STEADY-STATE ANALYSIS OF BIASED FILTERED-X ALGORITHMS FOR ADAPTIVE ROOM EQUALIZATION

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ABSTRACT

This paper provides an analysis of the steady-state behavior of two biased adaptive algorithms recently introduced for listening room compensation, the biased filtered-x normalized least mean squares (Fx-BNLMS) and the biased filtered-x improved proportionate NLMS (Fx-BIPNLMS). We give theoretical results that show that the biased algorithms can outperform the unbiased ones in terms of the mean square error, especially in low signal-to-noise ratio (SNR) scenarios. Moreover, for impulse responses exhibiting high sparseness, the improved proportionate algorithms achieve faster convergence than the standard NLMS. Thereby, the advantages of the Fx-BIPNLMS algorithm are justified theoretically in terms of the excess mean square error. Simulation results show that there is a relatively good match between theory and practice, especially for low μ values.

Index Terms— Steady-state analysis, proportionated algorithms, biased adaptive filtering, room equalization

1. INTRODUCTION

Adaptive filtering becomes an excellent tool for audio applications mainly when practical systems imply time-varying scenarios and the use of multiple loudspeakers and microphones [1, 2]. In the particular case of room equalization, massive computation requirements can be efficiently deal with iterative filters such as the adaptive filters instead of using fixed strategies. As an example, several contributions have been recently proposed in the time, frequency or wave domain [3, 4, 5, 6, 7, 8].

In this paper, and in the adaptive equalization (AE) context, we theoretically study how improved proportionate (IP) schemes combined with a biased strategy can be used to produce a good tradeoff between convergence speed and steady-state misadjustment, especially in situations with both low signal-to-noise ratios (SNR) and sparse optimal coefficient vectors. On the one hand, the IP adaptive filters can be very helpful to improve the filter convergence with a high or unknown degree of sparsity [9, 10, 11, 12]. To alleviate this problem in active noise control (ANC) applications, the filtered-x IP normalized least mean squares (Fx-IPNLMS) algorithm was presented in [13]. On the other hand, in [14] a simple scheme that biased the adaptive filter weights towards zero was proposed in order to reduce the steady-state mean square error (MSE) of adaptive filters for system identification. More recently, the biased Fx-IPNLMS (Fx-BIPNLMS) algorithm was proposed as a way to take advantage of both strategies, the IP and the biased scheme [3]. The biased filtered-x NLMS (Fx-BNLMS) algorithm was also developed and compared with the Fx-BIPNLMS, showing that the IP scheme can offer improved convergence capabilities. Although several simulations were provided in [3], we further study the performance of both biased filtered-x algorithms through theoretical analysis.

The aim of this contribution is to apply and adapt the approaches of [10, 14] to the steady-state analysis of two biased algorithms suitable for AE, see Fig. 1. Even though we follow the same methodology of [10] and [14], AE applications require ad hoc treatment, similar to [15] for ANC. This paper is organized as follows: In the next section we describe the Fx-BNLMS and the Fx-BIPNMLS algorithms for AE. The steady-state analysis of the adaptive algorithms is carried out in Sec. 3. Several examples that validate the analysis are provided in Sec. 4. Finally, Sec. 5 presents the main conclusions of the work.

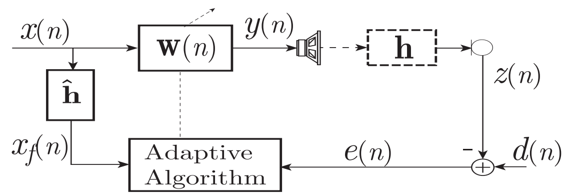

Fig. 1. Block diagram of an AE system.

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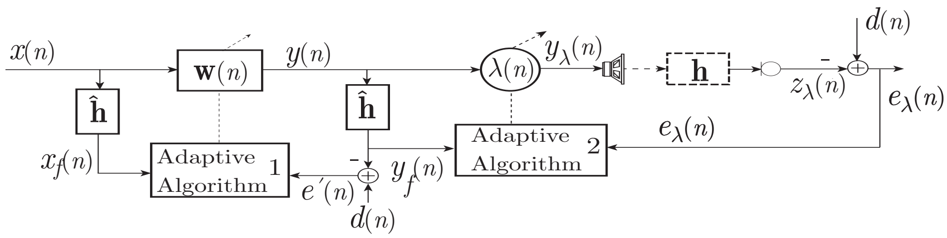

Fig. 2. Block diagram of a biased AE system.

