MAXIMUM LIKELIHOOD SNR ESTIMATION OVER TIME-VARYING FLAT-FADING SIMO CHANNELS

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ABSTRACT

In this paper, we propose a new signal-to-noise-ratio (SNR) maximum likelihood (ML) estimator over time-varying single-input multiple-output (SIMO) channels, for both data-aided (DA) and non-data-aided (NDA) cases. Unlike the classical techniques which assume the channel to be slowly time-varying and, therefore, considered as constant during the observation period, we address the more challenging problem of *instantaneous* SNR estimation over fast time-varying channels. The channel variations are locally tracked using a polynomial-in-time expansion. In the DA scenario, the ML estimator is developed in closed-form expression. In the NDA scenario, however, the ML estimates of the per-antenna SNRs are obtained iteratively, with very few iterations, using the expectation-maximization (EM) procedure. Our estimator is able to accurately estimate the *instantaneous* SNRs over a wide range of average SNR. We show through extensive Monte-Carlo simulations that the new estimator outperforms previously developed solutions.

1. INTRODUCTION

Many modern wireless communication systems require an accurate estimation of the signal-to-noise ratio (SNR) at the receiver. In fact, the *a priori* knowledge of the environment fading conditions is needed if the system is intended to operate at its optimal performance using various adaptive schemes. For instance, the information about the SNR is required in power control strategies, adaptive modulation and coding schemes as well as to provide the channel quality information for handoff schemes [1-3]. Roughly speaking, SNR estimation techniques can be mainly divided into two broad categories: data-aided (DA) techniques that rely during the estimation process on *a priori* known transmitted symbols (pilot sequences), and non-data-aided (NDA) techniques where the estimation process is applied blindly using completely unknown transmitted data.

In principle, DA approaches are often expected to provide more accurate estimates when the intended parameter is constant. However, in our case, the *instantaneous* SNR is not a constant parameter due to fast channel time-variations. Therefore, estimating it from pilot symbols only, that are usually placed far apart in time, results in inaccurate estimates. This is because the receiver is simply missing the details of channel variations between the pilot positions. Thus, estimating the channel coefficients at non-pilot positions using NDA approaches provides the receiver with stronger channel tracking capabilities. During the last two decades, SNR estimation has gained a great attention but most of the derived techniques were designed for single-input single-output (SISO) systems under constant channels [4, 5]. These include the fourth-order and sixth-order moment-based approaches of [4, 6] and [7], respectively, and the ML approach of [10]. More recently, there has been an emerging interest in estimating the SNR under spatial diversity. Indeed, a SIMO SNR estimator which exploits the fourth-order cross-moments between the antenna elements has been proposed in [9, 8] and an ML approach has been also introduced in [11-13].

Yet, all the aforementioned techniques (both moment-based and ML-based ones) were primarily designed for constant channels and their performances increasingly degrade with faster channel time variations related to the user's mobility [14]. Typically, current and future generation systems such as LTE, LTE-Advanced and beyond are expected to support communications at velocities that can reach 500 Km/h [15]. To the best of our knowledge, the only work that has so far considered the NDA ML SNR estimation under time-varying channels is [16], but again in the traditional case of SISO systems. As far as SIMO configurations are concerned, the only work that considered time-varying channels is [17] where a least-square (LS)-based approach was developed, relying on detected data in a decision-directed (DD) scheme.

In this paper we tackle, for the first time, the problem of ML *instantaneous* SNR estimation over time-varying SIMO channels, both for DA and NDA schemes. The channel is locally approximated using polynomial-in-time expension with few unknown coefficients. In contrast to the DA scenario where the ML estimator is derived in closed-form, the presence of the unknown symbols in the NDA case results in a very complicated likelihood function whose analytical maximization is intractable. Therefore, we resort to a more elaborate solution using the EM concept. We develop thereby an iterative technique that is able to converge to the *exact* NDA ML estimates within very few iterations. Simulation results show the distinct performance advantage offered by exploiting spatial diversity.

The remainder of this paper is structured as follows. In section 2, the system model is introduced. The new DA and NDA ML SNR estimators will be formulated in section 3. Simulation results will be presented in section 4, and finally some concluding remarks will be drawn out in section 5.

We also mention that some of the common notations will be used throughout this paper. In fact, Vectors and matrices are represented respectively in bold lower and upper case fonts respectively. Moreover, $\{.\}^T$ and $\{.\}^H$ denote the transpose and the Hermitian operators, respectively. The operators $\Re\{.\}$, $\Im\{.\}$ and $\{.\}^*$ return the real, imaginary parts and the conjugate of any complex quantity (scalar or vector).

