ONE-SHOT BLIND CFO ESTIMATION FOR OFDM WITH MULTI-ANTENNA RECEIVER

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ABSTRACT

In this paper, we propose a new blind carrier frequency offset (CFO) estimation method for orthogonal frequency division multiplexing (OFDM) with multi-antenna receiver. The proposed method is one-shot and works with only a single OFDM block, which differs from the existing works for multi-antenna receiver. We show that, the proposed method not only supports fully loaded transmission, but also outperforms the existing maximum likelihood estimator for multi-antenna receiver. The numeral results are provided to corroborate the proposed studies.

Index Terms— Orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), multi-antenna, one-shot.

1. INTRODUCTION

Carrier frequency offset (CFO) estimation for orthogonal frequency division multiplexing (OFDM) has drawn substantial research interests during the past few years. Different kinds of blind CFO estimators have been developed owing to the potential benefit of improving spectral efficiency. The CFO can be obtained based on the kurtosis-type criterion [1], constant modulus constellations [2, 3] or the presence of null subcarriers [4]. Especially, the methods in [1, 2, 3, 4] are one-shot estimators which need only a single OFDM block to work with.

Several blind CFO estimators have also been proposed which benefit from the multi-antenna redundancy at the receiver. The work in [5] is based on the trilinear decomposition, while the improved estimator in [6] exploits the multi-invariance property. Unfortunately, a relative large number of OFDM blocks should be collected in both [5] and [6]. More recently, [7] has proposed a blind maximum likelihood (ML) CFO estimator for OFDM system with multi-antenna receiver. However, the estimator [7] still requires at least two OFDM block durations, during which the channel between the transceivers should stay constant. Thus, the estimator [7] should be more vulnerable to the channel variations than the existing one-shot estimators [1, 2, 3, 4]. Moreover, the work of [7] has not considered the information that the channel length between the transceivers is usually bounded by the length of cyclic prefix (CP) in practical OFDM systems. Note that there exists the possibility to acquire even better performance than the ML estimator of [7] by exploiting this key information. This motivates our current work.

In this paper, we exploit the above information about the maximum channel length and propose a new blind CFO estimation method for OFDM with multi-antenna receiver. The proposed method is one-shot and works with only a single OFDM block, which differs from the existing works [5, 6, 7] for multi-antenna receiver. We further show that, the proposed method not only supports fully loaded transmissions, but also outperforms the existing ML estimator [7] for multi-antenna receiver.

2. SYSTEM MODEL

We consider an OFDM system with a total of N subcarriers sequentially indexed with $\{i\}$, $i = 1, 2, \cdots, N$. Assume that the transmitter has one antenna, whereas the receiver is equipped with M antennas. We consider the system is fully loaded for simplicity. The normalized CFO between the transceiver is denoted by ϕ, which is the ratio between the real CFO and the subcarrier spacing. Like [1, 7], we consider $\phi \in (-0.5, 0.5)$ in this paper.

Assume the channel between the transceivers stays constant within one OFDM block duration and may vary from block to block. Denote the length of CP by $L-1$. Then the multipath channel between the transmitter and the mth receive antenna during the gth block can be modeled as the following length-L vector:

$$\mathbf{h}_{m,g} = [h_{m,g}(1), h_{m,g}(2), \cdots, h_{m,g}(L)]^T. \quad (1)$$

Assume $\mathbf{h}_{m,g}$ contains only L_p non-zero elements with random positions, where L_p denotes the number of channel taps between the transceivers. Note that the position of each non-

This work is supported by the National Natural Science Foundation of China (NSFC) (No. 61302069 & 61302066 & 61172093 & 61172092 & 61102081), China Postdoctoral Science Foundation (No. 2013M530425), Shaanxi Province Postdoctoral Research Funding, and the Foundation for Innovative Research Groups of the NSFC (No. 61221063)

zero element in $\mathbf{h}_{m,g}$ stands for the delay of the corresponding channel tap. We assume these non-zero elements are i.i.d. complex Gaussian variables with zero mean and power $1/L_p$ such that the total power is normalized, i.e., $E[\|\mathbf{h}_{m,g}\|_F^2] = 1$. Denote $\mathbf{F}$ as the normalized $N \times N$ DFT matrix with its (i,j)th entry being $\mathbf{F}(i,j) = \frac{1}{\sqrt{N}} \mathrm{e}^{-\mathbf{j}2\pi(i-1)(j-1)/N}$. The frequency domain channel response between the transmitter and the mth receive antenna at all of the N subcarriers can be expressed as $\sqrt{N}\mathbf{F}_L\mathbf{h}_{m,g}$, where $\mathbf{F}_L \in \mathbb{C}^{N\times L}$ consists of the first L columns of $\mathbf{F}$.