2. BIASED ALGORITHMS FOR ADAPTIVE EQUALIZATION

An extended description of the Fx-IPNLMS for AE can be followed in [3]. In contrast to the Fx-NLMS which distributes the adaptation energy equally among all the filter coefficients, the Fx-IPNLMS assigns a different adaptation speed $\mu_l(n)$ to each coefficient according to,

$$w_l(n) = w_l(n-1)+\mu_l(n)e(n)x_f(n-l),\ l = 0, \ldots, L_w-1, \tag{1}$$

$$\mu_l(n) = \frac{\mu g_l(n)}{\delta + \sum_{k=0}^{L_w-1} g_k(n)x_f^2(n-k)}, \tag{2}$$

$$g_l(n) = (1-\kappa)\frac{1}{2L_w} + (1+\kappa)\frac{|w_l(n)|}{\varepsilon + 2\sum_k |w_k(n)|}, \tag{3}$$

where $w_l(n)$ is the lth coefficient of the L_w-length vector $\mathbf{w}(n)$, and $x_f(n)$ corresponds to the input signal $x(n)$ filtered through the L_h-length estimated impulse response $\hat{\mathbf{h}}$. Meanwhile, $g_l(n)$ is the adaptation gain factor of the lth filter coefficient and $\kappa \in [-1, 1]$ arranges from the NLMS algorithm ($\kappa = -1$) to $\kappa = 1$ for the proportionate NLMS algorithm.

The objective is to estimate the L_w-length optimal coefficient vector $\mathbf{w}_0$, such that the desired signal $d(n)$ will be the input signal with a suitable source-microphone delay $x(n-\tau)$. Thus, the vector $\mathbf{w}_0$ will correspond to the inverse of the acoustic channel, $\mathbf{w}_0 \otimes \mathbf{h} = \delta(n-\tau)$, where $\otimes$ denotes the discrete linear convolution. Therefore, the desired signal can be written as $d(n) = \mathbf{w}_0^T\mathbf{x}_f(n)$, where $\mathbf{x}_f(n)$ is a L_w vector with the input signal $x(n)$ filtered by the estimated impulse response $\hat{\mathbf{h}}$. However, this result is not achieved in practice and it is more realistic to use $d(n) = \mathbf{w}_0^T\mathbf{x}_f(n) + r(n)$, being $r(n)$ a Gaussian noise of zero mean and σ_r^2 variance, uncorrelated with the input signal.

The error signal $e(n)$ can be expressed as a function of the *a priori* error $e_a(n) = \widetilde{\mathbf{w}}^T(n-1)\mathbf{x}_f(n)$ and the coefficient error vector $\widetilde{\mathbf{w}}(n) = [\mathbf{w}_0 - \mathbf{w}(n-1)]$. Similarly to the methodology applied in [15] it holds,

$$\begin{aligned} e(n) &= d(n) - z(n) = \mathbf{w}_0^T\mathbf{x}_f(n) + r(n) - \mathbf{w}^T(n-1)\mathbf{x}_f(n) \\ &= e_a(n) + r(n). \end{aligned} \tag{4}$$

Regarding the biased schemes, the biased filtered-x AE system is explained in detail in [3] and illustrated in Fig. 2. The output error of the biased system $e_\lambda(n)$ is defined as $e_\lambda(n) = e_{a_\lambda}(n) + r(n)$, where the *a priori* error of the biased scheme $e_{a_\lambda}(n)$ can be written as a term of the previously defined unbiased *a priori* error $e_a(n)$ similarly to [14],

$$e_{a_\lambda}(n) = \lambda(n)e_a(n) + [1-\lambda(n)]\left[\mathbf{w}_0^T\mathbf{x}_f(n)\right]. \tag{5}$$