2. SYSTEM MODEL

We consider continuously transmitted symbols over frequency-flat time-varying SIMO channels. Assuming an ideal receiver with perfect time and frequency synchronization, the sampled baseband received signal over the $\{i^{th}\}_{i=1}^{N_r}$ antenna element (after matched filtering) can be expressed as:

$$y_i(nTs) = h_i(nT_s)\, x\,(nT_s) + w_i(nT_s), \;\; n = 1, 2, 3, \cdots \quad (1)$$

where at time index n, $x(n)$ is any constant-envelope (e.g., $M-$ary PSK) transmitted symbol, $y_i(n)$ is the corresponding received sample, and $h_i(n)$ is the time-varying *complex* channel gain. The noise components, $w_i(n)$, assumed to be temporally white and uncorrelated between antenna elements, are realizations of a zero-mean complex circular Gaussian process, with independent real and imaginary parts, each of variance σ^2. Moreover, the overall noise power $2\sigma^2$ is assumed

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to be the same over all the antenna branches (*uniform* noise). As stated previously, most of the traditional techniques assume the channel to be constant during the observation period, in which case one simply assumes $h_i(n) = h_i$. But, in most real applications, this assumption does not hold due to the mobility of the users. In these situations, a more convenient time-varying channel model is the polynomial in time [18]. In fact, using Taylor's theorem, the time variations of the channel coefficients can be captured (locally) through a polynomial-in-time expansion of order $(L-1)$ as follows:

$$h_i(t_n) = \sum_{m=0}^{L-1} c_i^{(m)} t_n^m + R_L^{(i)}(n), \; i = 1, 2, \cdots, N_r \tag{2}$$

where $c_i^{(m)}$ is the m^{th} coefficient of the channel polynomial approximation over the i^{th} receiving antenna branch, $\{t_n = nT_s\}_{n\geqslant 1}$ is the n^{th} time instant and T_s is the sampling period. The term $R_L^i(n)$ refers to the remainder of the Taylor series expansion. This remainder can be driven to zero under mild conditions, such as i) a sufficiently high approximation order $(L-1)$, or ii) a sufficiently small ratio NF_d/F_s (where F_s is the sampling rate and F_d is the maximum Doppler frequency) by choosing small local approximation window sizes N. Choosing a high approximation order (first condition) may result in numerical instabilities due to badly conditioned matrices (depending on the value of the sampling rate). To avoid this problem, we resort to the second solution which consists in choosing a small approximation window, so that $R_L^{(i)}(n)$ can be neglected to finally obtain the following approximation:

$$h_i(t_n) = \sum_{m=0}^{L-1} c_i^{(m)} t_n^m, \;\; i \;\; = \;\; 1, 2, \cdots, N_r, \tag{3}$$

In essence, our goal in this paper can be formulated as follows: given the received samples $\{y_i(n)\}_{n=1}^N$, and the statistical noise model, we will estimate the *instantaneous* SNR, in real time, which can be expressed as:

$$\rho_i = \frac{\sum_{n=1}^N |h_i(n)|^2}{N(2\sigma^2)} = \frac{\sum_{n=1}^N \big| \sum_{m=0}^{L-1} c_i^{(m)} t_n^m \big|^2}{N(2\sigma^2)}. \tag{4}$$

3. FORMULATION OF THE NEW ML SNR ESTIMATORS

3.1. DA ML SNR Estimator

In the DA scenario, only the received samples corresponding to pilot positions are used during the estimation process. We assume that the pilot symbols are transmitted periodically each $T_s' = N_p T_s$ where $N_p > 1$ is an integer quantifying the normalized (by T_s) time period between any two consecutive pilot positions. If the SNR is to be estimated from N' pilot symbols, we gather their corresponding received samples, over each antenna element, in a column vector $\mathbf{y}_{i,\text{DA}} = [y_i(T_s'), y_i(2T_s'), \cdots, y_i(N'T_s')]^T$. The corresponding channel coefficients are then obtained from (3) as follows:

$$h_i(t_n') = \sum_{m=0}^{L-1} c_i^{(m)} t_n'^m, \;\; i \;\; = \;\; 1, 2, \cdots, N_r, \tag{5}$$

where $t_n' = n\, T_s'$ for $n = 1, 2, \cdots, N'$. For ease of notation, we define the following vectors:

$$\begin{aligned} \mathbf{h}_i &= [h_i(T_s'), h_i(2T_s'), \cdots, h_i(N'T_s')]^T \\ \mathbf{w}_i &= [w_i(T_s'), w_i(2T_s'), \cdots, w_i(N'T_s')]^T \\ \mathbf{c}_i &= [c_i^{(0)}, c_i^{(1)}, \cdots, c_i^{(L-1)}]^T, \end{aligned} \tag{6}$$