Denote $S_{k,g}$, $k = 1, 2, \cdots, N$, as the N transmitted data symbols in the gth OFDM block. Define $N \times N$ diagonal matrix $\mathbf{E}\{\phi\} = \mathrm{diag}(1, \mathrm{e}^{\mathbf{j}2\pi\phi/N}, \cdots, \mathrm{e}^{\mathbf{j}2\pi(N-1)\phi/N})$ which represents the phase rotation introduced by the CFO. Denote $\mathbf{S}_g = [\bar{S}_{1,g}, \bar{S}_{2,g}, \cdots, \bar{S}_{N,g}]^T$ where $\bar{S}_{k,g} = \mathrm{e}^{\mathbf{j}\frac{2\pi(N+L-1)(g-1)\phi}{N}} S_{k,g}$. Then, in the gth OFDM block duration, the received time domain signal at the mth receive antenna after CP removal can be expressed by the following length-N vector:

$$\mathbf{y}_{m,g} = \sqrt{N}\mathbf{E}\{\phi\}\mathbf{F}^H \mathrm{diag}(\mathbf{S}_g)\mathbf{F}_L\mathbf{h}_{m,g} + \mathbf{n}_{m,g},$$

where $\mathbf{n}_{m,g}$ is the corresponding additive white Gaussian noise vector with $E[\mathbf{n}_{m,g}\mathbf{n}_{m,g}^H] = \sigma_n^2\mathbf{I}_N$ at the mth receive antenna in the gth OFDM block.

3. PROPOSED ESTIMATOR

We place the vectors $\mathbf{y}_{m,g}$, $m = 1, 2, \cdots, M$, next to each other and obtain the following $N \times M$ matrix:

$$\begin{aligned}\mathbf{y}_g =& [\mathbf{y}_{1,g}, \mathbf{y}_{2,g}, \cdots, \mathbf{y}_{M,g}] \\ =& \sqrt{N}\mathbf{E}\{\phi\}\mathbf{F}^H \mathrm{diag}(\mathbf{S}_g)\mathbf{F}_L\mathbf{h}_g + \mathbf{n}_g\end{aligned} \tag{2}$$

where $\mathbf{h}_g = [\mathbf{h}_{1,g}, \mathbf{h}_{2,g}, \cdots, \mathbf{h}_{M,g}] \in \mathbb{C}^{L\times M}$ and $\mathbf{n}_g = [\mathbf{n}_{1,g}, \mathbf{n}_{2,g}, \cdots, \mathbf{n}_{M,g}] \in \mathbb{C}^{N\times M}$ denotes the corresponding additive noise matrix. In the following, we omit the noise items in the received signal to ease the presentation. Perform CFO compensation with a trial value of ϕ, we obtain the following frequency domain signal after DFT conversion:

$$\begin{aligned}\mathbf{Y}_g\{\tilde{\phi}\} =& \mathbf{F}\mathbf{E}^H\{\tilde{\phi}\}\mathbf{y}_g \\ =& \sqrt{N}\mathbf{F}\mathbf{E}\{\phi - \tilde{\phi}\}\mathbf{F}^H \mathrm{diag}(\mathbf{S}_g)\mathbf{F}_L\mathbf{h}_g.\end{aligned}$$

We rewrite

$$\mathbf{F}_L = [\mathbf{f}_{L,1}, \mathbf{f}_{L,2}, \cdots, \mathbf{f}_{L,N}]^T,$$

$$\mathbf{Y}_g\{\tilde{\phi}\} = \left[\boldsymbol{\xi}_{1,g}\{\tilde{\phi}\}, \boldsymbol{\xi}_{2,g}\{\tilde{\phi}\}, \cdots, \boldsymbol{\xi}_{N,g}\{\tilde{\phi}\}\right]^T,$$

where $\boldsymbol{\xi}_{k,g}^T\{\tilde{\phi}\} \in \mathbb{C}^{1\times M}$ and $\mathbf{f}_{L,k}^T \in \mathbb{C}^{1\times L}$ denote the kth row vectors of $\mathbf{Y}_g\{\tilde{\phi}\}$ and $\mathbf{F}_L$, respectively.

