COMPRESSED SENSING FOR MAGNETIC RESONANCE IMAGES WITH PHASE VARIATIONS

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ABSTRACT

The application of compressed sensing (CS) to MRI has the potential to significantly reduce scan time. However, the quality of reconstructed images will be degraded when the MR images have strong phase variations. In the present paper, we propose a new CS method that is easy to implement and robust to phase variations on MR images. When the signal trajectory in *k*-space is symmetrical with respect to its origin, the k-space signal corresponding to the real and imaginary parts of the complex image can be synthesized independently by restricting the *k*-space signal to an even function or an odd function. The proposed method involves random but symmetrical *k*-space acquisition and independent reconstruction of the real and imaginary parts of images using the real-valued constraint. Several numerical experiments demonstrate that the proposed CS method provides a better quality of images compared to the other methods.

Index Terms— complex image, phase, MRI, sparse

1. INTRODUCTION

ThE application of compressed sensing (CS) to MRI has the potential for significant scan time reductions and therefore a large number of researches on CS have been reported and demonstrated so far[1,2]. In general, there exists a phase variation on the MR images due to inhomogeneous of static magnetic field or the difference of susceptibility in the human tissue. Large phase variations are sometimes appeared on the reconstructed images and they are locally enhanced intentionally to observe the blood flow or diffusion in the fascicles of nerve fiber. When MR image have rapid spatial phase variations, the quality of images in CS reconstruction will be degraded because the sparsity of MR images will be decreased due to fluctuation of its amplitude. CS use in applications with rapid spatial phase variations is challenging.

To meet this problem an iterative reconstruction technique with separate magnitude and phase regularization was proposed [3]. The regularizer for phase used in the cost function sometime fails in handle big jumps in the wrapped phase map. The problem lies in the point that it is not easy to estimate the rapid spatial phase variations with interleaved signal.

In this paper, we present an improved CS method which don't require the estimation of spatial phase variation and have the possibility to be applied to many CS reconstruction algorithms. When the signal trajectory on the *k*-space has a symmetrical relation with reference to the origin of *k*-space, the signal corresponding to the real and imaginary part of complex image can be reconstructed independently by restricting the real part or imaginary part of *k*-space signal to even function or odd function, respectively.

The application of CS to MR acquisition in Cartesian trajectory requires *k*-space random sampling for phase-encoding direction. Proposed CS method involves random but symmetrical *k*-space sampling, which allows the reconstruction of real- or imaginary-part of MR complex images independently, which is very easy to implement and can be applicable to many iterative reconstruction technique.

Several numerical experiments demonstrated that the proposed CS method provides a better quality of images compared to the other methods such as complex-value reconstruction using complex sparsifying transform function or reconstruction using estimated phase function. In addition, we compare the FREBAS transform[3] with wavelet transform as sparsifying transform function in the CS reconstruction procedure.

2. METHOD

Let the observed MR signal and spin density distribution be s(**k**) and $\rho(\mathbf{x})$, respectively, where **k** is a *k*-space vector and **x** is a space vector. The MR signal obtained in conventional Fourier transform imaging is given by Eq. (1),

$$s(\mathbf{k}) = \int \rho(\mathbf{x}) e^{-j\phi(\mathbf{x})} e^{-j(\mathbf{k}\cdot\mathbf{x})} d\mathbf{x} = \mathbf{F}\left[\rho(\mathbf{x}) e^{-j\phi(\mathbf{x})}\right] \tag{1}$$

where $\phi(\mathbf{x})$ is the function of phase variation due to imperfect of MRI equipment and the difference of susceptibility in the human tissue, and **F** is the operator of Fourier transform. Image reconstruction can be performed by applying the inverse Fourier transform to the signal **s**(**k**).

$$\rho(\mathbf{x}) e^{-j\phi(\mathbf{x})} = \mathbf{F}^{-1}\left[s(\mathbf{k})\right] \tag{2}$$

The real and imaginary-parts of complex image $\rho(\mathbf{x})\exp\{-j\phi(\mathbf{x})\}$ can be written as Eq.(3) and (4).

$$\mathbf{F}\left[\mathrm{Re}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}\right]\right]=\frac{1}{2}\mathbf{F}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}+\rho(\mathbf{x})e^{j\phi(\mathbf{x})}\right]$$
$$=\frac{1}{2}\left\{s(\mathbf{k})+s(-\mathbf{k})^{*}\right\} \quad (3)$$

$$\mathbf{F}\left[\mathrm{Im}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}\right]\right]=-\frac{j}{2}\mathbf{F}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}-\rho(\mathbf{x})e^{j\phi(\mathbf{x})}\right]$$
$$=-\frac{j}{2}\left\{s(\mathbf{k})-s(-\mathbf{k})^{*}\right\} \quad (4)$$

If a symmetrical relation with respect to the origin of k-space is provided, as shown in Eq. (3) and (4), then the real and imaginary parts of complex images can be reconstructed independently of each other. In the present paper, we focus on Cartesian grid sampling, which is the most widely used technique. In general, random point sampling is adopted for the phase encoding direction, because scan time reduction is exactly proportional to the under-sampling factor. In the proposed method, the sampling trajectory for the phase encoding direction is set so as to be random but is symmetrical with respect to the origin of k-space, which makes the calculation of Eq. (3) and (4) possible using the acquired under-sampled signal data.

