

# SPARSITY-INDUCING MODIFIED FILTERED-X AFFINE PROJECTION ALGORITHMS FOR ACTIVE NOISE CONTROL

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## ABSTRACT

This paper describes a novel technique for promoting sparsity in the modified filtered-x algorithms required for active noise control. The proposed algorithms are based on recent techniques incorporating approximations to the  $\ell_0$ -norm in the cost functions that are used to derive adaptive filtering algorithms. In particular, zero-attracting and reweighted zero-attracting filtered-x adaptive algorithms are developed and considered for active noise control problems. The results of simulations indicate that the proposed techniques improve the convergence of the existing modified algorithm in the case where the primary and secondary paths exhibit a degree of sparsity.

**Index Terms**— Active noise control, adaptive algorithms, sparsity-aware techniques.

## 1. INTRODUCTION

Active noise control (ANC) is a popular technique for removing noise from a system by using a variant of the system identification scenario to subtract the effect of a noise-generating plant from a signal [1]. ANC algorithms are based on classical adaptive algorithms, with provision made for the additional electroacoustic path between the filter output and measured error signal, known as the secondary path [2]. A common technique to account for this path is known as the filtered-x (FX) scheme, originally developed for least-mean square (LMS) algorithms in the context of adaptive control [3] and since developed for other classical algorithms [4, 5, 6].

The FX scheme eliminates potential algorithm instability caused by the additional delay in the secondary path [7], but is marred by slow convergence [8]. The modified filtered-x (MFX) scheme was introduced in [9], and improves upon the convergence of the FX algorithms by introducing additional steps to approximate the error signal. MFX schemes have since become a popular choice for ANC architectures. However, there is still room to improve the convergence speed.

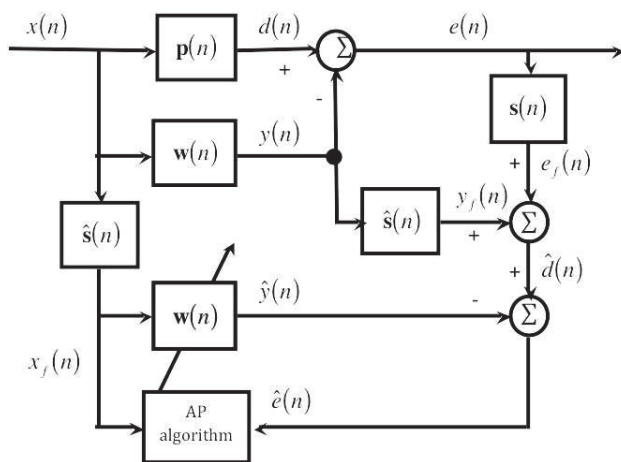
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Recent developments in the field of compressive sensing have led to the introduction of sparsity-promoting penalties in adaptive filtering algorithms [10, 11, 12, 13], producing zero-attracting (ZA) and reweighted zero-attracting (RZA) algorithms. These have been shown to provide faster convergence when the system in question has a degree of sparsity. This is often the case in the real systems encountered in ANC applications. To the knowledge of the authors, however, this powerful sparsity-inducing technique has not been considered with the modified filtered-x architecture used in ANC.

In this paper, sparsity-inducing techniques are incorporated into the filtered-x affine projection (FxAP) algorithm. The FxAP algorithm is selected since it is a popular algorithm for ANC applications, performing well in the presence of correlated signals, and allowing a trade-off between convergence speed and complexity to be easily controlled by varying the projection order  $K$ . A number of techniques have been introduced to improve the FxAP algorithm, including the introduction of the modified form, MFxAP [5], and adapting the system parameters over time (see [14] and references therein). This paper proposes zero-attracting and reweighted zero-attracting MFxAP algorithms. Experimental results demonstrate the superior performance of the proposed algorithms for systems with a moderate degree of sparsity.

The rest of this paper is structured as follows. In Section 2, the active noise control problem is stated and the MFxAP algorithm is briefly reviewed. In Section 3 the proposed algorithms are developed with reference to the MFxAP algorithm from which they derive. In Section 4, the results of simulation trials performed on these algorithms are presented and discussed. Section 5 presents conclusions and describes avenues of further research.