When the trial CFO value equals the real value, i.e., $\tilde{\phi} = \phi$, we have

$$\mathbf{Y}_g\{\phi\} = \sqrt{N}\mathrm{diag}(\mathbf{S}_g)\mathbf{F}_L\mathbf{h}_g. \tag{3}$$

It is then observed that, $\boldsymbol{\xi}_{k,g}\{\phi\}$ is linearly related to the vector $(\mathbf{f}_{L,k}^T\mathbf{h}_g)^T$ with only a scalar ambiguity. Hence, there holds $(\boldsymbol{\xi}_{k,g}^{\perp}\{\phi\})^H\mathbf{h}_g^T\mathbf{f}_{L,k} = \mathbf{0}$, $k = 1, 2, \cdots, N$, where $\boldsymbol{\xi}_{k,g}^{\perp}\{\phi\}$ denotes the $M \times (M-1)$ orthogonal complement matrix of $\boldsymbol{\xi}_{k,g}\{\phi\}$. Thus, we can further obtain

$$\sum_{k=1}^{N} \left\|(\boldsymbol{\xi}_{k,g}^{\perp}\{\phi\})^H\mathbf{h}_g^T\mathbf{f}_{L,k}\right\|_F^2 = 0. \tag{4}$$

Based on the fact that

$$\mathrm{Vec}((\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H\mathbf{h}_g^T\mathbf{f}_{L,k}) = (\mathbf{f}_{L,k}^T \otimes (\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H)\mathrm{Vec}(\mathbf{h}_g^T),$$

we have

$$\begin{aligned}&\sum_{k=1}^{N} \left\|(\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H\mathbf{h}_g^T\mathbf{f}_{L,k}\right\|_F^2 \\ =& \mathrm{Vec}(\mathbf{h}_g^T)^H \left(\sum_{k=1}^{N}(\mathbf{f}_{L,k}^* \otimes \boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})(\mathbf{f}_k^T \otimes (\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H)\right) \\ & \mathrm{Vec}(\mathbf{h}_g^T) \\ =& \mathrm{Vec}(\mathbf{h}_g^T)^H \left(\sum_{k=1}^{N}(\mathbf{f}_{L,k}^*\mathbf{f}_{L,k}^T) \otimes \left(\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\}(\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H\right)\right) \\ & \mathrm{Vec}(\mathbf{h}_g^T).\end{aligned} \tag{5}$$

Note that $\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\}(\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H = \mathbf{I}_M - \frac{\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}^H}{\|\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}\|_F^2}$. We then rewrite (5) into

$$\begin{aligned}&\sum_{k=1}^{N} \left\|(\boldsymbol{\xi}_{k,g}^{\perp}\{\tilde{\phi}\})^H\mathbf{h}_g^T\mathbf{f}_{L,k}\right\|_F^2 \\ =& \mathrm{Vec}(\mathbf{h}_g^T)^H\boldsymbol{\Xi}_g\{\tilde{\phi}\}\mathrm{Vec}(\mathbf{h}_g^T)\end{aligned} \tag{6}$$

where the matrix $\boldsymbol{\Xi}_g\{\tilde{\phi}\} \in \mathbb{C}^{ML\times ML}$ is expressed as

$$\boldsymbol{\Xi}_g\{\tilde{\phi}\} = \sum_{k=1}^{N}(\mathbf{f}_{L,k}^*\mathbf{f}_{L,k}^T) \otimes \left(\mathbf{I}_M - \frac{\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}^H}{\|\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}\|_F^2}\right). \tag{7}$$

Combing both (4) and (6), we arrive at

$$\mathrm{Vec}(\mathbf{h}_g^T)^H\boldsymbol{\Xi}_g\{\phi\}\mathrm{Vec}(\mathbf{h}_g^T) = 0, \tag{8}$$

which indicates that the matrix $\boldsymbol{\Xi}_g\{\phi\}$ is rank deficient when $\tilde{\phi}$ equals the real value ϕ.

On the other side, when $\tilde{\phi} \neq \phi$, the residual frequency offset leads to ICI. In this case, we have

$$\begin{aligned}\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\} =& a_{k,k}\{\phi - \tilde{\phi}\}\sqrt{N}\bar{S}_{k,g}\mathbf{h}_g^T\mathbf{f}_{L,k} \\ &+ \underbrace{\sum_{p=1,p\neq k}^{N} a_{p,k}\{\phi - \tilde{\phi}\}\sqrt{N}\bar{S}_{p,g}\mathbf{h}_g^T\mathbf{f}_{L,p}}_{\mathrm{ICI}}\end{aligned} \tag{9}$$

where $a_{p,k}\{\phi-\tilde{\phi}\} = \frac{1-e^{j2\pi(\phi-\tilde{\phi})}}{N(1-e^{j2\pi(p-k+\phi-\tilde{\phi})/N})}$. It is seen that when $\tilde{\phi} \neq \phi$, the ICI item may not be zero and will introduce additional vectors to space the column space of $\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}$. Note that the ICI item is usually regarded as the zero-mean white Gaussian noise [8]. This makes the vector $\boldsymbol{\xi}_{k,g}\{\tilde{\phi}\}$, $k = 1, 2, \cdots, N$, become hardly linearly related to the vector $\tilde{\mathbf{h}}_g^T \mathbf{f}_{L,k}$ with arbitrary trial matrix $\tilde{\mathbf{h}}_g \in \mathbb{C}^{L\times M}$. In other words, the equations $(\boldsymbol{\xi}^{\perp}_{k,g}\{\tilde{\phi}\})^H \tilde{\mathbf{h}}_g^T \mathbf{f}_k = \mathbf{0}$, $k = 1, 2, \cdots, N$, hardly hold with arbitrary trial matrix $\tilde{\mathbf{h}}_g$. This implies when $\tilde{\phi} \neq \phi$, there holds

$$\sum_{k=1}^{N} \left\|(\boldsymbol{\xi}^{\perp}_{k,g}\{\tilde{\phi}\})^H \tilde{\mathbf{h}}_g^T \mathbf{f}_{L,k}\right\|_F^2 > 0, \tag{10}$$

with arbitrary trial matrix $\tilde{\mathbf{h}}_g$. Then following the similar steps from (5) to (7), we know the rank deficient property of matrix $\Xi_g\{\tilde{\phi}\}$ hardly holds when $\tilde{\phi} \neq \phi$.

Based on the above discussions, the proposed estimator can be expressed as

$$\hat{\phi} = \min_{\tilde{\phi}} \lambda_{\min}(\boldsymbol{\Xi}_g\{\tilde{\phi}\}) \tag{11}$$

where $\lambda_{\min}(\cdot)$ denotes the minimal eigenvalue of the one matrix.

Remark 1: The proposed method does not rely on the exact knowledge about the number of channel taps. We only need to know the value of L, where $L-1$ stands for the maximum delay of channel taps between the transceivers. Note that the value of L is related to the length of CP and is known for both transceivers.

Remark 2: In the environment with time-varying channels, the channel hardly stay constant over multiple consecutive block durations. Consider the channel matrix $\mathbf{h}_g$ varies from block to block. Then, the proposed method with L_s block durations can be readily extended as follows

$$\hat{\phi} = \min_{\tilde{\phi}} \sum_{g=1}^{L_s} \lambda_{\min}(\boldsymbol{\Xi}_g\{\tilde{\phi}\}). \tag{12}$$

Next, we consider the special case where the channel can stay constant over continuous L_s block duration. In this situations, we denote $\mathbf{h} = \mathbf{h}_g$, $g = 1, 2, \cdots, L_s$, and then similar to (4), there holds

$$\sum_{g=1}^{L_s}\sum_{k=1}^{N} \left\|(\boldsymbol{\xi}^{\perp}_{k,g}\{\phi\})^H \mathbf{h}^T \mathbf{f}_{L,k}\right\|_F^2 = 0, \tag{13}$$

which is equivalent to

$$\mathrm{Vec}(\mathbf{h}^T)^H \Big(\sum_{g=1}^{L_s} \boldsymbol{\Xi}_g\{\phi\}\Big) \mathrm{Vec}(\mathbf{h}^T) = 0. \tag{14}$$

This indicates the matrix $\sum_{g=1}^{L_s} \boldsymbol{\Xi}_g\{\tilde{\phi}\}$ is rank deficient when $\tilde{\phi} = \phi$. Then, following similar steps from (9) to (11), the proposed estimator for this case can be expressed as

$$\hat{\phi} = \min_{\tilde{\phi}} \lambda_{\min}(\sum_{g=1}^{L_s} \boldsymbol{\Xi}_g\{\tilde{\phi}\}). \tag{15}$$

4. SIMULATIONS

In this section, we assess the proposed CFO estimation method from computer simulations. The total number of subcarriers is taken as $N = 64$. The length of CP is taken as 8, i.e., $L = 9$ is assumed. The normalized CFO is randomly generated from -0.4 to 0.4. The mean square error (MSE) of the normalized CFO estimation is adopted as the figure of merit. The signal-to-noise ratio (SNR) is defined as $E[|S_{k,g}|^2]/\sigma_n^2$. For comparison, we also included the results of the ML estimator in [7] and the method of [4], referred to as 'MLE' and 'MUSIC-like', respectively. For fairness, the multi-antenna redundancy at the receiver is also exploited in MUSIC-like as described in [7, eq. (21)]. Moreover, fully loaded transmission is assumed in both MLE and the proposed method, whereas 12 null subcarriers are reserved in MUSIC-like.