According to the CS theory, a signal ρ with a sparse representation in the basis Ψ as $\breve{\rho}=\Psi\rho$, can be recovered from the compressed measurements $s=\Phi\rho$, where Φ is an M x N measurement matrix (M<<N), if the Φ and Ψ are incoherent.

$$\mathbf{s}=\Phi\rho=\Phi(\Psi^{-1}\breve{\rho}),\qquad \|\rho\|_0=\mathrm{M}<<\mathrm{N} \quad (5)$$

The image is reconstructed from the under-sampled k-space data by solving the nonlinear optimization problem: minimize $\|\Psi\rho\|_1$ subject to $\|\mathbf{s}-\Phi\Psi^{-1}\breve{\rho}\|_2<\varepsilon$, where ε is a small constant.

Minimizing $\|\Psi\rho\|_1$, we use the IST based accelerated reconstruction technique, FISTA[4].

Let s (**k**) be the acquired MR signal data. Then, theoretically, CS reconstruction using $\mathbf{s}_r(\mathbf{k})=1/2\{\mathbf{s}(\mathbf{k})+\mathbf{s}(-\mathbf{k})^*\}$ instead of **s**, gives the real part of the complex image $\rho_r(\mathbf{x})$. Therefore, we can use the rather strong constraint that the image obtained in this method is a real-valued function in each iteration procedure, which can improve the quality of the resultant image. CS reconstruction using $\mathbf{s}_i(\mathbf{k})=-j/2\{\mathbf{s}(\mathbf{k})-\mathbf{s}(-\mathbf{k})^*\}$ gives the imaginary part of the complex image $\rho_i(\mathbf{x})$.

$$\mathbf{s}_r(\mathrm{k})=\frac{1}{2}\left\{\mathbf{s}(\mathbf{k})+\mathbf{s}(-\mathbf{k})^{*}\right\}\rightarrow\ \text{real-part of } \rho(\mathbf{x})$$

$$\mathbf{s}_i(\mathbf{k})=-\frac{j}{2}\left\{\mathbf{s}(\mathbf{k})-\mathbf{s}(-\mathbf{k})^{*}\right\}\rightarrow\ \text{imaginary-part of } \rho(\mathbf{x})$$

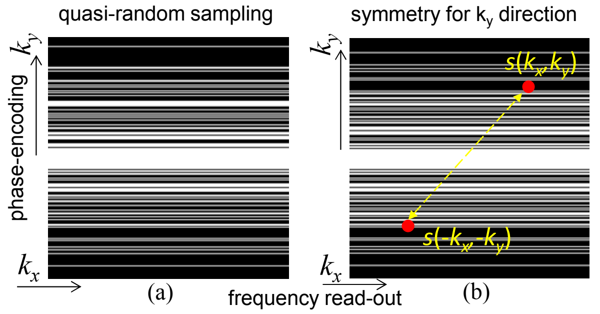

Fig. 1 Sampling trajectory in k-space for 35% signal case. Vertical direction is the phase-encoding direction. White lines show the trajectory on which signals are acquired. (a) quasi-random trajectory, (b) quasi-random but symmetrical for phase-encoding direction to utilize the proposed method

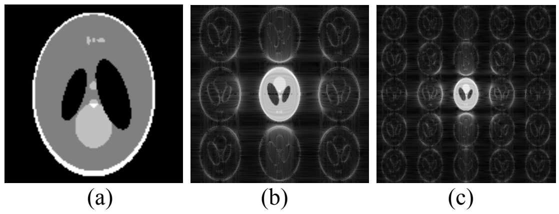

Fig. 2 Examples of FREBAS transform (a) input image, (b), (c) scaling parameter D=3 and 5, respectively.

Finally, we can obtain complex images by combining real and imaginary parts of images as $\rho(\mathbf{x})=\rho_r(\mathbf{x})+j\rho_i(\mathbf{x})$.