3. STEADY-STATE ANALYSIS

To analyze the steady-state performance of both adaptive filters, the Fx-BIPNLMS and the Fx-BNLMS, we use the excess mean square error, EMSE, defined as

$$\text{EMSE} = J_{ex,\lambda}(n) = E\left\{|e_{a_\lambda}(n)|^2\right\}. \tag{6}$$

Squaring $e_{a_\lambda}(n)$ in (5) and taking expectations as in [14], (6) can be rewritten using the EMSE term of the unbiased version $J_{ex}(n)$. Assuming that $\mathbf{w}(n)$ and $x(n)$ are uncorrelated, and in steady state $E\{\mathbf{w}(n)\} = \mathbf{w}_0$ as $n \longrightarrow \infty$, we get

$$J_{ex,\lambda}(\infty) = \lambda^2(\infty)J_{ex}(\infty)+[1-\lambda(\infty)]^2\,\mathbf{w}_0^T\mathbf{R}_{x_f}\mathbf{w}_0, \tag{7}$$

where $\mathbf{R}_{x_f}$ corresponds to $\mathbf{R}_{x_f} = E\left\{\mathbf{x}_f(n)\mathbf{x}_f^T(n)\right\}$.

Furthermore, to obtain the optimal value in steady state of the scaling factor $\lambda(\infty)$, the previous equation is derived with respect to λ and equaled to zero,

$$\lambda(\infty) = \frac{1}{1 + \frac{J_{ex}(\infty)}{\mathbf{w}_0^T\mathbf{R}_{x_f}\mathbf{w}_0}}. \tag{8}$$

For equalization applications, the use of the filtered-x schemes is the main difference with respect to that of [14]. In this case, the existence of the estimated channel response $\hat{\mathbf{h}}$ in the term $\mathbf{w}_0^T\mathbf{R}_{x_f}\mathbf{w}_0$ allows to suitably manipulate it as,

$$\begin{aligned} \mathbf{w}_0^T\mathbf{R}_{x_f}\mathbf{w}_0 &= \mathbf{w}_0^T E\left\{\hat{\mathbf{H}}^T\mathbf{x}(n)\mathbf{x}^T(n)\hat{\mathbf{H}}\right\}\mathbf{w}_0 \\ &= \left[\mathbf{w}_0^T\hat{\mathbf{H}}^T\right] E\left\{\mathbf{x}(n)\mathbf{x}^T(n)\right\}\left[\hat{\mathbf{H}}\mathbf{w}_0\right] \\ &= \left[\mathbf{w}_0^T\hat{\mathbf{H}}^T\right]\mathbf{R}_x\left[\hat{\mathbf{H}}\mathbf{w}_0\right], \end{aligned} \tag{9}$$

where $\mathbf{x}(n)$ is a vector with the last $L_w + L_h - 1$ samples of the input signal $x(n)$ and $\hat{\mathbf{H}}$ corresponds to the discrete linear convolution of the estimated channel response $\hat{\mathbf{h}}$, expressed in matrix form as a Toeplitz matrix of dimensions $(L_w + L_h - 1) \times L_w$,

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{h}(0) & 0 & & 0 \\ \hat{h}(1) & \hat{h}(0) & & 0 \\ \vdots & h(1) & \ddots & 0 \\ \hat{h}(L_h-1) & \vdots & & \hat{h}(0) \\ 0 & \hat{h}(L_h-1) & & \hat{h}(1) \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \hat{h}(L_h-1) \end{bmatrix} \quad (10)$$

If the equalization is perfectly achieved, the matrix vector product $\left[\hat{\mathbf{H}}\mathbf{w}_0\right]$ corresponds to a $(L_w + L_h - 1) \times 1$ column vector containing the delta function $\delta(n-\tau)$. Therefore, the term $\mathbf{w}_0^T \mathbf{R}_{x_f} \mathbf{w}_0$ can be simplified as

$$\mathbf{w}_0^T \mathbf{R}_{x_f} \mathbf{w}_0 = \sigma_x^2, \quad (11)$$

where σ_x^2 is the variance of the input signal.