Then, using (5) and taking into account the entire observation window, we can rewrite the channel approximation in a vector form as follows:

$$\mathbf{h}_i = \mathbf{T}\mathbf{c}_i, \;\; i = 1, 2, \cdots, N_r, \tag{7}$$

where $\mathbf{T}$ is a known Vandermonde matrix whose entries are given by $[\mathbf{T}]_{n,m} = t_n'^m$. Moreover, by defining $\mathbf{A} = \text{diag}\{x(T_s'), x(2T_s'), \cdots, x(N'T_s')\}$ we can rewrite the received samples on the i^{th} receiving antenna element in a $N'-$dimensional column vector as follows:

$$\mathbf{y}_{i,\text{DA}} = \mathbf{A}\mathbf{T}\mathbf{c}_i + \mathbf{w}_i = \mathbf{\Phi}\mathbf{c}_i + \mathbf{w}_i, \tag{8}$$

where $\mathbf{\Phi} = \mathbf{A}\mathbf{T}$ is a $(N' \times L)$ matrix. Now considering all the receiving antenna elements, and after stacking all the observation vectors $\mathbf{y}_{i,\text{DA}}$ one below another, into a single vector $\mathbf{y}_{\text{DA}} = [\mathbf{y}_{1,\text{DA}}^T \, \mathbf{y}_{2,\text{DA}}^T \, \cdots \, \mathbf{y}_{N_r,\text{DA}}^T]^T$, all the received samples can be written in the following vector-matrix form:

$$\mathbf{y}_{\text{DA}} = \mathbf{B}\mathbf{c} + \mathbf{w}, \tag{9}$$

where $\mathbf{c} = [\mathbf{c}_1^T \, \mathbf{c}_2^T \, \cdots \, \mathbf{c}_{N_r}^T]^T$ and $\mathbf{w}$ are LN_r- and $N'N_r$-dimensional column vectors, respectively, vectorized in the same way as $\mathbf{y}_{\text{DA}}$ and $\mathbf{B} = blkdiag\{\mathbf{\Phi}, \mathbf{\Phi}, ..., \mathbf{\Phi}\}$ is a $(N'N_r \times LN_r)$ block-diagonal matrix. The probability density function (pdf) of the observed signal, $\mathbf{y}_{\text{DA}}$, parameterized by $\boldsymbol{\theta}$ and $\mathbf{B}$, is given by:

$$p(\mathbf{y}_{\text{DA}}; \boldsymbol{\theta} | \mathbf{B}) \;\; = \;\; \frac{\exp\left\{-\frac{1}{2\sigma^2}[\mathbf{y}_{\text{DA}} - \mathbf{B}\mathbf{c}]^H [\mathbf{y}_{\text{DA}} - \mathbf{B}\mathbf{c}]\right\}}{(2\pi\sigma^2)^{N'N_r}}, \tag{10}$$

where $\boldsymbol{\theta} = [\mathbf{c}^T, \sigma^2]$ is a vector that contains all the unknown parameters. Then by taking the logarithm of (10) and dropping the constant terms, we obtain the DA log-likelihood function, $L_{\text{DA}}(\boldsymbol{\theta})$, as follows:

$$L_{\text{DA}}(\boldsymbol{\theta}) \; = -N'N_r \ln(\sigma^2) - \frac{1}{2\sigma^2}[\mathbf{y}_{\text{DA}} - \mathbf{B}\mathbf{c}]^H [\mathbf{y}_{\text{DA}} - \mathbf{B}\mathbf{c}]. \tag{11}$$

By differentiating (11) with respect to $\mathbf{c}$ and setting the result to zero we obtain the ML estimate of the polynomial coefficients over all the receiving antenna branches:

$$\widehat{\mathbf{c}}_{\text{DA}} = \left(\mathbf{B}^H\mathbf{B}\right)^{-1} \mathbf{B}^H \mathbf{y}_{\text{DA}}, \tag{12}$$

where $\mathbf{T}$ and $\mathbf{A}$ are known matrices and so is the matrix $\mathbf{B}$. In order to estimate the noise variance, we take the partial derivative of (11), with respect to σ^2, set the result to zero and substite $\mathbf{c}$ by $\widehat{\mathbf{c}}$ derived in (12), to finally obtain:

$$\widehat{\sigma^2}_{\text{DA}} \;\; = \;\; \frac{1}{2N'N_r} \|\mathbf{y}_{\text{DA}} - \mathbf{B}\widehat{\mathbf{c}}_{\text{DA}}\|^2 \tag{13}$$