3. EXPERIMENTS

MR normal volunteer images were collected using a Toshiba 1.5T MRI scanner. Flow-sensitive black blood images were acquired in order to obtain images that have locally varying phase distortions due to blood flow, as well as gently varying distortions due to static field inhomogeneities (TE/TR = 40/50 ms, 256×256 matrix, slice thickness: 1.5 mm×50 slices). The signal for the phase encoding direction, except for the central region, is randomly selected in order to simulate a given reduction factor, as shown in Fig. 1(a). Figure 1(b) shows the proposed CS trajectory in which echo signals were randomly selected but is symmetrical with respect to its origin. Reconstruction was performed using an iterative soft thresholding based algorithm, FISTA[4]. We used the Fresnel transformed domain band split transform (FREBAS)[3] shown in Fig.2 as a sparsifying function, which is a kind of directional image decomposition algorithm. FREBAS can be interpreted as analysis of the image by convolution of the image data with sinc functions having different modulation indices. Though FREBAS can be described as a type of convolution integral, the actual calculation executes the convolution in the Fourier domain, so the calculation cost is not so high. To improve the incoherency between the measurement matrix and sparsifying function, multi-step FREBAS domain

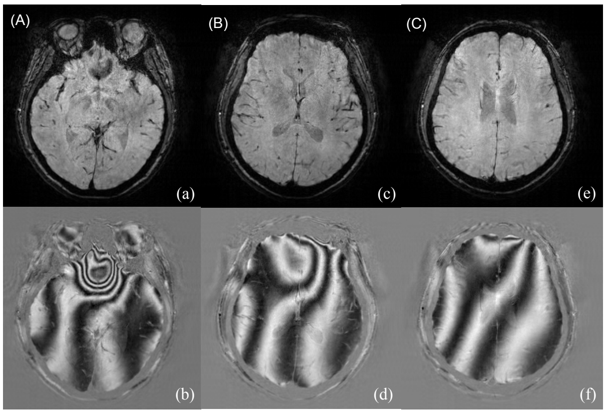

Fig. 3 Three types of MR images with phase variation used in the CS reconstruction experiments.

thresholding [5,6] is adopted, and the scaling parameters of FREBAS used in CS reconstructions are chosen 6, 9, respectively.

Figure 4 shows the comparison of peak-signal-to-noise ratio (PSNR) and structural similarity index (SSIM)[7] for 20 phase varied images between the proposed method (SymCS) and conventional method (CS-cmplx) using complex FREBAS transform. SSIM is framework for quality assessment based on the degradation of structural information. Since the FREBAS transform is a complex-valued transform and therefore complex images can be directly transformed to sparsified space, we attempted to reconstruct images without the use of the real-valued constraint or phase correction using quasi-random sampling shown in Fig.1(a) (CS-cmplx). We also attempted to reconstruct images in proposed method using a real-valued wavelet transform instead of FREBAS transform (SymCS-W). For wavelet-based CS reconstruction, 4-level dyadic decomposition with the Daubechies 4 wavelet was used. Figure 4 and 5 indicate that the real-valued wavelet transform is also applicable to the proposed method. Improved PSNR and SSIM are obtained in proposed method compared to other methods.

We compared the reconstructed images for the image shown in Fig. 3. For comparison, CS reconstruction using the real-valued constraint after correcting the phase variation using the estimated phase function that is provided by the central region of the *k*-space signal was performed (CS-PC). The best performances were obtained in the proposed method using FREBAS transform. Obtained images using wavelet-based CS tend to suffer from over-smoothing and loss of details of the subject. The directional selectivity of the wavelet transform involves only four components, (high, high), (high, low), (low, high) and (low, low), whereas FREBAS has many directional feature-tracking functions. This higher degree of directional representation contributes to the superior images in the proposed CS reconstruction method.

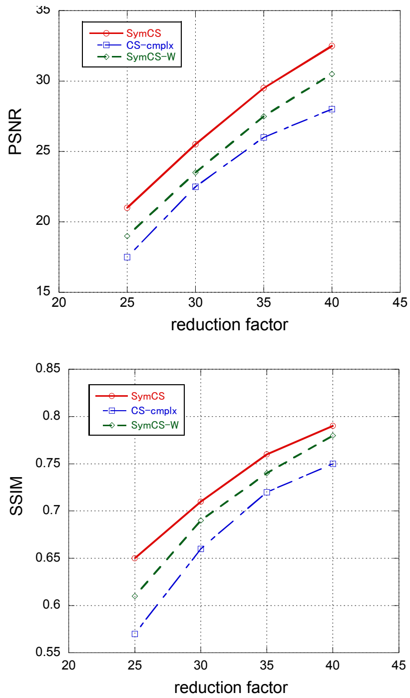

Fig. 4 Comparison of averaged PSNR and SSIM using 20 MR images between SymCS, CS-cmplx and SymCS-W.