**Notation:** Throughout this paper, uppercase boldface letters will be used to denote matrices, and lowercase boldface letters to denote vectors.  $E\{\cdot\}$  denotes statistical expectation,  $\text{sgn}\{\cdot\}$  is the signum function,  $\mathbf{I}$  is an identity matrix of appropriate dimensions, and  $\|\cdot\|_p$  denotes the  $p$ th norm of a vector. All vectors are column vectors. Following convention, the term  $\ell_0$ -norm and notation  $\|\cdot\|_0$  denotes the number of non-zero elements in a vector.



**Fig. 1.** Modified filtered-x AP structure.

## 2. PROBLEM STATEMENT AND REVIEW OF THE MFXAP ALGORITHM

In this section, the problem of active noise control is explained and the MFxAP algorithm is reviewed.

In the common case of feedforward ANC, an error signal is used to adjust an adaptive filter such that its output can be ‘subtracted’ from a signal using the principle of destructive interference, and hence eliminate or significantly reduce disturbing noise. However, the electroacoustic nature of ANC systems introduces a combined *secondary path*, since each component has an associated transfer function. It was shown in [7] that this can be split into two parts, one estimated as part of the plant, and one occurring after the summing junction and affecting the error signal, here denoted  $s(n)$ . The result is a delay in the system that may cause conventional adaptive algorithms to become unstable. The FX algorithms account for this by using an estimate of the secondary path,  $\hat{s}(n)$ , to filter the input signal  $x(n)$ , so that the algorithm input is the filtered signal  $x_f(n)$ . However, FX algorithms have strict limits on step size that negatively impact upon convergence speed [8]. The MFX algorithms improve upon this convergence speed by estimating the actual error signal, but can be improved further by introducing variable parameter or sparsity-inducing techniques such as those proposed here. The derivation of the MFXAP algorithm is reviewed below.

When deriving the conventional AP algorithm, the following expression is minimised:

$$\|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 \quad \text{s.t.} \quad \mathbf{d}(n) = \mathbf{U}(n)\mathbf{w}(n+1)$$

where  $\mathbf{U}(n)$  is a regressor matrix containing past values of

$x(n)$  as follows:

$$\mathbf{U}(n) = \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-L+1) \\ x(n-1) & x(n-2) & \cdots & x(n-L) \\ \vdots & \vdots & \ddots & \vdots \\ x(n-K+1) & x(n-K) & \cdots & x(n-L-K+2) \end{bmatrix}$$

The MFxAP algorithm is well suited to ANC as it does not require that  $\mathbf{d}(n)$  be available for its derivation. Instead, we set the condition to be

$$\hat{\mathbf{d}}(n) = \mathbf{U}_f(n)\mathbf{w}(n+1) \quad (1)$$

where, as seen in Fig. 1,  $\hat{\mathbf{e}}(n) = \hat{\mathbf{d}}(n) - \hat{\mathbf{y}}(n)$ . This uses a *filtered* regressor matrix,  $\mathbf{U}_f(n)$ , containing past values of  $x_f(n)$ . Therefore, the condition states that the *estimated* output for the next iteration should equal the *estimated* desired signal for the current iteration. Then, using a vector of Lagrange multipliers,  $\lambda$ , the cost function becomes:

$$J_1(n) = \|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 + \text{Re}\{[\hat{\mathbf{d}}(n) - \mathbf{U}_f(n)\mathbf{w}(n+1)]^H \lambda\} \quad (2)$$

In order to minimise the above cost function, we equate the gradient of  $J_1(n)$  with respect to  $\mathbf{w}^*(n+1)$  to zero:

$$\frac{\partial J_1(n)}{\partial \mathbf{w}^*(n+1)} = \mathbf{w}(n+1) - \mathbf{w}(n) - \mathbf{U}_f^H(n)\underline{\lambda} = 0$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mathbf{U}_f^H(n)\underline{\lambda} \quad (3)$$