Then, substituting (11) in both (7) and (8), we find that

$$\boxed{J_{ex,\lambda}(\infty) = \lambda^2(\infty) J_{ex}(\infty) + \left[1-\lambda(\infty)\right]^2 \sigma_x^2,} \quad (12)$$

$$\lambda(\infty) = \frac{1}{1 + \frac{J_{ex}(\infty)}{\sigma_x^2}}. \quad (13)$$

For the Fx-NLMS scheme and following the approach shown in [15] but in the room equalization context, a recursion for $\|\mathbf{w}(n)\|^2$ based on the energy conservation relation can be derived, where $\|\cdot\|$ denotes the Euclidean norm. This results in the EMSE for the Fx-NLMS algorithm,

$$J_{ex}(\infty) = \lim_{n\to\infty} E\left\{|e_a(n)|^2\right\} = \frac{\mu\sigma_r^2}{2-\mu}. \quad (14)$$

For the Fx-IPNLMS algorithm, we follow the approach of [10]. As before, the difference lies in the use of a filtered-x version of the input signal. Thus, doing some approximations in (2), (1) becomes,

$$w_l(n) = w_l(n-1) + \frac{\mu g_l(n)}{\sigma_{x_f}^2} e(n) x_f(n-l), \quad (15)$$

where $\sigma_{x_f}^2$ is the variance of the input signal $x(n)$ filtered through the estimated channel response $\hat{\mathbf{h}}$.

Finally, the EMSE for the Fx-IPNLMS is given by,

$$J_{ex}(\infty) = \sigma_{x_f}^2 \sum_{l=1}^{L_w} E\left\{\widetilde{w}_l^2(\infty)\right\} = \mu\sigma_r^2(n) \sum_{l=1}^{L_w} \frac{g_l(\infty)}{2-\mu g_l(\infty)}, \quad (16)$$

being $g_l(\infty) = (1-\kappa)\frac{1}{2L_w} + (1+\kappa)\frac{|w_{0,l}|}{\varepsilon + 2\sum_k |w_{0,k}|}$.

Note that for AE applications where at least a hundred of filter coefficients are needed for the inverse filter and $\sum_{l=1}^{L_w} g_l(n) \approx 1$, we can approximate $\sum_{l=1}^{L_w} \frac{g_l(\infty)}{2-\mu g_l(\infty)} = \sum_{l=1}^{L_w} \frac{1}{[2/g_l(\infty)]-\mu} \cong \sum_{l=1}^{L_w} g_l(\infty)/2 = 1/2$, as $2/g_l(\infty) \gg \mu$. Thus, (16) does not depend on the κ-value,

$$J_{ex}(\infty) = \frac{\mu\sigma_r^2}{2}. \quad (17)$$

Finally, (14) and (16) can be replaced in (12) to get the steady-state EMSE of the considered biased algorithms.

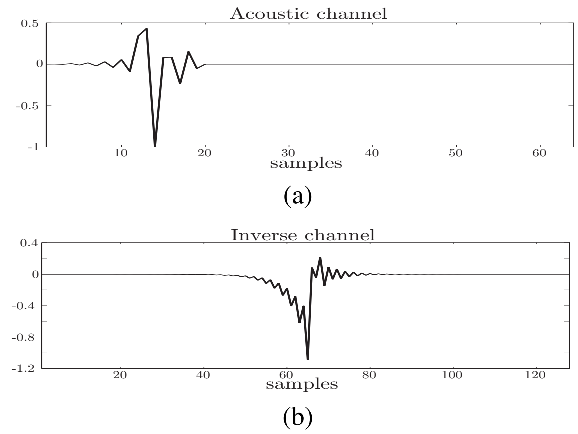

Fig. 3. (a) Acoustic channel; (b) Inverse of the channel.