The DA ML SNR estimator is obtained as $\widehat{\rho}_{i,\text{DA}} = ||\widehat{\mathbf{h}}_{i,\text{DA}}||^2 / N'(2\widehat{\sigma^2}_{\text{DA}})$ where $\widehat{\mathbf{h}}_{i,\text{DA}} = \mathbf{T}\widehat{\mathbf{c}}_{i,\text{DA}}$ contains the estimated channel coefficients.

3.2. NDA ML SNR Estimator

In this section, we assume no *a priori* knowledge about the transmitted symbols. To begin with, we mention that the formulation of the problem adopted in the previous section is problematic in the NDA scenario. In fact, as will be seen shortly, the EM algorithm averages at each iteration the likelihood function over all the possible values of the unknown transmitted symbols. Then by adopting the previous formulation, the EM algorithm would average over all the possible realizations of the matrix $\mathbf{B}$ that contains the transmitted sequence. This is a combinatorial problem with prohibitive complexity. In the DA scenario, this was not a problem since the matrix $\mathbf{B}$ is *a priori* known to the receiver. In this section, we reformulate our system differently so that the EM algorithm averages over the elementary symbols transmitted at separate time instants. To do so, we define[1] the

[1] We mention that $\mathbf{t}(n) = \mathbf{t}(nT_s)$. For ease of notation, we will from now on drop T_s in all similar quantities.

vector $\mathbf{t}(n) = [1, t_n, t_n^2, \cdots, t_n^{L-1}]^T$ and rewrite the channel model as follows:

$$h_i(t_n) \;=\; \sum_{m=0}^{L-1} c_i^{(m)} t_n^m \;=\; \mathbf{c}_i^T \mathbf{t}(n). \tag{14}$$

Then, we stack all the observed samples $\{y_i(n)\}_{i=1}^{N_r}$ at each time index n into a single vector, $\mathbf{y}$, which can be expressed as:

$$\mathbf{y}(n) = x(n)\,\mathbf{C}\,\mathbf{t}(n) + \mathbf{w}(n), \tag{15}$$

where $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{N_r}]^T$ and $\mathbf{w}(n) = [w_1(n), w_2(n), ..., w_{N_r}(n)]^T$. Based on (15), and when the transmitted symbols are assumed to be iid, the pdf of the received vector, $\mathbf{y}(n)$, conditioned on the transmitted symbol $x(n)$, can be expressed as:

$$p\big(\mathbf{y}(n);\boldsymbol{\theta}|x(n)=x_k\big) \;=\; \prod_{i=1}^{N_r} \frac{\exp\left\{-\frac{1}{2\sigma^2}\left|y_i(n) - x_k\mathbf{c}_i^T\mathbf{t}(n)\right|^2\right\}}{2\pi\sigma^2},$$

in which x_k can be one of M possible symbols that form the M-ary PSK constellation alphabet. Then averaging over this alphabet, the pdf of the received vector becomes:

$$p\big(\mathbf{y}(n);\boldsymbol{\theta}\big) \;=\; \frac{1}{M}\sum_{k=1}^{M}\prod_{i=1}^{N_r} \frac{\exp\left\{-\frac{1}{2\sigma^2}\left|y_i(n) - x_k\mathbf{c}_i^T\mathbf{t}(n)\right|^2\right\}}{2\pi\sigma^2}. \tag{16}$$

By investigating (16), it becomes clear that a joint maximization of the likelihood function, with respect to σ^2 and $\{\mathbf{c}_i\}_{i=1}^{N_r}$, is analytically intractable. Yet, this multidimensional optimization problem can be efficiently tackled using the EM concept. Indeed, the log-likelihood function of $\boldsymbol{y}(n)$ conditioned on x_k, $L(\boldsymbol{\theta}|x_k) = \ln\big(p(\mathbf{y}(n);\boldsymbol{\theta}|x(n) = x_k)\big)$ is given by:

$$\begin{aligned} L(\boldsymbol{\theta}|x_k) &= -N_r\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N_r}\left|y_i(n) - x_k\mathbf{c}_i^T\mathbf{t}(n)\right|^2 \\ &= -N_r\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N_r}\bigg(|y_i(n)|^2 + |x_k|^2\left|\mathbf{c}_i^T\mathbf{t}(n)\right|^2 \\ &\qquad - 2\Re\left\{y_i^*(n)x_k\mathbf{c}_i^T\mathbf{t}(n)\right\}\bigg). \end{aligned} \tag{17}$$

