNR. The output of CS reconstruction is then fused with MF to produce the final output.

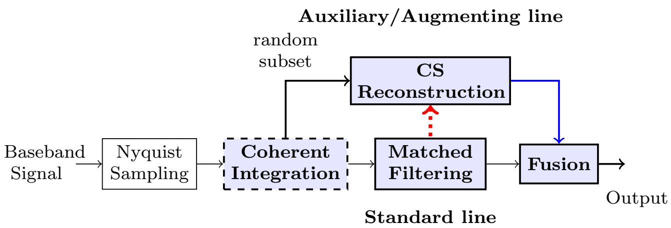


Fig. 1. Compressed Sensing and matched Filtering (CS-MF) framework. The highlighted blocks in this diagram are discussed in this paper.

We explain the implementation of CS using chirp signals in Section 2.1. A CS reconstruction algorithm based on CoSaMP guided by matched filtering is explained in Section 2.2. A note on fusing the output of MF and CS reconstruction is given in Section 2.3

2.1. Compressed sensing based on Chirp signals

We consider the implementation of compressed sensing based on linear frequency modulated (LFM) chirp signal as discussed in [8]. Consider a complex waveform f with carrier frequency ω_0. Let r and v denote the range and radial velocity of the target; this can be computed from the delay $\tau = 2r/c_0$ and Doppler shift $u = 2v\omega_0/c_0$, where c_0 refers to the velocity of light. With the reflectivity coefficient of the target be $x(\tau, u)$, the reflected signal is given by the following equation:

$$y(t) = \int\int x(\tau, u) f(t-\tau) e^{-2\pi i u t} du d\tau + w(t), \quad (1)$$

where $w(t)$ represents complex Gaussian baseband noise. The transmitted linear frequency modulated waveform f is given by:

$$f_{\text{LC}}(t) = \exp\left[2\pi i \left(\frac{\alpha_1}{2} t^2 + \omega_0 t\right)\right] I_T(t), \quad (2)$$

where $I_T(t)$ is the indicator function of duration $[0, T]$, which gives the duration of transmission. The reflected signal $y(t)$ is given by:

$$y(t) = \int\int x(\tau, u) f_{\text{LC}}(t-\tau) e^{-2\pi i u t} du d\tau, \quad (3)$$

and the sampled signal $y(t_j)$ is given by:

$$y(t_j) = f_{\text{LC}}(t_j) \sum_{k,l} x_{k,l} f_{\text{LC}}(-\tau_k) \exp[-2\pi i(\alpha_1 \tau_k + u_l) t_j],$$

with $\tau_k = k\Delta\tau$ and $u_l = (l - N/2)\Delta u$ where $k, l = 1, \dots, N$, represent the delay-Doppler grid. With the assumption:

$$\alpha_1 = \Delta u / N\Delta\tau, \quad (4)$$

we set up the following grid $\{\gamma_p\}$,

$$\gamma_p = \tau_k + u_l/\alpha_1 = (p + N - N^2/2)\Delta\tau, \quad (5)$$

where $p = k + N(l-1) \in \{1, \dots, N^2\}$. By choosing

$$T = \frac{NQ}{\Delta u}, \quad Q \in \mathbb{N}, \quad (6)$$

we can write the sampled signal $y(t_j)$ in the matrix form $\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{e}$. The individual elements of the matrix form is given by the following equations.

$$\mathbf{y}_j = \frac{y(t_j)}{f_{\text{LC}}(t_j)} e^{\pi i (2N - N^2) Q \widehat{t_j}}, \quad (7)$$

$$\mathbf{x}_p = x_{k,l} f_{\text{LC}}(-\tau_k), \quad (8)$$

$$F_{j,p} = \exp[-2\pi i Q p \widehat{t_j}] \in \mathbb{C}^{M \times N^2}. \quad (9)$$

By choosing t_j uniformly random over $[0, T]$, $j = 1, ..., M$, the sensing matrix $\mathbf{F}$ becomes random partial Fourier matrix of size $M \times N^2$.

In most cases, the random partial Fourier matrix $\mathbf{F}$ satisfies the restricted isometry condition, and hence can be used to reconstruct the sparse vector $\mathbf{x}$ via sparse reconstruction algorithms (greedy algorithms or basis pursuit denoising (BPDN)) [8–10].