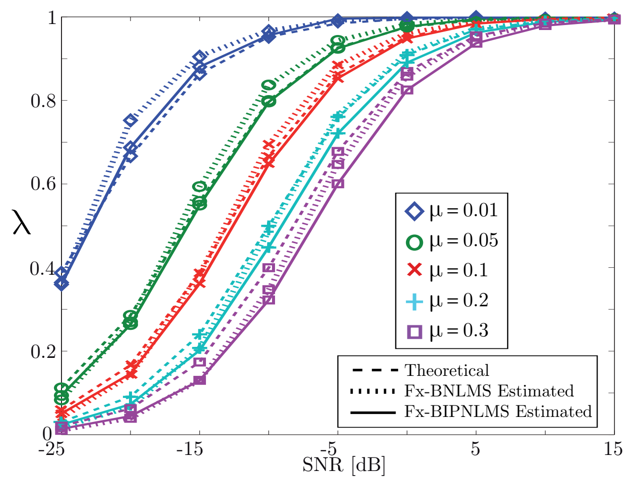

Fig. 4. Theoretical (dashed line) and estimated (solid line for the Fx-BIPNLMS and dotted line for the Fx-BNLMS algorithms) $\lambda(\infty)$ for different μ and SNR values.

4. SIMULATION RESULTS

In this section, theoretical predicted values are compared to the averaged estimated ones for the EMSE expressions and the scaling factor $\lambda(n)$ in steady state, for both the Fx-NLMS and the Fx-IPNLMS schemes and their biased versions.

Considering that the performance of the models is not dependent on the acoustic channel length considered, a 64-samples channel is used for simplicity. This channel has been measured in a real listening room [3], but artificially modified to get a quasi-sparse inverse impulse response, see Fig. 3. The optimal filter $\mathbf{w}_0$ in Fig. 3 (b) has been computed as the inverse filter of the acoustic channel showed in Fig. 3 (a) by using the least squares error method (LSE) [16]. A length twice the length of the acoustic channel and a delay $\tau = 78$ samples have been considered. In this context, the length used allows to obtain a good approximation without highly increasing the complexity of the algorithms.

Fig. 4 compares the theoretical results (in dashed line) for the scaling factor at steady state, $\lambda(\infty)$ in (13), with the

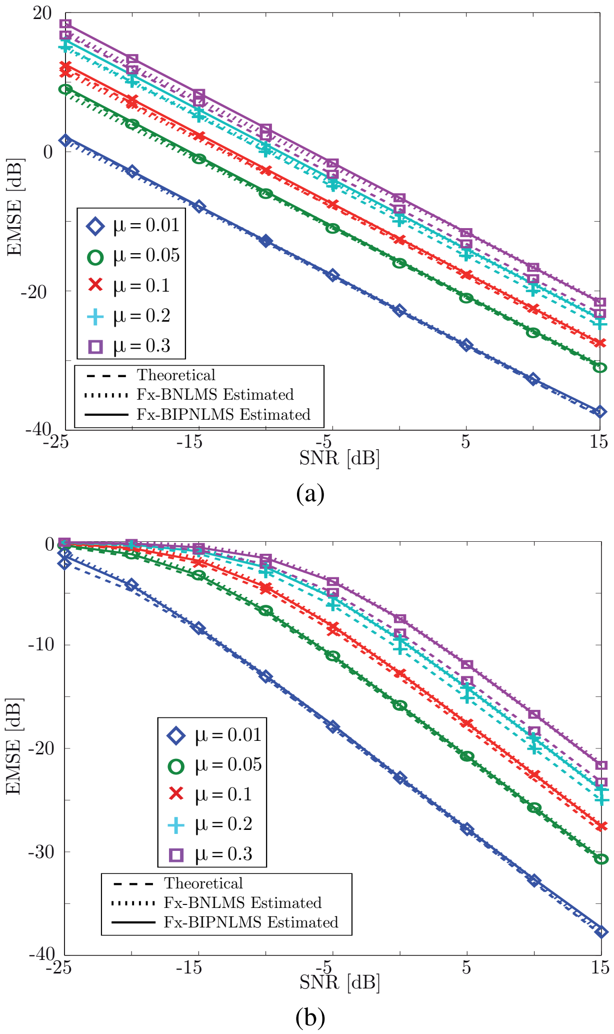

Fig. 5. Estimated (solid line for the IP type algorithms and dotted line for the NLMS ones) and theoretical (dashed line) EMSE curves in steady state:(a) For their unbiased versions $J_{ex}(\infty)$ [Eq. (16)]; and (b) for the biased ones $J_{ex,\lambda}(\infty)$ [Eq. (12)].