Substituting the above expression into (1), gives:

$$\hat{\mathbf{d}}(n) = \mathbf{U}_f(n)\mathbf{w}(n) + \mathbf{U}_f(n)\mathbf{U}_f^H(n)\underline{\lambda}$$

$$\hat{\mathbf{d}}(n) = \hat{\mathbf{y}}(n) + \mathbf{U}_f(n)\mathbf{U}_f^H(n)\underline{\lambda}$$

$$\hat{\mathbf{e}}(n) = \mathbf{U}_f(n)\mathbf{U}_f^H(n)\underline{\lambda}$$

$$\underline{\lambda} = [\mathbf{U}_f(n)\mathbf{U}_f^H(n)]^{-1}\hat{\mathbf{e}}(n) \quad (4)$$

Inserting (4) back into (3), and incorporating a step-size parameter,  $\mu$ , and a small positive constant,  $\delta$ , to avoid inverting a singular matrix, the MFxAP recursion is obtained:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{U}_f^H(n) [\mathbf{U}_f(n) \mathbf{U}_f^H(n) + \delta \mathbf{I}]^{-1} \hat{\mathbf{e}}(n) \quad (5)$$

### 3. PROPOSED SPARSITY-INDUCING FILTERED-X AFFINE PROJECTION ALGORITHMS

In this section, the proposed sparsity-inducing filtered-x affine projection algorithms are derived. In particular, the zero-attracting and reweighted zero-attracting strategies considered in [10, 12] are incorporated into a filtered-x structure, resulting in new algorithms for active noise control.

### 3.1. Zero-attracting MFxAP (ZA-MFxAP) algorithm

The ZA-MFxAP incorporates a sparsity-inducing penalty to attract coefficients towards zero. The obvious choice for such a penalty would be the  $\ell_0$ -norm. However, this is a discontinuous and non-convex function, making it costly and difficult to optimise mathematically. Zero-attracting algorithms use an  $\ell_1$ -norm penalty as an approximation, and the penalty is added to the cost function in (2) as follows:

$$J_2(n) = \|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 + \text{Re}\{[\hat{\mathbf{d}}(n) - \mathbf{U}_f(n)\mathbf{w}(n+1)]^H \underline{\Delta}\} + \alpha \|\mathbf{w}(n+1)\|_1 \quad (6)$$

As before, this cost function is minimised with respect to  $\mathbf{w}^*(n+1)$  and equated to zero, giving:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mathbf{U}_f^H(n)\underline{\Delta} - \alpha \text{sgn}\{\mathbf{w}(n+1)\}$$

Solving for  $\underline{\Delta}$  and incorporating  $\mu$  and  $\delta$  as before, and making the assumption that  $\text{sgn}\{\mathbf{w}(n+1)\} \approx \text{sgn}\{\mathbf{w}(n)\}$ , we obtain the ZA-MFxAP recursion:

$$\begin{aligned} \mathbf{w}(n+1) = & \mathbf{w}(n) + \mu \mathbf{U}_f^+(n) \hat{\mathbf{e}}(n) \\ & + \rho \mathbf{U}_f^+(n) \mathbf{U}_f(n) \text{sgn}\{\mathbf{w}(n)\} \\ & - \alpha \text{sgn}\{\mathbf{w}(n)\} \end{aligned} \quad (7)$$

where  $\rho = \mu\alpha$  is known as the zero-attraction strength, and  $\mathbf{U}_f^+(n) = \mathbf{U}_f^H(n)[\mathbf{U}_f(n)\mathbf{U}_f^H(n) + \delta\mathbf{I}]^{-1}$ . This is identical to (5) with the addition of two terms, controlled by  $\rho$ , which attract the coefficients towards zero.

### 3.2. Reweighted zero-attracting MFxAP (RZA-MFxAP) algorithm

The RZA-MFxAP algorithm uses a log-sum penalty in place of the  $\ell_1$ -norm, as this provides a closer approximation to the behaviour of the  $\ell_0$ -norm. Thus, the cost function becomes:

$$J_3(n) = \|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 + \text{Re}\{[\hat{\mathbf{d}}(n) - \mathbf{U}_f(n)\mathbf{w}(n+1)]^H \underline{\Delta}\} + \gamma \sum_{i=1}^L \log(1 + \epsilon |w_i(n)|) \quad (8)$$

Following a similar derivation procedure to ZA-MFxAP, the RZA-MFxAP recursion is given by:

$$\begin{aligned} \mathbf{w}(n+1) = & \mathbf{w}(n) + \mu \mathbf{U}_f^+(n) \hat{\mathbf{e}}(n) \\ & + \rho' \mathbf{U}_f^+(n) \mathbf{U}_f(n) \Psi(n) - \gamma \epsilon \Psi(n) \end{aligned} \quad (9)$$

where  $\Psi(n) = \frac{\text{sgn}\{\mathbf{w}(n)\}}{1 + \epsilon \|\mathbf{w}(n)\|}$ .