The EM algorithm consists of two major steps, the " expectation step" (E-step), in which the expectation of the likelihood function with respect to all the possible transmitted data x_k is computed, followed by the "maximization-step" (M-step), in which we maximize the output of the E-step with respect to the unknown parameters. The E-step is established as follows. Starting from an initial guess[2], $\widehat{\boldsymbol{\theta}}^{(0)}$, about the unknown parameter vector, the objective function is updated iteratively according to:

$$Q(\boldsymbol{\theta}|\widehat{\boldsymbol{\theta}}^{(q-1)}) = \sum_{n=1}^{N} E_{x_k}\left\{L(\boldsymbol{\theta}|x_k)\big|\widehat{\boldsymbol{\theta}}^{(q-1)}, \mathbf{y}(n)\right\}, \tag{18}$$

where $E_{x_k} = \{.\}$ is the expectation over all possible transmitted data x_k, and $\widehat{\boldsymbol{\theta}}^{(q-1)}$ is the estimated parameter vector at the $(q-1)^{th}$ iteration. After some algebraic manipulations, it can be shown that:

$$\begin{aligned} Q(\boldsymbol{\theta}|\widehat{\boldsymbol{\theta}}^{(q-1)}) = -NN_r\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N_r}\Bigg(M_2^i + \\ \sum_{n=1}^{N}\left|\mathbf{c}_i^T\mathbf{t}(n)\right|^2 - 2\sum_{n=1}^{N}\beta_i^{(q-1)}(n)\Bigg), \end{aligned} \tag{19}$$

[2]The initialization issues will be discussed at the end of this section.

where $M_2^{(i)} = E\{|y_i(n)|^2\}$ is the second-order moment of the received samples over the i^{th} receiving antenna element, with:

$$\beta_i^{(q-1)}(n) = \sum_{k=1}^{M} P_{k,n}^{(q-1)}\Re\left\{y_i(n)x_k\mathbf{t}^T(n)\widehat{\mathbf{c}}_i^{(q-1)}\right\}, \tag{20}$$

in which $P_{k,n}^{(q-1)}$ is the *a posteriori* probability of x_k at iteration $(q-1)$, $P_{k,n}^{(q-1)} = P(x_k|\mathbf{y}(n);\widehat{\boldsymbol{\theta}}^{(q-1)})$, which can be computed using the Bayes formula:

$$P_{k,n}^{(q-1)} = \frac{P[x_k]P\left(\mathbf{y}(n)\big|x_k;\widehat{\boldsymbol{\theta}}^{(q-1)}\right)}{P\left(\mathbf{y}(n);\widehat{\boldsymbol{\theta}}^{(q-1)}\right)}. \tag{21}$$

Since the transmitted symbols are iid, i.e., $P[x_k] = 1/M$, we have:

$$P\left(\mathbf{y}(n);\widehat{\boldsymbol{\theta}}^{(q-1)}\right) = \frac{1}{M}\sum_{k=1}^{M} P\left(\mathbf{y}(n)\big|x_k;\widehat{\boldsymbol{\theta}}^{(q-1)}\right). \tag{22}$$

In a nutshell, the M-step can be fulfilled by finding the parameters' values that maximize the output of the E-step:

$$\widehat{\boldsymbol{\theta}}^{(q)} = \arg\max_{\boldsymbol{\theta}}\left\{Q\left(\boldsymbol{\theta}\big|\widehat{\boldsymbol{\theta}}^{(q-1)}\right)\right\}. \tag{23}$$

