New Subspace-Based Blind Channel Estimation for Orthogonally Coded MIMO-OFDM Systems

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***Abstract*— Two novel subspace-based blind channel estimation methods for orthogonally coded multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems with more than two transmit antennas are proposed in this paper. The proposed methods exploit the null space induced by the orthogonal space-time block codes (OSTBC). They can be applied to systems with one or more receive antennas. Simulation results show that accurate estimates can be obtained.**

***Index Terms*—OFDM, MIMO, orthogonal space-time block code (OSTBC), blind channel estimation**

I. Introduction

In recent years, space-time block coding (STBC) has emerged as a powerful approach to exploit spatial diversity and to combat fading in multiple-input multiple-output (MIMO) wireless communication systems. In 1998, Alamouti's code for two transmit antennas [1] has been incorporated as a simple and efficient transmit diversity technique. STBC with orthogonal designs for any number of transmit antennas was proposed in [2]. The combination of orthogonal space-time block codes (OSTBC) and orthogonal frequency division multiplexing (OFDM), or simply OSTBC-OFDM, has drawn much attention because it attains the maximum transmit diversity and has a simple maximum likelihood (ML) receiver structure. It is known that the performance of MIMO-OFDM systems critically depend on the accuracy of the estimated channel responses. Many channel estimation methods have been proposed in the past. Our paper focuses on blind channel estimation.

Most of the existing blind MIMO channel estimation methods can only deal with flat fading channels [3] [4]. The OFDM system converts the frequency-selective MIMO channel into many flat-fading subcarriers in MIMO-OFDM systems. The subcarrier-wise approach may suffer from a prohibitively high computational complexity when the number of subcarriers is large. A coherent approach was proposed in [5] for blind channel estimation for orthogonally coded MIMO-OFDM systems with more than 2 transmit antennas. Because the approach is based on convex optimization and amounts to solving a semidefinite program (SDP), its computational cost may be rather high. In [6], the authors proposed a lower computational-complexity method. The method gives a good performance but it works only when there are more than one receive antenna.

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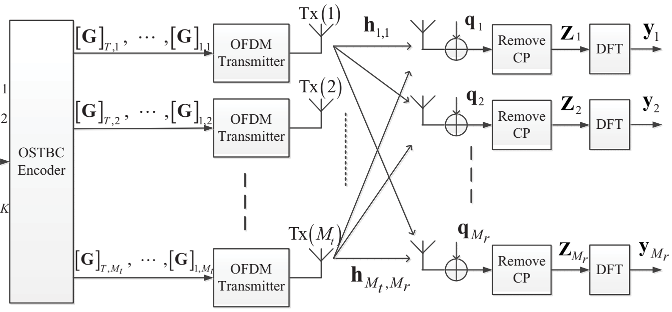

Fig. 1: Block diagram of OSTBC-OFDM system

In this paper, we propose two new algorithms for blind channel estimation. In the first part, we propose a new subspace-based method for blind channel estimation in MIMO-OFDM systems with more than 2 transmit antennas. The proposed method exploits the null space induced by the OSTBCs with rate less than one. The method works even when there is only one receive antenna. In the second part, we introduce a modification of the proposed method so that fewer received blocks are needed. The minimum number of received blocks needed can be as small as one for most practical systems. Numerical simulation is also given to verify the performance of the proposed methods.

This paper is organized as follows. The system model of OSTBC-OFDM is introduced in Section II. In Section III, we propose a subspace-based method that can estimate channel using noise subspace. In Section IV, we propose a modified estimation method which requires fewer received blocks. Simulation results are presented in Section V and concluding remarks are provided in Section VI.

Notation: In this paper, the symbols $\mathbf{A}^T$, $\mathbf{A}^*$, and $\mathbf{A}^\dagger$ denote the transpose, the complex conjugate, and the conjugate-transpose of matrix $\mathbf{A}$ respectively. The symbol $\otimes$ denotes the Kronecker product and $\|\cdot\|_F$ denotes the Frobenius norm.

II. System model

In this paper, we consider MIMO-OFDM systems employing OSTBC. The block diagram of OSTBC-based MIMO-OFDM system model is shown in Fig. 1, where M_t and M_r denote the numbers of transmit and receive antennas respectively. For each OFDM transmitter, let N and L be respectively the size of the discrete Fourier transform (DFT) and the length of the cyclic prefix (CP). Let us denote the transmission matrix of the OSTBC by the $T \times M_t$ matrix $\mathbf{G}$.

For example, the 4×3 transmission matrix for 3 transmit antennas with 3 information symbols is

$$\mathbf{G}_3(k) = \begin{bmatrix} s_1(k) & s_2(k) & s_3(k) \\ -s_2^*(k) & s_1^*(k) & 0 \\ s_3^*(k) & 0 & -s_1^*(k) \\ 0 & -s_3^*(k) & s_2^*(k) \end{bmatrix} \quad (1)$$

for $k = 0, 1, \ldots, N-1$, where k denotes subcarrier index and $s_i(k)$ is the ith modulation symbol transmitted on the kth subcarrier. The OSTBC in (1) has a rate of $R = 3/4$.

Assume that the channel orders do not exceed the CP length L and they do not vary during the transmission time of T. Then at the receiver, the kth subcarrier received signal of the m_rth antenna is given by

$$y_{m_r,t}(k) = \sum_{m_t=1}^{M_t} H_{m_t,m_r}(k)\,[\mathbf{G}(k)]_{t,m_t} + q_{m_r,t}(k) \quad (2)$$

for $0 \le k < N$, $1 \le t \le T$, and $1 \le m_r \le M_r$, where H_{m_t,m_r} is the frequency response of the channel between the m_tth transmit antenna and the m_rth receive antenna, and $q_{m_r,t}(k)$ is the noise which is assumed to be an additive white Gaussian noise (AWGN) with variance σ_n^2. For clarity, in this paper we derive our results for the case of 3 transmit antennas and 1 receive antenna ($M_t = 3$ and $M_r = 1$). The proposed method can easily be extended to other OSTBCs with $M_t \ge 3$. Extension to the case of $M_r > 1$ will be explained at the end of each section.

III. Proposed Blind Channel Estimation

Consider an OSTBC-OFDM system with $M_t = 3$ and $M_r = 1$. Utilizing (1) and (2), the 4 received signals, $y_{m_r,t}(k)$ for $t = 1, 2, 3, 4$, at the kth subcarrier in (2) can be written as (For simplicity, we drop the subscript m_r.)

$$\mathbf{y}(k) \triangleq \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \end{bmatrix} = \mathbf{H}(k) \underbrace{\begin{bmatrix} s_1(k) \\ s_2(k) \\ s_3(k) \\ s_1^*(k) \\ s_2^*(k) \\ s_3^*(k) \end{bmatrix}}_{\mathbf{s}(k)} + \underbrace{\begin{bmatrix} q_1(k) \\ q_2(k) \\ q_3(k) \\ q_4(k) \end{bmatrix}}_{\mathbf{q}(k)}, \quad (3)$$

where

$$\mathbf{H}(k) = \begin{bmatrix} H_1(k) & H_2(k) & H_3(k) & 0 & 0 & 0 \\ 0 & 0 & 0 & H_2(k) & -H_1(k) & 0 \\ 0 & 0 & 0 & -H_3(k) & 0 & H_1(k) \\ 0 & 0 & 0 & 0 & H_3(k) & -H_2(k) \end{bmatrix}. \quad (4)$$

Note that one OSTBC-OFDM block consists of 4 OFDM symbols for $t = 1, 2, 3, 4$. Assume that we have collected J received blocks (TJ OFDM symbols) and let the superscript $^{(i)}$ denote the block index. Stacking the J blocks together, we form

$$\mathbf{Y}(k) \triangleq \begin{bmatrix} \mathbf{y}^{(1)}(k) & \mathbf{y}^{(2)}(k) & \cdots & \mathbf{y}^{(J)}(k) \end{bmatrix}. \quad (5)$$

Utilizing (3), we can write

$$\mathbf{Y}(k) = \mathbf{H}(k) \underbrace{\begin{bmatrix} \mathbf{s}^{(1)}(k) \cdots \mathbf{s}^{(J)}(k) \end{bmatrix}}_{\mathbf{S}(k)} + \underbrace{\begin{bmatrix} \mathbf{q}^{(1)}(k) \cdots \mathbf{q}^{(J)}(k) \end{bmatrix}}_{\mathbf{Q}(k)}. \quad (6)$$

Assuming that the signal and noise are uncorrelated, we have

$$\mathbf{Y}(k)\mathbf{Y}^\dagger(k) = \mathbf{H}(k)\mathbf{S}(k)\mathbf{S}^\dagger(k)\mathbf{H}^\dagger(k) + \mathbf{Q}(k)\mathbf{Q}^\dagger(k). \quad (7)$$

Note that the 4×6 matrix $\mathbf{H}(k)$ has rank equal to 3 only. There exists a nonzero vector

$$\mathbf{v}(k) = \begin{bmatrix} v_0(k) & v_1(k) & v_2(k) & v_3(k) \end{bmatrix}^T \quad (8)$$

such that $\mathbf{v}^\dagger(k)\mathbf{H}(k)\mathbf{S}(k)\mathbf{S}^\dagger(k)\mathbf{H}^\dagger(k)\mathbf{v}(k) = 0$. Suppose that J is large enough so that $\mathbf{S}(k)\mathbf{S}^\dagger(k)$ has full rank. It implies that $\mathbf{v}(k)$ satisfies $\mathbf{v}^\dagger(k)\mathbf{H}(k) = \mathbf{0}$, which is equivalent to:

$$\underbrace{\begin{bmatrix} v_0^*(k) & 0 & 0 \\ 0 & v_0^*(k) & 0 \\ 0 & 0 & v_0^*(k) \\ 0 & v_1^*(k) & -v_2^*(k) \\ -v_1^*(k) & 0 & v_3^*(k) \\ v_2^*(k) & -v_3^*(k) & 0 \end{bmatrix}}_{\triangleq \mathbf{V}^*(k)} \begin{bmatrix} H_1(k) \\ H_2(k) \\ H_3(k) \end{bmatrix} = \mathbf{0}. \quad (9)$$

Combining all N subcarriers, we have

$$\underbrace{\begin{bmatrix} \mathbf{V}^*(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^*(1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{V}^*(N-1) \end{bmatrix}}_{\triangleq \tilde{\mathbf{V}}^*} \tilde{\mathbf{h}}^f = \mathbf{0}, \quad (10)$$

where $\tilde{\mathbf{h}}^f = [H_1(0), H_2(0), H_3(0), H_1(1), H_2(1), H_3(1), \ldots, H_1(N-1), H_2(N-1), H_3(N-1)]^T$. $\tilde{\mathbf{h}}^f$ can be written as

$$\tilde{\mathbf{h}}^f = \begin{bmatrix} \mathbf{I}_N \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \mathbf{I}_N \otimes \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \mathbf{I}_N \otimes \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^f \\ \mathbf{h}_2^f \\ \mathbf{h}_3^f \end{bmatrix}, \quad (11)$$

where $\mathbf{h}_i^f = \begin{bmatrix} H_i(0) & H_i(1) & \cdots & H_i(N-1) \end{bmatrix}^T$. Assume that the channel orders do not exceed the CP length L. Then we can write

$$\begin{bmatrix} \mathbf{h}_1^f \\ \mathbf{h}_2^f \\ \mathbf{h}_3^f \end{bmatrix} = (\mathbf{I}_3 \otimes \mathbf{W}_{L+1}) \underbrace{\begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix}}_{\mathbf{h}}, \quad (12)$$

where $\mathbf{W}_{L+1}$ is the first $L+1$ columns of the DFT matrix $\mathbf{W}$ and the $(L+1) \times 1$ vectors $\mathbf{h}_i = \begin{bmatrix} h_i(0) & h_i(1) & \cdots & h_i(L) \end{bmatrix}^T$ contain the channel impulse responses for $i = 1, 2, \ldots, M_t$. Therefore, (10) can be rewritten as

$$\tilde{\mathbf{V}}^* \tilde{\mathbf{W}}_{L+1} \mathbf{h} = \mathbf{0}, \quad (13)$$

where (Note that we have used the formula $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC}\otimes\mathbf{BD})$ to get the following expression)

$$\tilde{\mathbf{W}}_{L+1} = \left[\mathbf{W}_{L+1}\otimes\begin{bmatrix}1\\0\\0\end{bmatrix} \; \mathbf{W}_{L+1}\otimes\begin{bmatrix}0\\1\\0\end{bmatrix} \; \mathbf{W}_{L+1}\otimes\begin{bmatrix}0\\0\\1\end{bmatrix}\right]. \quad (14)$$

The channel $\mathbf{h}$ can be obtained by (13).

In practice, there is the channel noise $\mathbf{Q}(k)$ in (6). We can estimate $\mathbf{v}(k)$ by

$$\hat{\mathbf{v}}(k) = \arg\min_{\|\mathbf{v}(k)\|_F^2=1} \mathbf{v}^\dagger(k)\mathbf{Y}(k)\mathbf{Y}^\dagger(k)\mathbf{v}(k) \quad (15)$$

and form the matrix $\tilde{\mathbf{V}}$ using the estimates $\hat{\mathbf{v}}(k)$. Then the channel can be estimated (up to a scalar ambiguity) as

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|_F^2=M_t} \mathbf{h}^\dagger\tilde{\mathbf{W}}_{L+1}^\dagger\tilde{\mathbf{V}}^T\tilde{\mathbf{V}}^*\tilde{\mathbf{W}}_{L+1}\mathbf{h}. \quad (16)$$

Remarks:

(i) It can be verified (proof omitted due to space limitation) that the channel matrix $\mathbf{H}(k)$ in (4) does not have full row rank when the underlying OSTBC has rate $R < 1$. It was proved in [7] that the rate of complex orthogonal designs for more two transmit antennas is always less than one. Therefore, the proposed method can be applied to all OSTBC-OFDM systems with more than two transmit antennas.

(ii) When $M_r \geq 2$, we can stack all received signals with the kth subcarrier. So (3) becomes

$$\check{\mathbf{y}}(k) \triangleq \begin{bmatrix}\mathbf{y}_1(k)\\\mathbf{y}_2(k)\\\vdots\\\mathbf{y}_{M_r}(k)\end{bmatrix} = \underbrace{\begin{bmatrix}\mathbf{H}_1(k)\\\mathbf{H}_2(k)\\\vdots\\\mathbf{H}_{M_r}(k)\end{bmatrix}}_{\check{\mathbf{H}}(k)}\mathbf{s}(k) + \underbrace{\begin{bmatrix}\mathbf{q}_1(k)\\\mathbf{q}_2(k)\\\vdots\\\mathbf{q}_{M_r}(k)\end{bmatrix}}_{\check{\mathbf{q}}(k)}, \quad (17)$$

where $\mathbf{y}_i(k)$, $\mathbf{H}_i(k)$, and $\mathbf{q}_i(k)$ are the corresponding terms at the ith receive antenna. By the result of remark (i), $\check{\mathbf{H}}(k)$ does not have full row rank when $M_t \geq 3$. Following a similar derivation, we can obtain an expression of channel estimate $\hat{\mathbf{h}}$ similar to (16).

(iii) For some OSTBCs, the dimension of the left null space of $\mathbf{H}(k)$ can be larger than 1. In this case, there exist more than one linearly independent vector $\mathbf{v}_1(k), \mathbf{v}_2(k), \ldots, \mathbf{v}_m(k)$ such that $\mathbf{v}_i^\dagger(k)\mathbf{H}(k) = \mathbf{0}$. Then we can form $\tilde{\mathbf{V}}_1^*, \tilde{\mathbf{V}}_2^*, \ldots, \tilde{\mathbf{V}}_m^*$ in (10) for each null vector. Hence, the channel can be estimated by

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|_F^2=M_t} \mathbf{h}^\dagger\tilde{\mathbf{W}}_{L+1}^\dagger\left(\sum_{i=1}^m \tilde{\mathbf{V}}_i^T\tilde{\mathbf{V}}_i^*\right)\tilde{\mathbf{W}}_{L+1}\mathbf{h}. \quad (18)$$

(iv) Assume that the channels $\mathbf{h}_1$, $\mathbf{h}_2$, and $\mathbf{h}_3$ have no common factor and at least one of the channel orders is equal to the CP length L. Then it can be shown that the channels can be uniquely identified up to a scalar ambiguity, or equivalently, $\hat{\mathbf{h}} = c\mathbf{h}$ for some scalar c. The proof is omitted due to the limitation of space.

IV. A MODIFIED ESTIMATION METHOD WITH FEWER RECEIVED BLOCKS

The method proposed in Section III works only when the 6×6 matrix $\mathbf{S}(k)\mathbf{S}^\dagger(k)$ in (7) have full rank. Thus, the minimum number of the received blocks is $J = 6$. Since there are T OFDM symbols for each space-time block, we need to collect $6T$ OFDM symbols for channel estimation. In this section, we propose a modified method for channel estimation with fewer received blocks.

Let us rewrite (9) as

$$\begin{aligned} v_0^*(k)H_1(k) = v_0^*(k)H_2(k) = v_0^*(k)H_3(k) &= 0,\\ v_1^*(k)H_2(k) &= v_2^*(k)H_3(k),\\ v_1^*(k)H_1(k) &= v_3^*(k)H_3(k),\\ v_2^*(k)H_1(k) &= v_3^*(k)H_2(k). \end{aligned} \quad (19)$$

Assume that $H_1(k)$, $H_2(k)$, and $H_3(k)$ are not all zero (i.e., the channels have no common zero at the DFT frequencies.) From (19), we have the relation

$$\begin{bmatrix}v_0^*(k)\\v_1^*(k)\\v_2^*(k)\\v_3^*(k)\end{bmatrix} = \alpha_k\underbrace{\begin{bmatrix}0&0&0\\0&0&1\\0&1&0\\1&0&0\end{bmatrix}}_{\triangleq\mathbf{P}}\begin{bmatrix}H_1(k)\\H_2(k)\\H_3(k)\end{bmatrix}, \quad (20)$$

where α_k is a nonzero scalar. In the absence of noise, we get $\mathbf{v}^\dagger(k)\mathbf{Y}(k)\mathbf{Y}^\dagger(k)\mathbf{v}(k) = \mathbf{0}$. Utilizing (20), we obtain

$$\begin{bmatrix}H_1^*(k) & H_2^*(k) & H_3^*(k)\end{bmatrix}\mathbf{P}^\dagger\mathbf{Y}^*(k)\mathbf{Y}^T(k)\mathbf{P}\begin{bmatrix}H_1(k)\\H_2(k)\\H_3(k)\end{bmatrix} = \mathbf{0}. \quad (21)$$

Stacking all vectors together, (21) can be written as

$$\left(\tilde{\mathbf{h}}^f\right)^\dagger(\mathbf{I}_N\otimes\mathbf{P})^\dagger\boldsymbol{\Omega}(\mathbf{I}_N\otimes\mathbf{P})\tilde{\mathbf{h}}^f = \mathbf{0}, \quad (22)$$

where $\tilde{\mathbf{h}}^f$ is defined in (10) and

$$\boldsymbol{\Omega} \triangleq \begin{bmatrix}\mathbf{Y}^*(0)\mathbf{Y}^T(0) & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{Y}^*(1)\mathbf{Y}^T(1) & \ddots & \vdots\\ \vdots & \ddots & \ddots & \mathbf{0}\\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{Y}^*(N-1)\mathbf{Y}^T(N-1)\end{bmatrix}. \quad (23)$$

From (14), we define a $NT\times M_t(L+1)$ matrix

$$\mathbf{F} \triangleq (\mathbf{I}_N\otimes\mathbf{P})\tilde{\mathbf{W}}_{L+1}. \quad (24)$$

Utilizing (11), (12), (22), and (24), we conclude that the channel (up to a scalar ambiguity) can be estimated as

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|_F^2=M_t} \mathbf{h}^\dagger\mathbf{F}^\dagger\boldsymbol{\Omega}\mathbf{F}\mathbf{h}. \quad (25)$$

Notice that a necessary condition for the uniqueness is that the rank of $\boldsymbol{\Omega}$ is larger than or equal to $M_t(L+1)-1$. Assume that we have collected J received blocks to form $\boldsymbol{\Omega}$. Then the rank of $\boldsymbol{\Omega}$ is $N\times\min\{J, M_t\}$, so the necessary condition becomes $\min\{J, M_t\} \geq (M_t(L+1)-1)/N$. In practice, N is

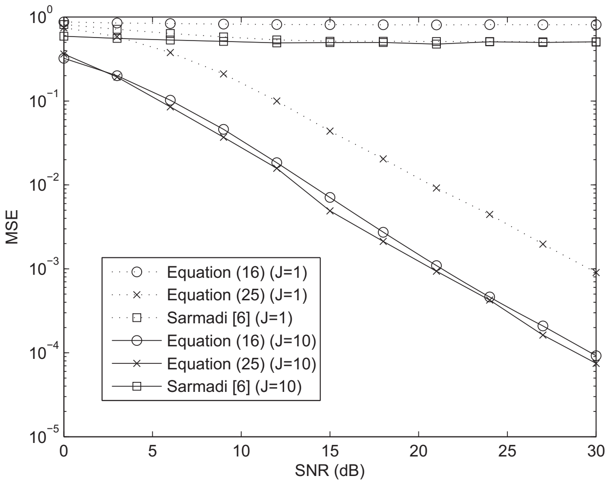

Fig. 2: Comparison of the MSE performance with single receive antenna ($M_r = 1$)

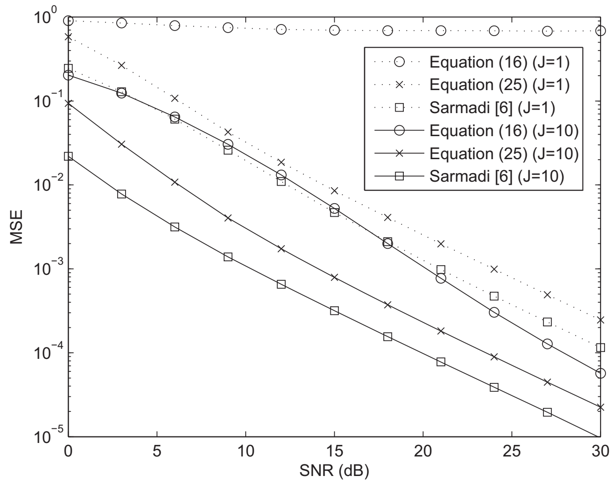

Fig. 3: Comparison of the MSE performance with multiple receive antennas ($M_r = 3$)

usually much larger than L. Therefore, the minimum number of blocks needed is

$$J \geq \frac{M_t(L+1)-1}{N}.$$

For many systems, this minimum number can be as small as $J = 1$.

Remark: The method can be extended to the case of $M_r > 1$ by stacking together all the channel taps for $m_r = 1, 2, \ldots, M_r$. The details are omitted due to the limitation of space.

V. Simulation results

We assume that the channel does not change while channel estimation is performed. Channel taps are generated as independent and identically distributed (i.i.d.) random variables. The channel noise is AWGN and the transmission symbols are modulated by 16 points quadrature amplitude modulation (16-QAM). Two cases of the numbers of blocks are considered (i) $J = 1$; (ii) $J = 10$.

The OFDM block size is $N = 64$, and the length of CP is $L = 4$. The number of transmit antennas is $M_t = 3$, and the rate $R = 3/4$ OSTBC in (1) is used.

The mean square error (MSE) is defined as

$$\text{MSE} = \frac{1}{N_{runs}} \sum_{i=1}^{N_{runs}} \frac{\|c\hat{\mathbf{h}} - \mathbf{h}\|_F^2}{\|\mathbf{h}\|_F^2},$$

where the factor $c = (\hat{\mathbf{h}}^\dagger \mathbf{h})/(\|\hat{\mathbf{h}}\|_F^2)$ and $N_{runs} = 2000$ denotes the total number of Monte Carlo trials.

First assume that the number of receive antenna is $M_r = 1$. In Fig. 2, we plot the performances of the subspace-based methods in (16) and (25), and Sarmadi's algorithm of [6]. We observe that Sarmadi's algorithm does not work in $M_r = 1$ case. For the case of one block ($J = 1$), (16) can not work since the matrix $\mathbf{S}(k)\mathbf{S}^\dagger(k)$ in (7) does not have full rank. On the other hand, the simulation shows that the modified method (25) requires only one receive OSTBC-OFDM block to obtain good estimation performance. For the case of ten blocks ($J = 10$), the performance of (25) is little better than that of (16).

Next we consider the case of multiple receive antennas $M_r = 3$. Fig. 3 shows the results. We find that Sarmadi's algorithm can work in this case and the performance is better than our methods. The gain is approximately 3dB. For the case of one block ($J = 1$), (16) can still not work. For the case of ten blocks ($J = 10$), the performance of the modified method (25) is better than that of (16). The gain is about 3dB as well. Therefore, the modified method not only needs fewer blocks but also has a better performance when the block number is the same ($J = 10$).

VI. Conclusions

In this paper, we propose two novel blind channel estimation methods. The first method can work for all OSTBC-OFDM systems with 3 or more transmit antennas and no matter how many receive antennas there are. The second method can estimate channel with fewer received blocks. Simulation results verify the performances of the proposed methods.

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