First, we consider the channel stays constant over continuous L_s block duration. The proposed method is taken as (15) for this case. The CFO estimation MSE performance of our method versus SNR is shown in Fig. 1, where the solid and dashed curves correspond to the results with $M = 2$ and $M = 4$ receive antennas, respectively. The channel with $L_p = 9$ taps is assumed in this example. From the results, we can make the following observations:

First, as expected, with a single OFDM block duration, i.e., $L_s = 1$, the estimation performance of our method can be improved as the SNR increases, which demonstrates the validity of our proposed one-shot method.

Second and more importantly, it is seen that, our method could outperform the other two competitors in this example. Specifically, when $L_s = 2$ block durations are available, we can observe a performance gap more than 2 dB between our method and MLE under the moderate and high SNR region. As we have discussed earlier, this is because MLE does not exploit the key information about maximum channel length, although it is based on ML criterion. On the other hand, we see that our method also obtains much better than MUSIC-like, even 12 null subcarriers are reserved in latter. Especially, the performance gap between our method and MUSIC-like becomes as large as 7 dB under the moderate and high SNR region when $M = 4$ antennas are adopted at the receiver.

We then evaluate the performance of our method in fast time-varying channels. Denote ξ_d as the maximum Doppler frequency normalized by subcarrier spacing. We increase ξ_d from 0 to 0.1 and display the results in Fig. 2. We consider

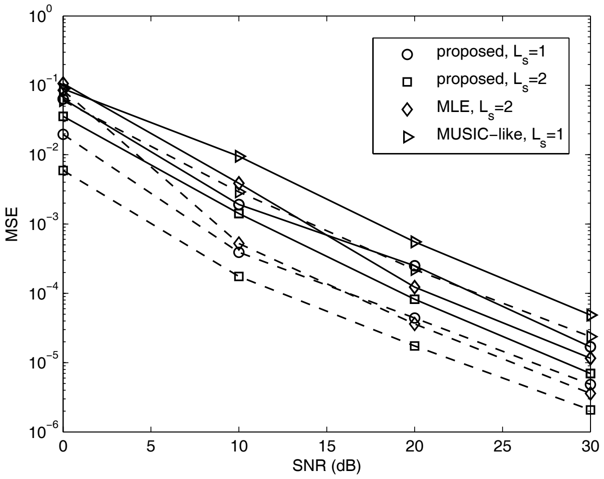

Fig. 1. The CFO estimation performance comparison. The solid and dashed curves correspond to the results with $M = 2$ and $M = 4$ receive antennas, respectively.

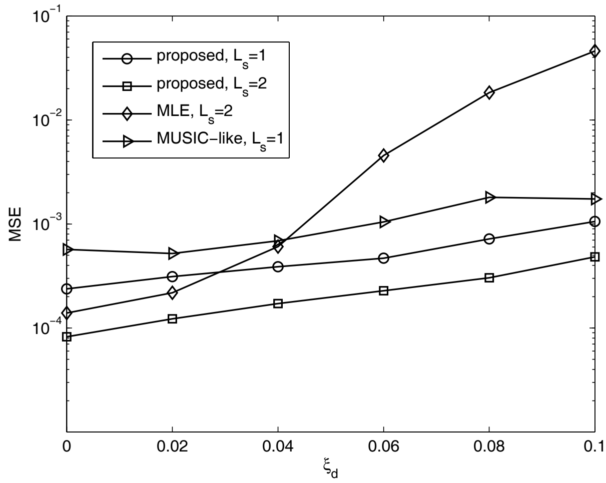

Fig. 2. The CFO estimation performance comparison versus the maximum normalized Doppler frequency.

$M = 2$, SNR= 20 dB and the channel is with $L_p = 9$ taps. The sum-of-sinusoids statistical simulation model proposed in [9] is adopted. The proposed method is taken as (12) in the case of time-varying channels. The results of MLE and MUSIC-like are also included for comparison. As expected, the increase of maximum Doppler frequency degrades the performance of all estimators. It is seen clearly that, the performance of MLE declines much more quickly than MUSIC-like and our method. This indicates that MLE is much more sensitive to time-varying channel than MUSIC-like and our method. This is because MLE requires that the channel stay constant over at least two block durations, whereas both MUSIC-like and our method are one-shot estimators.

5. CONCLUSIONS

In this paper, we have developed an one-shot blind CFO estimation method for OFDM with multi-antenna receiver. The proposed method supports fully loaded systems, and also outperforms existing ML estimator for multi-antenna receiver especially in time-varying channels. The simulation results have been provided, which corroborate the proposed studies.

6. REFERENCES

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