To that end, we rewrite (19) as function of $\Re\{\mathbf{c}_i\}$ and $\Im\{\mathbf{c}_i\}$ and differentiate the result with respect to these new parameters. Then by setting it to zero, we obtain their ML estimates from which the intended ML estimate of the complex channel coefficients $\widehat{\mathbf{c}}_{i,\text{NDA}}^{(q)} = \Re\{\widehat{\mathbf{c}}_i\} + j\Im\{\widehat{\mathbf{c}}_i\}$ is obtained as follows:

$$\widehat{\mathbf{c}}_{i,\text{NDA}}^{(q)} = \left(\sum_{n=1}^{N}\mathbf{t}(n)\mathbf{t}^T(n)\right)^{-1}\left(\sum_{n=1}^{N}\lambda_{i,n}^{(q-1)}\mathbf{t}(n)\right), \tag{24}$$

where:

$$\lambda_{i,n}^{(q-1)} = \left(\sum_{k=1}^{M}P_{k,n}^{(q-1)}x_k^*\right)y_i(n) \tag{25}$$

and

$$\widehat{\sigma^2}_{\text{NDA}}^{(q)} = \frac{1}{2NN_r}\sum_{i=1}^{N_r}\left(M_2^{(i)} + \eta_n^{(q-1)}\right). \tag{26}$$

In (26), $\eta_n^{(q-1)}$ is given by:

$$\eta_n^{(q-1)} \;=\; \sum_{n=1}^{N}\left(\left|\mathbf{t}^T(n)\widehat{\mathbf{c}}_i^{(q-1)}\right|^2 - 2\beta_i^{(q-1)}(n)\right). \tag{27}$$

We mention that in order to find the ML estimates of the channel coefficients, we rewrite (18) in terms of their real and imaginary parts and the n differentiate it with respect to these new parameters (18). Differencing with respect to $\{\mathbf{c}_i\}_{i=1}^{N_r}$ directly is difficult, since the partial derivatives are taken with respect to complex vectors. We enclose the details in the Appendix. After few iterations, (10 iterations), the EM algorithm converges to the exact NDA ML estimates $\widehat{\mathbf{c}}_{i,\text{NDA}}$ and $\widehat{\sigma^2}_{\text{NDA}}$ of $\mathbf{c}_i$ and σ^2, respectively. Finally, the NDA ML SNR estimator is obtained by injecting the estimated values (24) and (26) in (4).

Now recall that the EM algorithm is iterative in nature and, therefore, its performance is closely tied to the initial guess $\widehat{\boldsymbol{\theta}}^{(0)}$. We will see in the next section that when it is not appropriately initialized, its performance is severely degraded especially at high SNR levels. Yet, an appropriate initial guess about the polynomial coefficients, $\widehat{\mathbf{c}}$, and the

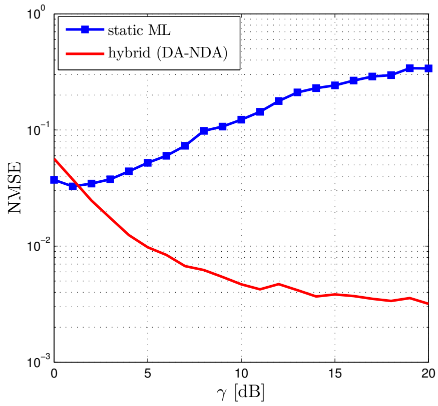

Fig. 1. NMSE for the "*static ML*" and the "*hybrid*" EM-based estimator ($L = 4$) vs. the average SNR γ, with $N_r = 2$, $N = 100$, $F_dT_s = 0.007$, and $Fs = 24300$ Hz.

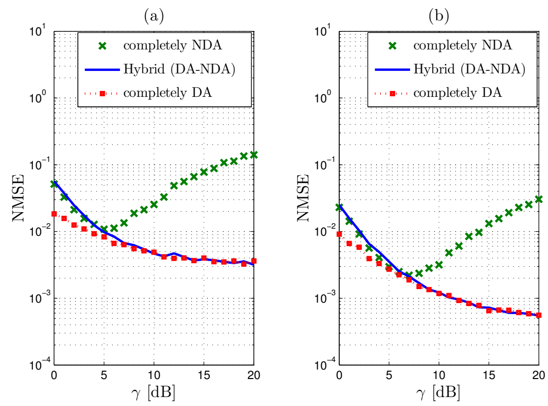

Fig. 2. NMSE for the completely DA, the *hybrid* and the completely NDA EM-based estimators ($L = 4$) vs. the average SNR γ, with $N = 100$, $F_dT_s = 0.007$, and $Fs = 24300$ Hz, for (a) $N_r = 2$, (b) $N_r = 4$.

noise variance, $\widehat{\sigma^2}$, can be obtained using very few pilot symbols within each approximation window by applying the DA ML estimator developed in the previous subsection. Therefore, we will henceforth use two different designations for the new EM-based estimator according to the initialization procedure. We shall refer to it as "*completely NDA*" if it is initialized *arbitrarily* and as "*hybrid*" when it is *appropriately* initialized using the DA estimator of the previous subsection.