averaged estimated ones (in solid line for the Fx-BIPNMLS schemes and in dotted line for the Fx-BNLMS ones) for different values of both SNR and μ. For the improved proportionated algorithms, $\kappa = -0.5$ has been chosen as recommended in [9]. Each curve representes a different μ value and for various SNR values along x-axis, keeping constant $\sigma_x^2 = 1$ and varying σ_r^2. Fig. 4 shows that for high values of SNR the scaling factor is close to 1, and from (12) $J_{ex,\lambda}(\infty) = J_{ex}(\infty)$. For low SNR and high μ values $\lambda(\infty)$ tends to 0. It can be observed that the estimated results for the Fx-BIPNLMS agree with the theoretical ones especially for low μ values.

As was developed in the theoretical analysis, the EMSE expression (16) can be approximated by (17) providing the same theoretical results of $J_{ex}(\infty)$ for both the Fx-IPNLMS and the Fx-NLMS algorithms. Fig. 5 (a) shows the EMSE

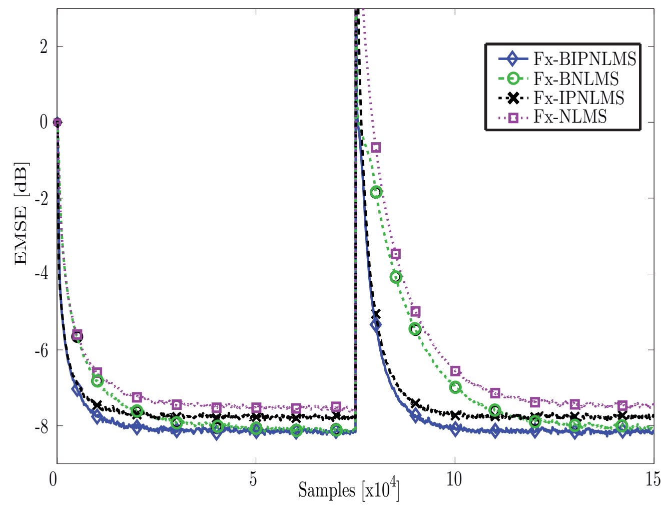

Fig. 6. EMSE evolution with time-varying channel for the Fx-IPNLMS and Fx-NLMS algorithms and their biased versions.

value in dB for the unbiased algorithms, $J_{ex}(\infty)$ in (16). Estimated results (solid line for the Fx-IPNLMS and dotted line for the Fx-NLMS algorithms) fall close to the theoretical ones (dashed line), their values increase with σ_r^2 and μ, and are almost similar to the theoretical ones. Fig. 5 (b) shows the $J_{ex,\lambda}(\infty)$ in (12) for the Fx-BIPNLMS and the Fx-BNLMS algorithms. Also the simulated results agree with the theoretical ones, but are upper limited by 0dB as the SNR decreases. As in Fig. 4, a worst behavior is obtained for high μ values. Although perfect secondary path estimates have been considered in the present simulations, the derived models predict quite accurately the simulated results with not very high modelling errors.

The second experiment shows the ability of the biased IP scheme to improve the convergence performance of the normalized version. Fig. 6 shows the EMSE evolution for the different algorithms with $\mu = 0.1$ and a low value of SNR$= -5$dB, thus the biased versions achieve lower EMSE values. Furthermore, after $75,000$ samples the channel $\mathbf{h}$ slightly changes and thus its inverse filter $\mathbf{w}_0$, showing the ability of the adaptive algorithms to follow system variations.

5. CONCLUSIONS

The steady-state analysis of two biased filtered-x algorithms, the Fx-BIPNLMS and Fx-BNLMS algorithms, has been presented. The novelty of this work is the theoretical model of the biased schemes applied to adaptive equalization that involves the use of filtered-x structures.

Simulation results show a good match between the theoretical expressions and the estimated values, especially for low μ values. The biased algorithms outperform their unbiased versions for low SNR values. Moreover, the Fx-BIPNLMS algorithm exhibits a better convergence performance than the Fx-BNLMS implementation.

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