In this case the strength of the zero-attraction is controlled by  $\rho' = \mu\gamma\epsilon$  where  $\epsilon$  is known as the shrinkage magnitude. The RZA-MFxAP is more selective than the ZA-MFxAP, and attracts coefficients proportional to  $1/\epsilon$  toward zero most strongly. Therefore careful tuning of  $\epsilon$  can reduce the bias of the estimation procedure.

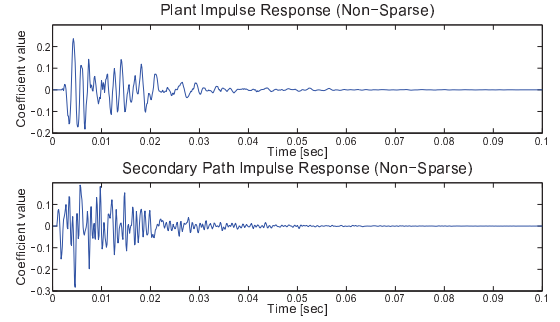


Fig. 2. Non-sparse plant and secondary paths.

## 4. RESULTS

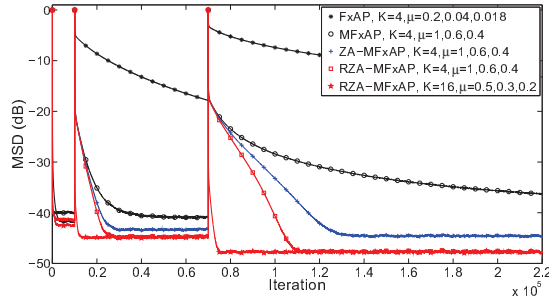
This section compares the results of simulation trials for the proposed algorithms outlined in (7) and (9) with those of the conventional FxAP and MFxAP algorithms. The results are averaged over 50 simulation trials.

Three types of primary path were used: a non-sparse path (density 785/800), illustrated in Fig. 2, a partially-sparse path formed by arbitrarily setting the majority of the coefficients in the non-sparse path to zero (density 73/800) and a sparse path in which the fourth coefficient is set to one and all remaining coefficients to zero (density 1/800). Three types of secondary path with similar densities were generated in the same way from the non-sparse secondary path in Fig. 2, with  $\hat{s}(n) = s(n)$ . A poorer estimate of  $s(n)$  is known to degrade convergence speed, although it has been shown in [15] that in rare cases  $\hat{s}(n) \neq s(n)$  may improve algorithm performance.

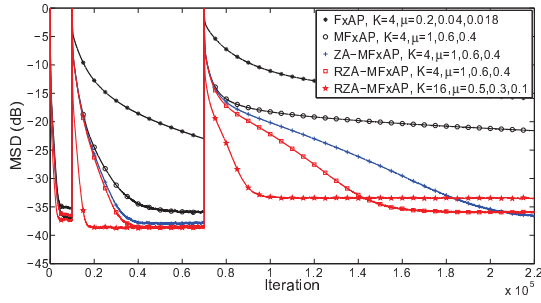
For each primary path formulation, the algorithms were run for 220,000 iterations with the secondary path set as sparse at the start of the experiment, changed to partially-sparse at iteration 10,000 and to non-sparse at iteration 70,000. These values were found to provide sufficient time for the proposed algorithms to converge. Figures 3-5 show mean-square deviation (MSD) convergence curves for each of these experiments. Each secondary path change requires that  $\mu$  and the parameter  $\epsilon$  in the RZA-MFxAP algorithm be retuned, with the requisite values given in the figures. The values  $\delta = 0.002$  and  $\rho = 0.0000001$  were found to be suitable for almost all applications, with  $\rho$  requiring retuning only for the ZAMFxAP algorithm in Fig. 5.

The first 10,000 iterations in each figure illustrate algorithm performance when the secondary path is sparse. In all cases, the convergence speeds of conventional and proposed algorithms are similar, with the higher-order RZA-MFxAP algorithms showing a slight improvement upon FxAP and MFxAP when the plant is partially sparse as in Fig. 4.

It is seen in Fig. 3 that as the secondary path density increases, the convergence speed of the FxAP and MFxAP decreases significantly. The ZA-MFxAP and RZA-MFxAP



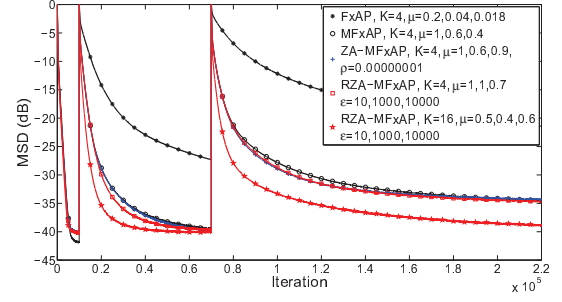
**Fig. 3.** MSD results for sparse plant ( $\rho = 0.0000001$  and  $\epsilon = 10$  unless stated otherwise).



**Fig. 4.** MSD results for partially-sparse plant ( $\rho = 0.0000001$  and  $\epsilon = 10$  unless stated otherwise).

with projection order 4 improve upon these algorithms by converging within far fewer iterations, but the higher-order RZA-MFxAP algorithm reduces convergence time to a very low number of iterations and also attains lower steady-state MSD than the other algorithms, indicating that the comparative density of the secondary path has a less significant effect on this algorithm than conventional ANC algorithms.

The performance of the algorithms when the plant is partially sparse can be seen in Fig. 4. When the secondary path is semi-sparse, the proposed algorithms show clear improvement - in terms of both MSD convergence speed and steady-state performance - over the traditional MFxAP, and vastly outperform the FxAP algorithm. As the secondary path density increases further in the third section of Fig. 4, the performance of the MFxAP is significantly degraded, whereas the proposed algorithms continue to perform far better, with the RZA-MFxAP - particularly at higher order - showing the fastest convergence. It should be noted that every algorithm initially converges at the same rate, but the RZA-MFxAP algorithms increase in convergence speed as the selective aspect begins work on coefficients proportional to  $1/\epsilon$  [10]. Steady-state differences are clearly visible between RZA-MFxAP orders in this section of Fig. 4. This is primarily due to the tuning of  $\epsilon$ , which has a significant impact upon convergence



**Fig. 5.** MSD results for non-sparse plant ( $\rho = 0.0000001$  and  $\epsilon = 10$  unless stated otherwise).

speed if reduced from the selected value. It would therefore be worthwhile to consider algorithms incorporating a variable shrinkage magnitude parameter as an improvement to the algorithms proposed here, allowing steady-state performance to be improved without negatively affecting convergence rate.

The results of running the algorithms for a non-sparse plant can be seen in Fig. 5. In this case, while the proposed algorithms continue to significantly outperform the FxAP, the performance gain over the MFxAP algorithm is less pronounced. This is because the plant is non-sparse, and therefore the sparsity-inducing technique does not give the proposed algorithms an advantage when finding an estimate. Although the difference is less notable than in previous cases, the proposed algorithms do outperform the MFxAP somewhat - particularly the RZA-MFxAP with a projection order of 16 - due to the presence of close-to-zero coefficients in the plant impulse response.

These results demonstrate that in any case where a degree of sparsity is to be expected in the primary or secondary path, the best convergence speed and steady-state error can be obtained by using the proposed RZA-MFxAP algorithm, particularly at a relatively high projection order such as 16. A higher order may be undesirable from a complexity point of view, but since the algorithms have a complexity on the order of  $L^2$  and  $K \ll L$ , the increase in complexity will be small in proportion to the performance gain. Further work might incorporate recent improvements to sparse techniques [16].

## 5. CONCLUSION

This paper has proposed adaptive algorithms for exploiting sparsity in modified filtered-x algorithms required in active noise control problems. The proposed algorithms have incorporated zero-attracting and reweighted zero-attracting strategies into filtered-x adaptive algorithms. The results of simulations have shown that the proposed techniques improve the convergence of the existing modified algorithm in the case where the primary and secondary paths exhibit a degree of sparsity.

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