4. SIMULATION RESULTS

In this section, we assess the performance of our new DA and NDA ML *instantaneous* SNR estimators via Monte-Carlo simulations using 5000 runs. The estimator performance is evaluated in terms of normalized (by the average SNR) mean square error (NMSE), which is defined as $\text{NMSE}(\widehat{\rho_i}) = E\{(\rho_i - \widehat{\rho_i})^2\}/\gamma^2$ where $\gamma = E\{|x(n)|^2\}/(2\sigma^2)$ is the average SNR per symbol. But since the constellation energy is assumed to be normalized to one, i.e., $E\{|x(n)|^2\} = 1$, γ is simply given by $\gamma = 1/(2\sigma^2)$. We begin by showing that the existing NDA ML SNR estimator [11] that was primarily designed for static or slowly time-varying SIMO channels fails completely to estimate the instantaneous SNR for fast time-varying channels. For the sake of clarity, we refer to the ML estimator of [11] as "*static ML*". The performance of the two estimators is compared in Fig. 1. It is clearly seen that the "*static ML*" estimator exhibits a noticeable performance degradation when applied to time-varying channels. This stems from the fact that this estimator is erroneously approximating the fast time-varying channels (over each antenna element) as piecewise constant, over each approximation window. The new estimator is, however, able to track the channel variations more accurately and therefore its performance improves steadily with the average SNR. In Fig. 2, we examine the impact of initialization on the EM-based algorithm. We plot the NMSE of its "*completely NDA*" version when it is simply initialized by ones, i.e., $\widehat{\boldsymbol{\theta}}^{(0)} = [1, 1, \cdots, 1]^T$ as well as the "*hybrid*" version initialized by $\widehat{\boldsymbol{\theta}}^{(0)} = [\widehat{\mathbf{c}}_{\text{DA}}^T, \widehat{\sigma^2}_{\text{DA}}]^T$. Note that the DA initial guesses are obtained from $N' = 10$ pilot symbols over each approximation window of size $N = 100$ symbols (i.e., $N' = 0.1N$). In LTE systems, for instance, a pilot symbol is transmitted each 7 OFDM symbols corresponding to $N' = N/7 = 0.1429N$ [19]. We also plot in the same figure the performance of the "*completely DA*" estimator where all the symbols are assumed to be perfectly known (pilots) (i.e., $N' = N$).

It is clearly seen that the performance of the EM algorithm is critically affected when it is not *appropriately* initialized. In fact, with bad initialization, the iterative algorithm may converge to a local maximum. Fortunately, this problem which is actually inherent to all iterative algorithms, is circumvented by taking advantage of the presence of pilot symbols by computing an accurate initial guess using the DA procedure developed also in this paper. In doing so, the new EM-based estimator coincides with the *completely DA* approach over a wide range of the SNR. This means that it provides the *exact* ML estimates that would be theoretically obtained in the hypothetical scenario where all the transmitted symbols are perfectly known (*completely DA*). In Fig. 3. we compare the performance of the *completely DA* and the *hybrid* ML estimators, for different antenna-array sizes. Even in the special case of a SISO system ($N_r = 1$), the proposed estimator yields sufficiently accurate SNR estimates. It is also seen that increasing the number of receiving antenna elements improves substantially the estimator's performance, since it helps in accurately estimating the noise variance that is assumed to be constant across all the antenna elements.

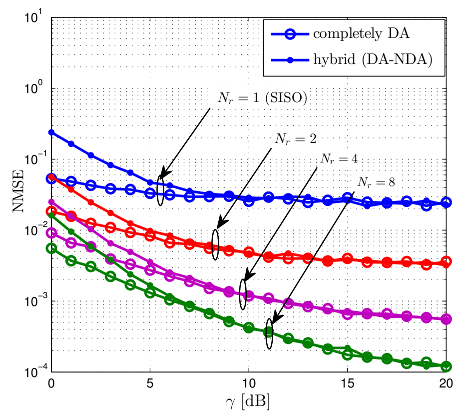

Fig. 3. NMSE for the completely DA and the *hybrid* (DA-NDA) EM-based estimator ($L = 4$) vs. the average SNR γ, for different numbers of receiving antennas with $N = 100$, $F_dT_s = 0.007$, and $Fs = 24300$ Hz.

