

# NEW FEEDBACK ACTIVE NOISE CONTROL SYSTEM WITH IMPROVED PERFORMANCE

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## ABSTRACT

In many practical active noise control (ANC) applications, the undesired primary noise consists of multiple harmonic-related tones, which can be effectively reduced by internal model control (IMC) feedback ANC systems. In this paper, a new feedback ANC system is proposed with improved convergence rate and noise reduction when the frequency separation between two adjacent tones is small. In the proposed system, adaptive notch filter (ANF) is used to estimate the frequencies of the multi-tone noise, based on which the reference signals are internally generated and regrouped to increase the frequency separation in each channel. Compared with the conventional IMC feedback ANC system, the proposed system converges faster, has a better tracking capability when the frequencies of the primary noise vary, and is also less sensitive to impulsive noise. Computer simulations are conducted for both synthesized tonal noise and motorcycle engine noise to show the improved performance of the proposed system.

**Index Terms**— Feedback active noise control system, adaptive notch filter, frequency separation, convergence rate, frequency estimation

## 1. INTRODUCTION

Active noise control (ANC) [1-3] is an effective technique for reducing low-frequency acoustic noise, where traditional passive methods [4, 5] are bulky, costly, and ineffective. In many practical ANC applications, the primary noise is usually periodic and thus, contains multiple tones at the fundamental frequency and several harmonic frequencies.

To reduce the multi-tone noise, Glover proposed a direct-form multi-frequency feedforward ANC system [6], which uses a sum of tones as the reference signal to an adaptive filter with length much larger than two. However, there are many practical ANC applications whereby the reference signal is not available due to the physical limitation of microphone placement or inaccuracy in the reference signal generation. For these cases, feedback ANC

systems [7, 8] can be used, in which the reference signal is internally synthesized based on the measured error signal.

It was found in [9] that the convergence rate of Glover's feedforward ANC system [6] is affected by the frequency separation between two adjacent tones. Hence, a direct/parallel feedforward ANC system was proposed in [9], where all the frequency components of the reference signal are partitioned to different channels to increase the frequency separation. Frequency separation effects and step size bound of the conventional internal model control (IMC) feedback ANC systems were analyzed in [10] and [11, 12], respectively. However, to solve the slow convergence problem in the conventional IMC feedback ANC system due to small frequency separation, the approach used in [9] for the feedforward ANC system cannot be directly copied.

In this paper, an improved feedback ANC system is proposed to solve the above-mentioned problem, assuming that the number of tones in the primary noise is known. Inspired by the idea presented in [13] and [14], in the proposed system, the frequencies of the multi-tone noise are estimated in real time by using infinite-impulse-response (IIR) adaptive notch filter (ANF) [15]. Based on the estimated frequencies, the reference signals are internally generated and grouped to different channels so that the frequency separation in each channel is increased, and thus, the convergence rate and noise reduction are improved. Furthermore, the proposed new feedback ANC system is found to be less sensitive to impulsive noise.

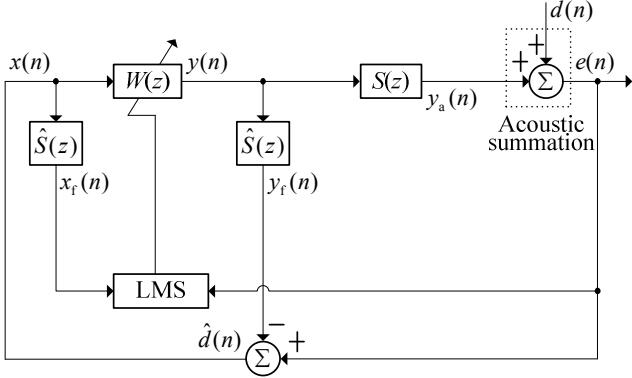
## 2. CONVENTIONAL IMC FEEDBACK ANC SYSTEM

The block diagram of the conventional IMC feedback ANC system is shown in Fig. 1, which estimates the undesired primary noise  $d(n)$  based on the measured error signal  $e(n)$  and adaptive filter output signal  $y(n)$ , and use it as the reference signal  $x(n)$  for the adaptive filter  $W(z)$ .

As shown in Fig. 1, the internally synthesized reference signal  $x(n)$  is expressed as

$$x(n) \equiv \hat{d}(n) = e(n) - y_f(n), \quad (1)$$

where  $n$  is the time index, and  $y_f(n)$  is the estimated anti-



**Fig. 1.** Block diagram of the conventional IMC feedback ANC system

noise signal. The output signal  $y(n)$  of the adaptive filter  $W(z)$ , also referred to as the canceling noise, is obtained as

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \quad (2)$$

where  $\mathbf{x}(n) = [x(n) \ x(n-1) \ \dots \ x(n-L+1)]^T$  is the reference signal vector,  $L$  denotes the adaptive filter length, and  $\mathbf{w}(n) = [w_0(n) \ w_1(n) \ \dots \ w_{L-1}(n)]^T$  is the adaptive weight vector, which is updated based on the filtered-X least-mean square (FXLMS) algorithm as follows:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{x}_f(n) e(n), \quad (3)$$

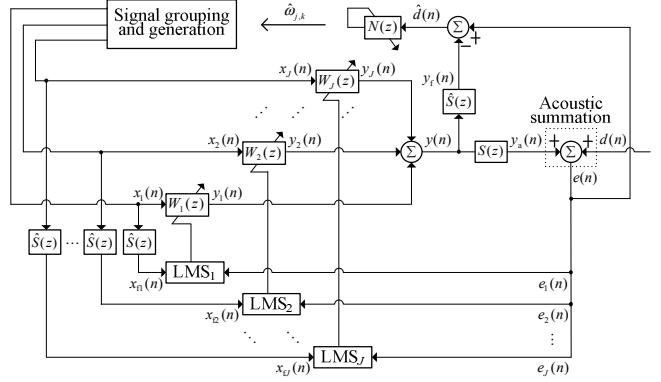
where  $\mu$  is the step size, and  $\mathbf{x}_f(n) = [x_f(n) \ x_f(n-1) \ \dots \ x_f(n-L+1)]^T$  is the filtered reference signal vector.

### 3. PROPOSED NEW FEEDBACK ANC SYSTEM

When aiming at reducing multi-tone noise, the conventional IMC feedback ANC system has some drawbacks. Its convergence rate is slow if the frequency separation is small [10]. The frequency separation of its reference signal is not controllable. Additionally, it is very sensitive to impulsive noise, which can even cause it to become unstable [1].

To solve these problems, we propose an alternative approach in generating the reference signal for the feedback ANC system. The block diagram of the proposed new feedback ANC system is shown in Fig. 2. Similar to the conventional IMC feedback ANC system, an estimate of the primary noise  $d(n)$  is firstly obtained based on (1) and denoted as  $\hat{d}(n)$ . In this paper, it is assumed that the secondary path model  $\hat{S}(z)$  is perfect, and thus, the primary noise model  $\hat{d}(n)$  is perfect as well.

Following that, the total frequencies of the primary noise model  $\hat{d}(n)$  are estimated using the frequency estimation approach proposed in [15]. Assume that the undesired primary noise  $d(n)$  is expressed as



**Fig. 2.** Block diagram of the proposed new feedback ANC system

$$d(n) = d'(n) + v(n), \quad (4)$$

where  $v(n)$  denotes the additive white Gaussian noise, and

$$d'(n) = \sum_{k=1}^D A_k \cos(\omega_k n + \phi_k), \quad (5)$$

where  $D$  represents the total number of tones to be canceled, and  $A_k$ ,  $\omega_k$ , and  $\phi_k$  are the amplitude, frequency, and phase, respectively, of the  $k$ -th frequency component of  $d(n)$ . The ANF  $N(z)$  shown in Fig. 2 is to obtain accurate estimation of the frequencies of  $\hat{d}(n)$ . It is constructed in a cascaded form with following transfer function:

$$N(z) = \prod_{k=1}^D N_k(z), \quad (6)$$

where  $N_k(z)$  is the  $k$ -th second-order IIR filter and its transfer function is expressed as

$$N_k(z) = \frac{1 - 2z^{-1} \cos(k\omega) + z^{-2}}{1 - 2rz^{-1} \cos(k\omega) + r^2 z^{-2}}, \quad (7)$$

where the pole radius  $r$  is a positive number close to but less than 1, which determines the notch bandwidth. The fundamental frequency  $\hat{\omega}_1(n+1)$  is estimated iteratively as:

$$\hat{\omega}_1(n+1) = \hat{\omega}_1(n) - \mu' \alpha_D(n) \beta_D(n), \quad (8)$$

where  $\mu'$  is the step size,  $\alpha_D(n)$  is the filter output at the  $D$ -th section, and  $\beta_D(n)$  is the gradient term of  $\alpha_D(n)$  defined as

$$\beta_D(n) = \frac{\partial \alpha_D(n)}{\partial \hat{\omega}(n)}. \quad (9)$$

Denoting the estimated fundamental frequency after convergence as  $\hat{\omega}_1$ , the harmonic frequencies are found as

$$\hat{\omega}_k = k\hat{\omega}_1, \quad k = 1, 2, \dots, D. \quad (10)$$

Based on the above estimated frequencies, the reference signals are internally generated. Unlike the conventional IMC feedback ANC system, all the generated frequency components in the proposed system are grouped to different channels so that the frequency separation is increased, resulting in decreased eigenvalue spread, and thus, faster convergence rate according to [1, 10]. The number of tones per channel  $K$  and the number of channels  $J$  are related as

$$K = \left\lceil \frac{D}{J} \right\rceil, \quad (11)$$

where  $\lceil \cdot \rceil$  is the ceiling function [16]. The selection of  $J$  is important, which directly affects the performance of the proposed system. Larger  $J$  leads to larger frequency separation in each channel, and thus, faster convergence of the overall system, but higher computational cost because every channel requires the filter operation by secondary path model [1, 9]. In practice, a compromise has to be made on  $J$  so that a balance between performance and cost is achieved.

Thus, the reference signal in the  $i$ -th channel  $x_i(n)$  is obtained as

$$x_i(n) = \cos(\hat{\omega}_i n) + \cos(\hat{\omega}_{i+J} n) + \dots + \cos(\hat{\omega}_{i+(K-1)J} n), \quad (12)$$

where  $i = 1, 2, \dots, J$ . In this way, the frequency separation is increased from  $\hat{\omega}_i$  to  $J\hat{\omega}_i$  in all the channels. The output signal of the  $i$ -th adaptive filter is computed as

$$y_i(n) = \mathbf{w}_i^T(n) \mathbf{x}_i(n), \quad i = 1, 2, \dots, J. \quad (13)$$

where  $\mathbf{w}_i(n)$  are the coefficients of the  $i$ -th adaptive filter. Hence, as shown in Fig. 2, the canceling noise is obtained as

$$y(n) = \sum_{i=0}^J y_i(n), \quad i = 1, 2, \dots, J. \quad (14)$$

#### 4. SIMULATION RESULTS

In this section, simulations are conducted to prove the improved performance of the proposed new feedback ANC system over the conventional IMC feedback ANC system. The sampling rate  $f_s$  is fixed at 2 kHz. For synthesized tonal signal as the primary noise, there are eight tones. Their frequencies are 20 Hz, 40 Hz, 60 Hz, 80 Hz, 100 Hz, 120 Hz, 140 Hz and 160 Hz. For the proposed new feedback ANC system, all the eight tones are grouped into two channels. The first channel contains the frequencies of 20 Hz, 60 Hz, 100 Hz and 140 Hz, and the second channel contains the frequencies of 40 Hz, 80 Hz, 120 Hz and 160 Hz. In this way, the frequency separation in each channel is increased from 20 Hz to 40 Hz. To compare these two systems when the frequencies of the primary noise vary, it is assumed that at the end of the forth second, the eight frequencies change gradually to 50 Hz, 100 Hz, 150 Hz, 200 Hz, 250 Hz, 300 Hz, 350 Hz and 400 Hz. From the start of the seventh second, all the frequencies maintain at the new values.

The variation in the frequency separation is very common in practical applications. Take the motorcycle engine noise as an example. It is well-known that the fundamental frequency of motorcycle engine noise is related to the engine revolutions per minute (RPM) [17] as

$$\text{fundamental frequency} = \frac{N_c}{2} \times \frac{\text{RPM}}{60} \text{ Hz}, \quad (15)$$

where  $N_c$  is the number of cylinders, and the remaining term (RPM/120) in (15) is called the firing frequency. Hence, based on (15), the frequency separation, which is the

same as the fundamental frequency [17], is also related to RPM. Thus, in practical applications, when the speed of the vehicle varies, the frequency separation of the generated engine noise varies accordingly. Therefore, it is very important that the proposed system is able to track the variation of all the frequencies. If there is any significant discrimination between the frequencies of the reference signal and primary noise, the noise reduction performance will be degraded severely [18]. The frequency tracking capability of the proposed new feedback ANC system is shown in Fig. 3, when the variance of the pure multi-tone noise  $d'(n)$  and the white Gaussian noise  $v(n)$  are equivalent. It can be clearly seen that the proposed new feedback ANC system is able to accurately estimate all the frequencies, and track their changes. Since these estimated frequencies are accurate, the generated reference signals for all the channels will be accurate as well.

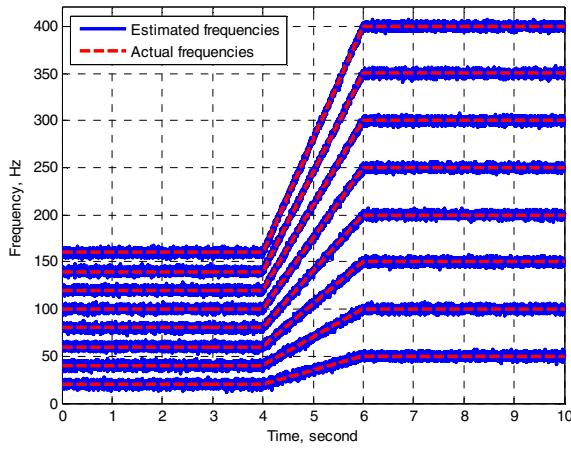
Figure 4 shows the comparison between the conventional IMC and proposed new feedback ANC systems. It is found that the proposed system has a better tracking capability and is able to maintain a lower residual error.

The convergence rates of the conventional and proposed systems for practical motorcycle noise under different frequency separations are shown in Fig. 5. Similar to the case of synthesized tonal noise, the first eight tones of the motorcycle noise are considered, and they are grouped into two channels. The x-axis shows the different frequency separations of the motorcycle noise, and the corresponding motorcycle speeds. The convergence time is defined as the time when the MSE has dropped by 8.69 dB [1], which indicates that the error has dropped to  $1/e^2$  of its initial value. It is found that the proposed system gives a faster convergence than the conventional IMC feedback ANC system. This is because in the proposed system, the frequency separation increases due to the regrouping of all the frequency components in the reference signal, resulting in a smaller eigenvalue spread, and thus, faster convergence.

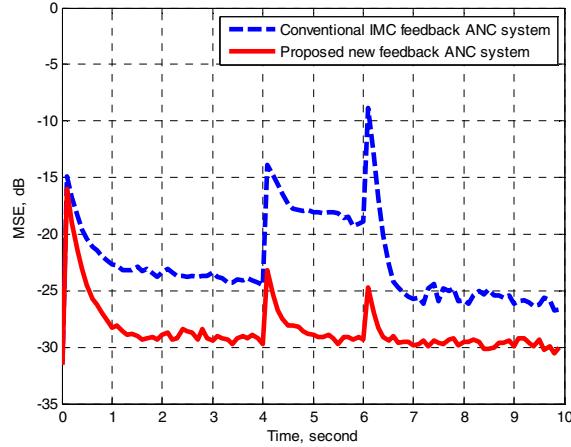
For the impulsive noise case, the impulsive noise is modeled as an exponentially decayed signal that occurred at the end of the fifth second as

$$v(n) = \begin{cases} 0 & \text{for } n < 5 \cdot f_s, \\ 10e^{-0.08n} \sin(2\pi \cdot 0.25n) & \text{for } n \geq 5 \cdot f_s. \end{cases} \quad (16)$$

In (16),  $f_s$  is used to represent second. From Fig. 6, it is obvious that the proposed new feedback ANC system is much less sensitive to impulsive noise. This is because in the conventional IMC feedback ANC system shown in Fig. 1, the reference signal includes the impulsive noise picked up by the error sensor. Hence, the adaptive filter is affected by the impulsive noise. However, in the proposed new feedback ANC system, the frequency estimation procedure is not affected by the impulsive noise. Thus, its reference signal is not affected. Therefore, the proposed feedback ANC system is less sensitive to impulsive noise.



**Fig. 3.** Frequency tracking capability of the proposed new feedback ANC system

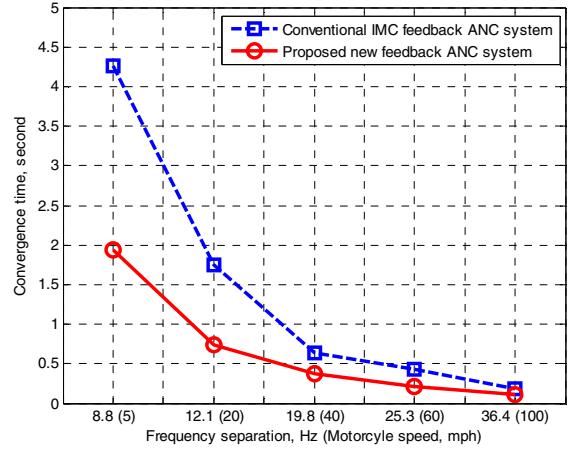


**Fig. 4.** MSE curves for the conventional IMC and proposed new feedback ANC systems when the noise frequencies vary

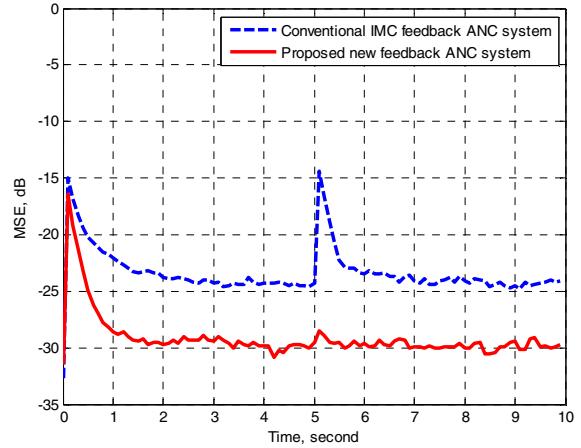
Compared with the conventional IMC feedback ANC system, the proposed new feedback ANC system incurs an increase in the computational load, due to the introduced frequency estimation block and increased number of secondary path models used. However, as the frequency separation is increased in all the  $J$  channels, to achieve the same convergence rate, the total number of taps of all the  $J$  adaptive filters will be reduced [9]. Hence, the overall increased computational load is not significant, compared with the very much improved convergence rate.

## 5. CONCLUSIONS

In this paper, a new feedback ANC system with improved performance was presented. Assuming that the number of tones in the primary noise is known, the proposed system first estimates all the frequencies appearing in the primary noise, followed by generating the reference signals based on



**Fig. 5.** Convergence time vs. frequency separation for the conventional IMC and proposed new feedback ANC systems for practical motorcycle noise



**Fig. 6.** Effect of impulsive noise on the conventional IMC and proposed new feedback ANC systems

the estimated frequencies. The internally generated reference signals are then separated into different channels so that the frequency separation in each channel is increased, and thus, the convergence rate of the overall system is improved significantly. The proposed system is also less sensitive to impulsive noise, and has a better tracking capability if the frequencies of the primary noise vary with time.

## 6. RELATION TO PRIOR WORK

This paper applies the direct/parallel structure presented in [9] for the feedforward ANC system into the conventional IMC feedback ANC system to increase its frequency separation so that its convergence rate and noise reduction are improved. For successful control of the frequency separation, the idea of using frequency estimation in ANC systems [13, 14], and IIR adaptive notch filter [15] are adopted as well.

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