Symmetrical signal acquisition reduces the randomness of sampling and therefore reduces the mutual incoherence between the sampling operator and the Fourier transform operator. However, simple reconstruction scheme which does not require the estimation of spatial phase variation in the image reconstruction formula contributes to smaller reconstruction era due to the presumption error of phase variation. In addition, the use of the real-valued constraint effectively remove the error components of the images which form the imaginary-part of temporally reconstructed images and therefore, the resultant images have improved PSNR and smaller artifacts.

4. CONCLUSIONS

A new CS technique that is robust to phase variations on MR images is proposed. The proposed method does not require the estimation of phase variation function and can reconstruct the real and imaginary parts of images independently using the simple iterative thresholding algorithm. Several numerical experiments demonstrated that the proposed CS method provides better-quality images for the images having rather strong phase variation compared to the methods using a complex-valued sparsifying transform function or reconstruction with phase correction by applying estimated phase functions to the images.

5. ACKNOWLEDGMENT

We would like to thank T. Kimura for providing the MR images.

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	SymCS (proposed)	SymCS-W	CS-cmplx	CS-PC
Phase variation	Yes	Yes	Yes	Yes
sparsifying func.	FREBAS	wavelet	FREBAS	FREBAS
signal trajectory	symmetrical	symmetrical	random	random
Real-value constraint	Yes	Yes	No	Yes
Phase correction	No	No	No	Yes

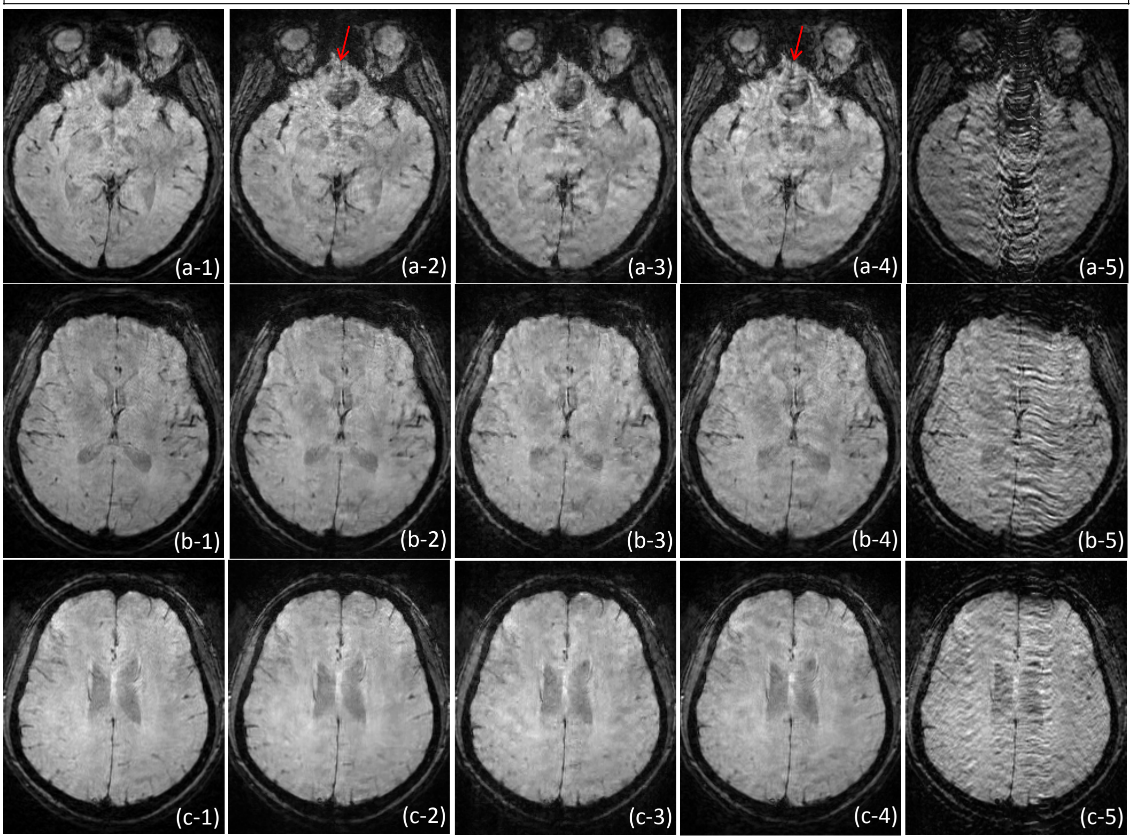

Fig. 5 Comparison of reconstructed images with other reconstruction methods; SymCS is a CS using symmetrical *k*-space trajectory and FREBAS transform, SymCS-W is similar to SymCS but use wavelet transform, CS-cmplx uses complex-valued FREBAS transform but is no use of real-value constraint with random trajectory, CS-PC adopts a phase correction using an estimated phase distribution. (a-1),(b-1),(c-1) are the fully-scanned images.