5. CONCLUSION

In this paper, we proposed a new *instantaneous* SNR estimator over time-varying flat fading SIMO channels, using a polynomial-in-time expansion. In the DA scenario, the SNR estimator was derived in closed form, whereas, in the NDA case, we proposed a solution that is based on the expectation-maximization concept. Moreover, the new iterative estimator converges within very few iterations to the exact ML estimates when it is initialized with some few pilot symbols. Computer simulations show that our newly developed estimator is able to reach the optimal performance over a wide range of average SNR.

6. REFERENCES

[1] N. C. Beaulieu, A. S. Toms, and D. R. Pauluzzi, “Comparison of four SNR estimators for QPSK modulations”, *IEEE Commun. Letters*, vol. 4, no. 2, pp. 43-45, Feb. 2000.

[2] K. Balachandran, S. R. Kadaba, and S. Nanda, “Channel quality estimation and rate adaption for cellular mobile radio”, *IEEE J. Sel. Areas Commun.*, vol. 17, no. 7, pp. 1244-1256, Jul 1999.

[3] T. A. Summers and S. G. Wilson, “SNR mismatch and online estimation in turbo decoding”, *IEEE Trans. Commun.*, vol. 46, no. 4, pp. 421-423, Apr. 1998.

[4] D. R. Pauluzzi and N. C. Beaulieu, “A comparison of SNR estimation techniques for the AWGN channel”, *IEEE Trans. Commun.*, vol. 48, no. 2, pp. 1681-1691, Oct. 2000.

[5] M. Turkboylari and G. L. Stuber, “An efficient algorithm for estimating the signal-to-interference ratio in TDMA cellular systems”, *IEEE Trans. Commun.*, vol. 46, no. 6, pp. 728-731, June 1998.

[6] P. Gao and C. Tepedelenlioglu, “SNR estimation for non-constant modulus constellations”, *IEEE Trans. Signal Process.*, vol. 53, no. 3, pp. 865-870, Mar 2005.

[7] R. Lopez-Valcarce and C. Mosquera, “Sixth-order statistics-based non-data-aided SNR estimation”, *IEEE Commun. Letters*, vol. 11, no. 4, pp. 351-353, Apr. 2007.

[8] A. Stéphenne, F. Bellili, and S. Affes, “Moment-based SNR estimation over linearly-modulated wireless SIMO channels”, *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 714-722, Feb. 2010.

[9] A. Stéphenne, F. Bellili, and S. Affes, Moment-based SNR estimation for SIMO wireless communication systems using arbitrary QAM, *in Proc. 41st Asilomar Conference on Signals, Systems and computers*, Pacific Grove, CA, USA, 2007, pp. 601-605.

[10] A. Das (Nandan), “NDA SNR estimation: CRLBs and EM based estimators”, *in Proc. IEEE TENCON’08*, pp. 1-6, 2008.

[11] M. A. Boujelben, F. Bellili, S. Affes, and A. Stéphenne, “SNR estimation over SIMO channels from linearly modulated signals”, *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6017-6028 , Dec. 2010.

[12] M. A. Boujelben, F. Bellili, S. Affes, and A. Stéphenne, EM algorithm for non-data-aided SNR estimation of linearly-modulated signals over SIMO channels, *in Proc. of IEEE GLOBECOM*, Honolulu, Hawaii, USA, 2009, pp. 1-6.

[13] A. Das and B. D. Rao, “SNR and noise variance estimation for MIMO systems”, *IEEE Trans. Signal Process.*, vol. 60, no. 8, pp. 3929-3941, Aug. 2012.

[14] G. L. Stüber, *Principles of Mobile Communication*, Boston, MA: Kluwer, 1996.

[15] T. Gao and B. Sun, “A high-speed railway mobile communication system Based on LTE”, *in Proc. ICEIE 2010*, vol. 1, pp. 414-417, 2010.

[16] A. Wiesel, J. Goldberg, and H. Messer-Yaron, “SNR estimation in time-varying fading channels”, *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 841-848, May 2006.

[17] F. Bellili, A. Stphenne, and S. Affes, SNR estimation of QAM-modulated transmissions over time-varying SIMO channels, *in Proc. IEEE Int. Symp. Wireless Commun. Syst.*, Reykjavik, Iceland, 2008, pp. 199-203.

[18] C. Anton-Haro, J. A. R. Fonollosa, C. Fauli, and J. R. Fonollosa, “On the inclusion of channel’s time dependence in a hidden Markov model for blind channel estimation”, *IEEE Trans. Veh. Technol.*, vol. 50, no. 3, pp. 867-873, May 2001.

[19] 3GPP TS 36.211: 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation.