ROBUST JOINT CFO AND FAST TIME-VARYING CHANNEL TRACKING FOR MIMO-OFDM SYSTEMS

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ABSTRACT

In this paper, a robust joint carrier frequency offset (CFO) and fast time-varying channel tracking algorithm for MIMO-OFDM systems is proposed. The proposed method is robust to noise statistics and noise distribution uncertainties. The extended H_∞ filter (EHF) is employed for the joint tracking process as it does not require any knowledge of noise statistics or noise distribution, unlike the conventional extended Kalman filter (EKF). To reduce computational complexity, the tracking of fast time-varying channel gains is facilitated by the basis expansion model (BEM). Simulation results show that the performance of the proposed algorithm without knowledge of noise statistics matches the performance of the existing algorithm under Gaussian noise and is better than that of the existing algorithm under non-Gaussian noise.

Index Terms**—**Channel tracking, carrier frequency offset, extended H_∞ filter, multiple-input-multiple-output, orthogonal frequency-division multiplexing.

1. INTRODUCTION

MIMO-OFDM systems with multipath fading channels are vulnerable to the effects of carrier frequency offset (CFO). CFO is caused by Doppler shift or oscillator frequency mismatch between the transmitter and the receiver.

Existing works on joint CFO and fast time-varying channel estimation for MIMO-OFDM systems with doubly selective channel based on the extended Kalman filter (EKF) are of particular interest as they are suited for instantaneous estimation of time-varying channel gains. Joint CFO and channel tracking for MIMO-OFDM system using the EKF is considered in [1] where tracking is in frequency domain. However, the EKF has the disadvantage of requiring knowledge of noise statistics and needs the noise to be Gaussian. Practical communication systems may be affected by abrupt noise components that results from various phenomena such as car ignition systems, unprotected electric switches and lightning in the atmosphere. These phenomena leads to non-Gaussian noise [2]. Recently, a joint tracking method based on the extended H_∞ filter (EHF) is introduced in [3]. Unlike the EKF, the EHF does not require knowledge of noise distributions and is more robust to unknown disturbances. However, the EHF-based method in [3] is designed for slow time-varying channel. For slow time-varying channel, the channel gains are assumed to be invariant within an OFDM symbol, thus the channel gains can be easily tracked from symbol to symbol. On the other hand, the time variations of the channel gains within an OFDM symbol are significant for fast time-varying channel, thus the channel gains need to be tracked for every sampling interval in an OFDM symbol. This process can be very complex due to the large number of samples in an OFDM symbol. Thus, the method in [3] cannot be trivially extended to track fast time-varying channel gains.

In this paper, we address this problem by designing a joint CFO and fast time-varying channel tracking algorithm for MIMO-OFDM systems based on the incorporation of the basis expansion model (BEM) into the EHF. BEM is used to model the fast time-varying channel with a set of BEM coefficients to overcome the need to track the channel gains at every sampling interval. To the best of our knowledge, there is no algorithm that is designed based on the combination of EHF and BEM in the existing literature. A method to improve the performance of the proposed algorithm when time-multiplexed pilots are used is also proposed where data can be detected concurrently with the joint tracking process. Performance of the joint tracking process under non-Gaussian noise is investigated for MIMO-OFDM system. Despite its importance for modeling many practical environments, non-Gaussian noise has not been considered in the analysis of joint CFO and channel tracking. The proposed algorithm is compared with the existing EKF-based algorithm, which is the best available algorithm currently for performing joint CFO and fast time-varying channel estimation for MIMO-OFDM systems [1]. Simulation results show that the proposed algorithm (without needing noise statistics) is capable of matching the performance of the existing EKF-based algorithm under Gaussian noise and is also capable of outperforming the existing method under non-Gaussian noise.

2. SYSTEM MODEL

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Consider a MIMO-OFDM system with N_T transmit antennas and N_R receive antennas. The received signal can be expressed as:

$$y_{k,n}^{(\mathrm{r})} = \sum_{t=1}^{N_T} \sum_{l=0}^{L-1} e^{\frac{j2\pi(n-1)\varepsilon_k^{(tr)}}{N}} h_{k,n}^{(tr,l)} s_{k,\{n-l\} \bmod N}^{(t)} + w_{k,n} \tag{1}$$

for $t = 1,\ldots$, $r = 1,\ldots$, $l = 0,\ldots$, and $n = 0,\ldots$. k refers to the k-th OFDM symbol block, n refers to the n-th sample, N is the number of samples in one OFDM symbol and L is the number of propagation paths. The notation tr denotes the transmission from transmit antenna t to receive antenna r. $s^{(t)}$ is the transmitted signal, $h^{(tr)}$ is the multipath channel gains, $\varepsilon^{(tr)}$ is the normalized CFO with respect to subcarrier spacing and w is the observation noise with zero mean and noise variance σ_w^2.

BEM is used to model the fast time-varying channel gains in an OFDM symbol with a set of slow time-varying coefficients. BEM reduces the number of parameters to be estimated as the BEM coefficients can be estimated once every OFDM symbol instead of every sampling interval. The channel gains over an entire OFDM symbol for a particular path, $\tilde{\mathbf{h}}$ $^{(tr,l)}_{k,-N_{cp}},\ldots$ $]^T$, can be expressed as:

$$\tilde{\mathbf{h}} \quad {}^{(tr,l)}_{k} + \boldsymbol{\xi}_k^{(tr,l)} \tag{2}$$

where N_{cp} is the cyclic prefix length, $\boldsymbol{\xi}_k^{(tr,l)}$ is the modelling error, $\mathbf{c}_k^{(tr,l)}$ are the BEM coefficients, $\mathbf{B}$ is a N_{OFDM} x N_c matrix containing the N_c basis functions and $N_{OFDM} = N_{cp} + N$. There are various BEM designs such as Polynomial - BEM (P-BEM), generalized complex exponential - BEM (GCE-BEM), and Karhunen-Loeve – BEM (KL-BEM) [4, 5, 6, 7].

It can be noted that the observation model in [1] is implemented in frequency domain while the BEM modelling of the channel gains is in time domain. This necessitates the incorporation of the elements of the Fourier matrix in the frequency domain observation model which leads to higher computational complexity of the method in [1]. In this paper, time domain observation model is used. Let $\mathbf{y}_k = \left[y_{k,0}^1,\ldots \quad \ldots \quad \ldots \right.$. The time domain observation equation with BEM is:

$$\mathbf{y}_k = \mathbf{g}(\mathbf{x}_k) = \boldsymbol{\Gamma}_k \mathbf{c}_k + \mathbf{w}_k \tag{3}$$

where $\mathbf{c}_k$ and $\boldsymbol{\Gamma}_k$ are defined as:

$$\mathbf{c}_k = \left[\mathbf{c}_k^{(11)^T},\ldots \quad \ldots \right. \tag{4}$$

$$\mathbf{c}_k^{(tr)} = \left[\mathbf{c}_k^{(tr,0)^T},\ldots \quad \right]^T \tag{5}$$

$$\boldsymbol{\Gamma}_k = \mathrm{blkdiag}\left\{ \boldsymbol{\Gamma}_k^{(1)},\ldots\ \boldsymbol{\Gamma} \right. \tag{6}$$

$$\boldsymbol{\Gamma}_k^{(r)} = \left[\boldsymbol{\Gamma}_k^{(1r)},\ldots\ \boldsymbol{\Gamma} \quad \right] \tag{7}$$

$$\boldsymbol{\Gamma}_k^{(tr)} = \left[\boldsymbol{\Gamma}_k^{(tr,0)},\ldots\ \boldsymbol{\Gamma} \quad \right] \tag{8}$$

$$\boldsymbol{\Gamma}_k^{(tr,l)} = \left[\mathbf{E}_k^{(tr)} \mathrm{diag}\left\{ \tilde{\mathbf{J}} \right. \right. \quad \ldots \quad \mathrm{ag}\left\{ \tilde{\mathbf{J}} \quad \right] \tag{9}$$

$$\tilde{\mathbf{J}} \quad {}_{-N_{cp}:N_{OFDM}-1,d} \tag{10}$$

$$\mathbf{S}_k^{(t,l)} = \left[\mathbf{S}_k^{(t)} \right]_{0:N\text{-}1,l} \tag{11}$$

$$\left[\mathbf{S}_k^{(t)} \right]_{\tilde{r}} \quad = s_{k,\{\tilde{r}}^{(t)} \tag{12}$$

for $d = 0,\ldots$, $l = 0,\ldots$, $\tilde{r}$ $\ldots$ and $\tilde{r}$ $\ldots$. $\mathbf{E}_k^{(tr)}$ is the diagonal frequency offset matrix which contains $\varepsilon_k^{(tr)}$. The notation $\mathrm{diag}\{\cdot\}$ and $\mathrm{blkdiag}\{\cdot\}$ refers to diagonal and block diagonal matrices respectively. The observation noise $\mathbf{w}_k$ has zero mean and covariance $\mathbf{R} = \sigma_w^2 \mathbf{I}$.

3. PROPOSED METHOD

The limitations of the EKF-based method in [1] are that the noise in the state-space model must be Gaussian and noise statistics must be known. Hence, its performance will degrade when noise statistics are unavailable or when non-Gaussian noise is present. The EHF provides a superior alternative to the EKF as it does not require knowledge of noise statistics and the noise need not be Gaussian.

We propose an EHF-based algorithm with the incorporation of BEM into the algorithm. The $N_T N_R (N_c L + 1)$x1 state vector consists of BEM coefficients and CFO:

$$\mathbf{x}_k = \left[\mathbf{c}_k^T, \varepsilon_k^{(11)},\ldots \quad \ldots \right. \tag{13}$$

Since the BEM coefficients and the CFO are varying slowly in time, they can be modelled as an autoregressive (AR) model of order one:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{v}_k \tag{14}$$

where $\mathbf{v}$ is a Gaussian noise with zero mean and covariance $\mathbf{Q}$. The state transition matrix and noise covariance can be expressed as:

$$\mathbf{A} = \mathrm{blkdiag}\left\{ \mathbf{A}^{(c,tr,l)},\ldots \quad ,\mathbf{A}^{(\varepsilon)} \right\} \tag{15}$$

$$\mathbf{A}^{(\varepsilon)} = \mathrm{diag}\left\{ \alpha^{(\varepsilon)},\ldots \right. \tag{16}$$

$$\mathbf{Q} = \mathrm{blkdiag}\left\{ \mathbf{Q}^{(c,tr,l)},\ldots \quad ,\mathbf{Q}^{(\varepsilon)} \right\} \tag{17}$$

$$\mathbf{Q}^{(\varepsilon)} = \mathrm{diag}\left\{ \sigma_\varepsilon^2,\ldots \right. \tag{18}$$

$\mathbf{A}^{(c,tr,l)}$ and $\mathbf{Q}^{(c,tr,l)}$ can be obtained by solving the Yule-Walker equations. The EHF is designed to minimize the estimation error, $\mathbf{e}_k = \mathbf{x}_k - \hat{\mathbf{x}}_k$, under the worse possible effect of the noises $\mathbf{w}$ and $\mathbf{v}$ [3, 8]. $\hat{\mathbf{x}}_k$ is the estimated state vector. Then, the cost function to be minimized is [3, 8]:

$$J = \frac{\sum_{k=1}^{K} \mathbf{e}_k^H \mathbf{e}_k}{\mathbf{e}_0^H \mathbf{P}_0^{-1} \mathbf{e}_0 + \sum_{k=1}^{K} \left[\mathbf{w}_k^H \mathbf{W}_k^{-1} \mathbf{w}_k + \mathbf{v}_k^H \mathbf{V}_k^{-1} \mathbf{v}_k \right]} \tag{19}$$

where K is the total number of OFDM symbols considered. The symmetric, positive-definite matrices $\mathbf{P}_0, \mathbf{W}_k$ and $\mathbf{V}_k$ are weighting matrices. The weighting matrices can be set to be equal to the initial estimation error, observation noise covariance and state noise covariance respectively if those information are available. Let $J < 1/\gamma$, where γ is a parameter chosen by the designer that controls the level of noise attenuation. This optimization can be solved using the game theory approach [8, 9]. The following are the recursive equations of our proposed method:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}\hat{\mathbf{x}}_{k-1|k-1} \tag{20}$$

$$\mathbf{P}_{k|k-1} = \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^H + \mathbf{V}_k \tag{21}$$

$$\begin{aligned} \mathbf{K}_k = \mathbf{P}_{k|k-1} \big(& \mathbf{I} - \gamma \mathbf{P}_{k|k-1} \\ & + \mathbf{G}_k^H \mathbf{W}_k^{-1} \mathbf{G}_k \mathbf{P}_{k|k-1} \big)^{-1} \mathbf{G}_k^H \mathbf{W}_k^{-1} \end{aligned} \tag{22}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{y}_k - \hat{\mathbf{\Gamma}}_k \hat{\mathbf{c}}_k \right) \tag{23}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} \left(\mathbf{I} - \gamma \mathbf{P}_{k|k-1} + \mathbf{G}_k^H \mathbf{W}_k^{-1} \mathbf{G}_k \mathbf{P}_{k|k-1} \right)^{-1} \tag{24}$$

where:

$$\mathbf{G}_k = \left[\left. \frac{\partial \mathbf{g}(\mathbf{x}_k)}{\partial \mathbf{c}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|k-1}}, \left. \frac{\partial \mathbf{g}(\mathbf{x}_k)}{\partial \boldsymbol{\varepsilon}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|k-1}} \right] \tag{25}$$

Note that the computation of the Jacobian $\mathbf{G}$ is different from the existing EHF algorithm in [3] due to the incorporation of BEM. The condition $0 < \gamma < \gamma_{\max}$ must be satisfied to ensure the positive-definiteness of $\mathbf{P}_{k|k-1}$ where:

$$\gamma_{\max} = \text{eigen}_{\max} \left[\left(\mathbf{P}_{k|k-1}^{-1} + \mathbf{G}_k^H \mathbf{W}_k^{-1} \mathbf{G}_k \right)^{-1} \right] \tag{26}$$

The notation $\text{eigen}_{\max}$ refers to the maximum eigenvalue. Since $\mathbf{Q}$ can be obtained by solving the Yule-Walker equations, $\mathbf{V}_k$ is set as $\mathbf{V}_k = \mathbf{Q}$. As the observation noise covariance is considered to be unavailable in this paper, $\mathbf{W}_k$ is estimated as $\mathbf{W}_k = \hat{\sigma}_w^2 \mathbf{I}$ where:

$$\hat{\sigma}_w^2 = \frac{1}{N_R N k} \sum_{k=1}^{K} \left(\mathbf{y}_k - \hat{\mathbf{\Gamma}}_k \hat{\mathbf{c}}_k \right)^H \left(\mathbf{y}_k - \hat{\mathbf{\Gamma}}_k \hat{\mathbf{c}}_k \right) \tag{27}$$

Next, we propose a method to improve the performance of the algorithm when time-multiplexed pilots are used. Consider that $N/4$ pilots are inserted at the beginning of every OFDM symbol. Since the BEM coefficients are considered to be constant for every OFDM symbol, the pilots can be used to directly estimate the BEM coefficients and CFO of an OFDM symbol using (20)-(24). After obtaining the channel and CFO estimates, a MMSE equalizer is implemented in frequency domain to equalize and detect the $3N/4$ data samples in that OFDM symbol. The MMSE equalizer used in this paper is similar to the one in [10]. The estimated data samples are then used to update the estimated BEM coefficients and CFO for the same symbol by repeating (22)-(24), where the estimated data samples is used instead of the pilots this time. The entire process will be repeated iteratively for every OFDM symbol to track the channel gains and CFO.

4. SIMULATION RESULTS

For our simulations, we consider a MIMO-OFDM system with $N_t = 2$, $N_R = 2$, $N = 128$, $N_{cp} = 16$, $L = 6$, carrier frequency $f_c = 5\text{GHz}$, $1/T_s = 2$ MHz, and $K = 2000$. Rayleigh fading channel with i.i.d. paths and Jakes spectrum is considered. There are 6 channel taps with relative power of [-6.48, -5.48, -6.87, -8.92, -10.51, -11.61] dB. The mobile velocity considered is $v^{(mobile)} = 300$ km/h, corresponding to a maximum Doppler shift of 1389 Hz. Performance comparison of the proposed algorithm with existing methods is done by evaluating the normalized mean square error (NMSE) of the estimation.

We first compare the performance of the proposed method when using different BEM designs (P-BEM, GCE-BEM and KL-BEM) and different N_c to determine the optimal N_c and BEM design. The NMSE performance is calculated based on the difference between the true channel gains and channel gains recovered from the estimated BEM coefficients, which ensures that both BEM modelling and estimation errors are taken into consideration. For a fixed N_c, we found that the NMSE performance for all BEM designs are almost similar at different SNR. Thus, we chose P-BEM as no a priori information is required. Varying N_c between 2 and 5 does not affect the NMSE performance much. For Gaussian noise, the performance when $N_c = 2$ is slightly better than the rest at low SNR while $N_c = 4$ is best at high SNR. For non-Gaussian noise, the performance when $N_c = 3$ is best at low SNR and $N_c = 4$ is best at high SNR. We select $N_c = 3$ for optimal performance between the different SNRs and noise distributions.

Next, the performance of the proposed EHF-based method is compared against the EKF-based method in [1]. For both algorithms, we set $\alpha^{(\varepsilon)} = 0.9999$, $\sigma_\varepsilon^2 = 0.0001$, while $\mathbf{A}^{(c)}$ and $\mathbf{Q}^{(c)}$ are obtained through Yule-Walker equations. The choice of $\gamma = 0.005$ is determined through

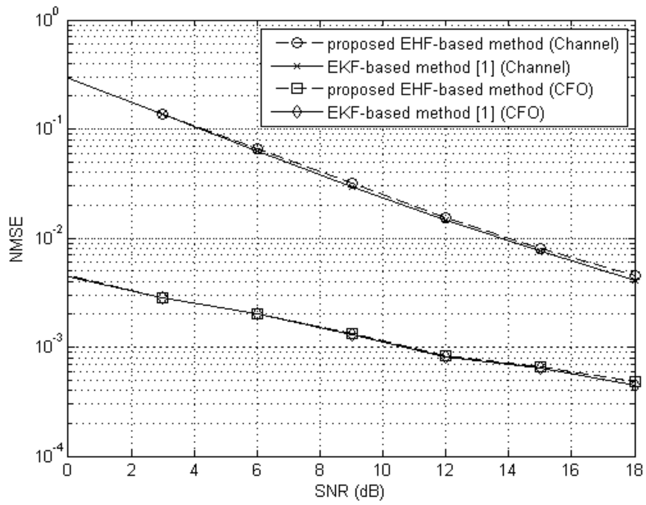

Fig. 1: NMSE of channel and CFO tracking versus SNR under Gaussian observation noise.

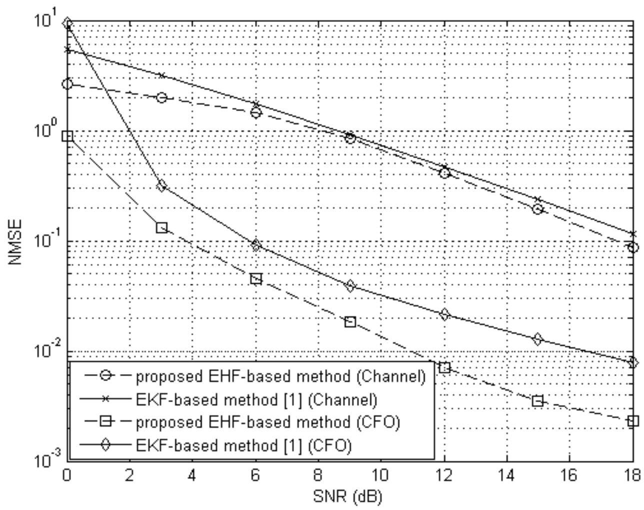

Fig. 2: NMSE of channel and CFO tracking versus SNR under non-Gaussian observation noise

experimentation. For the method in [1], all noise statistics are assumed to be available. For the proposed method, state noise covariance is assumed to be known while observation noise knowledge is unavailable. For initialization, $\hat{\mathbf{x}}_{0|0} = \mathbf{0}$ and $\mathbf{P}_{0|0} = \text{blkdiag}\left\{\mathbf{R}_c\left[0\right], \sigma_\varepsilon^2\mathbf{I}\right\}$ where $\mathbf{R}_c[0]$ is the BEM coefficients correlation matrix [1]. Both algorithms are using time-multiplexed pilots, with pilots length of $N/4$. Fig. 1 shows the plot of NMSE performance of channel and CFO tracking versus the signal-to-noise ratio (SNR) under Gaussian observation noise. It can be seen that the proposed EHF-based method can match the performance of the EKF-based method for both channel and CFO although the proposed method does not have knowledge of observation noise statistics.

Next, non-Gaussian noise is considered. For our simulations, a Gaussian mixture noise [2] is considered

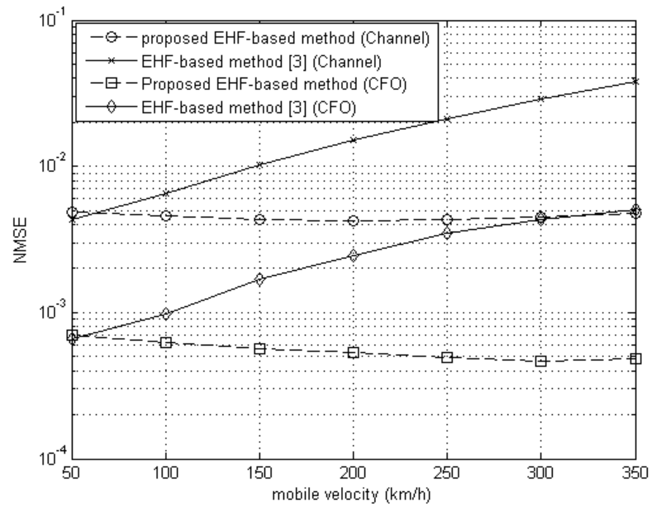

Fig. 3: NMSE of channel and CFO tracking versus mobile velocity.

where probability of impulsive noise occurrence is $\beta = 0.2$ and the ratio of the impulsive component variance to the nominal component is $\kappa = 10$. Fig. 2 shows the NMSE plot versus SNR under non-Gaussian observation noise. It can be seen that the proposed EHF-based method outperforms the existing EKF-based method as the existing method requires the observation noise to be Gaussian while the proposed method does not have such restrictions.

Finally, we compare the performance of the proposed method against the EHF-based method in [3] which is designed for slow time-varying channel. Fig. 3 shows NMSE versus increasing mobile velocity for SNR=18 dB under Gaussian observation noise. It can be seen that the performance of the method in [3] degrades for high mobile velocities as the channel is varying faster at high mobile velocities. On the other hand, the performance of the proposed method does not degrade for high velocities. Any method designed for slow time-varying channel is likely to suffer similar performance degradation due to the assumption of invariant channel within an OFDM symbol.

5. CONCLUSION

In this paper, we have presented a robust joint CFO and fast time-varying channel tracking algorithm for MIMO-OFDM system. The EHF-based algorithm utilizes BEM to track channel gains that vary significantly within an OFDM symbol. The proposed algorithm does not require knowledge of noise statistics and is more robust to unknown noise. The proposed algorithm is shown to match the performance of existing method under Gaussian noise and outperforms the existing method under non-Gaussian noise.

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