2.2. CS reconstruction by Modified CoSaMP (M-CoSaMP)

Compressive Sampling Matching Pursuit (CoSaMP) [10] is a greedy, iterative sparse signal recovery algorithm which provides an approximation of the sparse signal $\mathbf{x}$ (eq.(8)) from the compressed measurements $\mathbf{y}$ (eq. (7)). First, a proxy for the signal to be estimated is formed using the measurement matrix $\mathbf{F}$ and the compressed measurements $\mathbf{y}$. CoSaMP estimates the target signal iteratively from the signal proxy. At each iteration, the current approximation $\mathbf{a}^k$ induces a residual $\mathbf{v}$ containing the part of the target signal to be approximated. The signal estimate and residual will get updated in each iteration; the residual determines the tentative signal support for the every subsequent iteration.

The matched filtering estimate of the signal $\mathbf{x}$ (referred as $\mathbf{x}_{MF}$) is not sparse and suffers from side-lobes of the ambiguity function. However, since it is reasonable to assume that $\mathbf{x}$ is sparse, CoSaMP can be used to solve for $\mathbf{x}$. Both the MF and CoSaMP output provide helpful information about the target location. Hence the MF output $\mathbf{x}_{MF}$ can be used to obtain a better signal proxy in CoSaMP. We consider normalized MF output Ω_{MF} to weigh the CoSaMP signal proxy $\mathbf{F}^H\mathbf{v}$, where $\mathbf{F}^H$ refers to the Hermitian transpose of $\mathbf{F}^H$. The weighting is expected to improve the overall performance of CoSaMP, especially when the SNR is low. Pseudo-code for this algorithm is provided in Alg. 1.

Algorithm 1: Pseudo-code for modified Compressive Sampling Matching Pursuit (M-CoSaMP) reconstruction algorithm.

Input: Measurement Matrix $\mathbf{F}$, noisy measurement vector $\mathbf{y}$, sparsity level s, MF weighting Ω_{MF}
Output: An s-sparse approximation $\mathbf{a}$ of the signal $\mathbf{x}$
Intialize: $\mathbf{a}^0 \leftarrow 0$; $\mathbf{v} \leftarrow \mathbf{y}$; $k \leftarrow 0$;
$\mathbf{u} = (\mathbf{F}^H\mathbf{v}) \cdot \Omega_{MF}$ // Enhancement

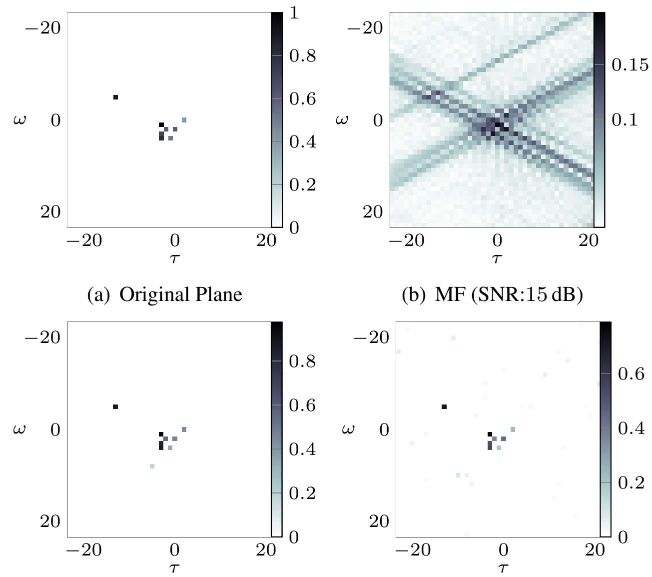
repeat
$k \leftarrow k + 1$
$\Omega \leftarrow \mathbf{supp}(\mathbf{u}_{2s})$ // Identify $2s$ large comp.
$T \leftarrow \Omega \cup \mathbf{supp}(\mathbf{a}^{k-1})$ // Merge Supports
$\mathbf{b}_{|T} \leftarrow \mathbf{F}_T^{\dagger}\mathbf{y}$
$\mathbf{b}_{|T^c} \leftarrow 0$
$\mathbf{a}^k \leftarrow \mathbf{b}_s$ // Prune to s large comp.
$\mathbf{v} \leftarrow \mathbf{u} - \mathbf{F}^H\mathbf{a}^k$
$\mathbf{u} = \mathbf{F}^H\mathbf{v}$
until *stopping criteria is met*

2.3. Fusing MF and Modified CoSaMP (M-CoSaMP+MF)

In matched filtering, the radar return (eq. (1)) is correlated with the time-frequency shifted versions of the transmitted signal f_{LC}, and hence the delay-Doppler plane contains the self-ambiguity function of f_{LC} centered at the target locations (τ, ω) scaled by the reflection coefficient $x(\tau, \omega)$. In case of multiple targets, the delay-Doppler plane is a superposition of the self-ambiguity function at the location of the targets. When two targets are sufficiently close, overlapping ambiguity functions leads to loss of ability to distinguish individual targets.

Even though the target plane reconstructions of MF and CS can be analyzed separately, fusing them can help in improving the resolution and removing the false alarms. This fusion will help to emphasize the targets detected by both MF and CS whereas suppressing false alarms. Especially at low SNR scenarios, CS output degrades and a careful inspection is needed to reduce the false alarms, which is accomplished by fusing the output of matched filtering.

In fusion, we use MF output as a guide to select the significant part of the CS output to obtain the overall output of CS-MF framework. We retain the subset of the CS output corresponding to the locations with the largest amplitude in the MF output. The output of CS is set to zero at locations where the MF output is relatively small. Specifically, we retain the N rows in the CS output corresponding to the largest amplitudes in the MF output along the Doppler dimension. This is explained in more detail in Section 3.



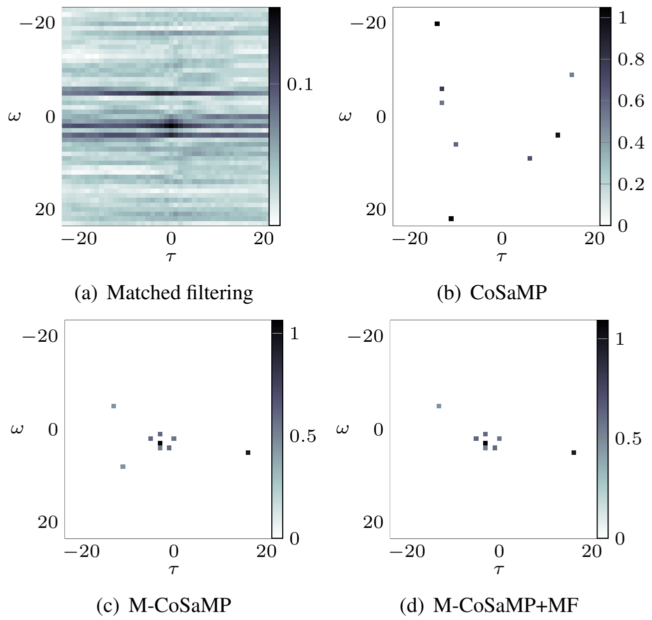
(a) Original Plane (b) MF (SNR:15 dB)
(c) CoSaMP (SNR:15 dB) (d) BPDN (SNR:15 dB)

Fig. 2. Original target scene, matched filtering, and CS reconstruction for single-pulse SNR of 15 dB. (a) Original target scene, (b) Matched filtering, (c) CoSaMP reconstruction, and (d) BPDN reconstruction.

3. RESULTS AND DISCUSSION

The Compressed sensing-Matched filtering (CS-MF) framework is illustrated using simulations on a time-frequency grid of size 47×47. We consider eight targets with varying amplitudes, distributed as one isolated target and seven clustered targets; the original target plane is shown in the Fig. 2(a). We assume that all the targets exactly occupy the grid positions. We consider the following experimental parameters for the LFM Chirp based CS : $N = 47$, $M = 47$, $\Delta\tau = 1$, $\Delta u = 1/M$, $Q = 47$; sparsity level is set to $s \approx M/(2 \log N^2)$ [10]. All simulations are averaged over 100 noise realizations to maintain consistency.

In Fig. 2, we compare CS Radar reconstructions with matched filtering. Matched filtering reconstruction of the original target scene is shown in Fig. 2(b); in this case the chirp rate is set to $\alpha_1/2 = 25$ [11, 12], and the target scene is reconstructed by an up-chirp followed by a down-chirp. Chirp based CS radar explained in the Sec. 2.1 is used to reconstruct the target plane by CoSaMP [10] and basis pursuit denoising [9] algorithms; the corresponding reconstructions are shown in Figs. 2(c) & (d), respectively. A comparison clearly reveals that CS reconstruction is able to exactly resolve the targets located in the adjacent cells individually, whereas MF approximately locates them. When noise increases, CS degrades quickly and often produces unreliable estimates whereas MF still locates the targets approximately.



(a) Matched filtering (b) CoSaMP

(c) M-CoSaMP (d) M-CoSaMP+MF

Fig. 3. Delay-Doppler reconstruction for the SNR of 0 dB for (a) Matched filtering, (b) CoSaMP, (c) CoSaMP with modified signal proxy, (d) (c) along with MF weighting.

The CS-MF framework explained in the Sec. 2, imposes limitations on the chirp signal parameters. This limitation reduces Bandwidth-Time (BT) product of the chirp signal to less than unity, thus making it unsuitable for pulse compression [13]. We show the obtained MF reconstruction of the target plane in Fig. 3(a); an inspection clearly highlights the resolution loss along time (delay) in comparison with a typical MF output with a higher BT product (Fig. 2(b)). Nevertheless, we stick to a lower BT product in CS-MF framework with the intent to realize MF and CS in single-pulse return.

The CS-MF framework helped in obtaining a satisfactory operation at low SNR scenarios; this is demonstrated in the Figs. 3(b), (c) & (d) for single-pulse SNR value of 0 dB. CoSaMP reconstruction, given in Fig. 3(b), failed completely. On the other hand, Fig. 3(c) shows that the M-CoSaMP reconstruction detects every target at the same SNR, except a few outliers. We extract K rows having largest amplitude along the Doppler direction from the MF output represented by the set $\{I\}$. We retain the rows indicated by the set $\{I\}$ in CS reconstruction and remove the rest of rows, whose output is shown in Fig. 3(d). By this simple procedure, target detection is enhanced and the false alarms in CS output can be substantially reduced.

In Fig. 4, we summarize the normalized root-mean-square distortion (NRMS) with the single-pulse SNR for the algorithms discussed in this paper. We define NRMS, an error measure comparing the original and the reconstructed target

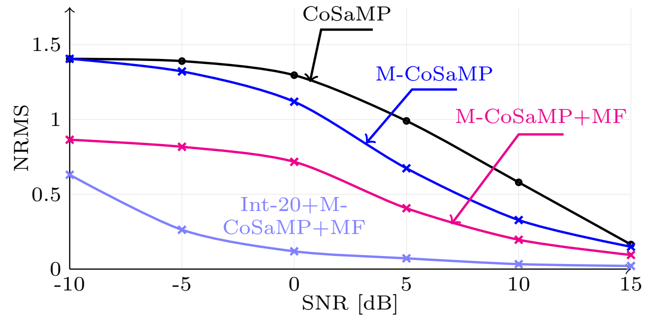


Fig. 4. Normalized root-mean-square distortion of the reconstructed target plane with single-pulse SNR.

plane by the following equation:

$$\mathrm{NRMS}(\mathbf{x}, \hat{\mathbf{x}}) = \sqrt{\frac{\sum_i (x_i - \hat{x}_i)^2}{\sum_i x_i^2}}, \tag{10}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ refers to original and reconstructed target plane, respectively. The curves shows that M-CoSaMP gives a higher improvement at higher SNRs compared to sub-zero SNRs; this is particularly because the MF output is also affected by noise which in turn degrades the reconstruction by M-CoSaMP. Nevertheless, fusing MF output gives a significant reduction of error particularly due to the reduction of the false alarms, which is also confirmed by visual inspection. Furthermore, a large reduction in error is achieved by operating the CS-MF framework after integration; this is confirmed from the substantial reduction in NRMS with an integration over 20 pulses (Int-20+M-CoSaMP+MF; Fig. 4).

4. CONCLUSION

In this paper, we proposed a framework to fuse compressed sensing and matched filtering that operates on single-pulse radar return. We realized compressed sensing using chirp signals and proposed an enhanced CS reconstruction algorithm (M-CoSaMP) guided by matched filtering. The numerical results show that M-CoSaMP outperforms traditional CoSaMP around zero and positive SNRs. However, the performance of the M-CoSaMP approaches CoSaMP at very high noise levels (very low SNRs). This is because the output of matched filtering which acts as a guide degrades with noise as well. Overall, the numerical results indicate that CS-MF framework perform better than conventional CS reconstruction algorithms.

Realizing the CS-MF framework in a single-pulse radar imposes a limitation on the bandwidth-time product of the transmitted chirp signal; this is a major concern as it hampers pulse compression and hence the resolution in matched filtering. Therefore, more theoretical and numerical studies are needed to understand and implement this framework in a practical radar system.

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