CHANNEL ESTIMATION FOR LTE AND LTE-A MU-MIMO UPLINK WITH A NARROW TRANSMISSION BAND

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ABSTRACT

The 3GPP LTE and LTE-A standards provide for multi-user multi-input multi-output (MU-MIMO) transmission in the uplink. But the property of reference signals (RSs) is such that care must be exercised in channel estimation to minimize the interference among different antenna channels. Typical DFT- and DCT-based channel estimation methods have low complexity but may yield relatively high inter-channel interference, especially when user equipments (UEs) are allocated narrow transmission bands. Methods that can attain better channel separation often have much higher computational requirements. In this paper, we propose an MU channel estimation technique that seeks to minimize an L_1 norm of error. It does not need second-order statistics as the linear minimum mean-square error (LMMSE) technique, has good numerical properties, and yields a performance between the least-square (LS), the DFT, and the LMMSE techniques.

Index Terms— Channel estimation, MU-MIMO, LTE, LTE-A, uplink, L_1-norm.

1. INTRODUCTION

The 3GPP LTE and LTE-A standards provide for multi-user multi-input multi-output (MU-MIMO) transmission in the uplink to boost the spectrum efficiency in the condition where user equipments (UEs) have fewer antennas than the base station (termed evolved Node B, or eNB). LTE and LTE-A both adopt single-carrier frequency-division multiple access (SC-FDMA) in the uplink. Consider the physical uplink shared channel (PUSCH). To facilitate channel estimation, there is one reference signal (RS) symbol per slot, where each slot contains 7 or 6 SC-FDMA symbols depending on whether the system uses normal or extended cyclic prefix (CP) [1]. For each UE, the RS occupies the same subcarriers as that allocated to the UE for data transmission. In MU-MIMO, the RSs transmitted from different UE antennas are the same except that each has a different linear phase shift in the frequency domain so that the channel responses associated with different UE antennas can be distinguished in channel estimation. Specifically, the phase shifts are such that the channel impulse responses (CIRs) associated with different UE antennas are evenly spaced out over the length of an SC-FDMA symbol's useful time period. Hence these CIRs can be clearly separated as long as their lengths are properly limited.

A popular family of channel estimation methods for multicarrier systems, including orthogonal frequency-division multiplexing (OFDM) and SC-FDMA, are the DFT- and DCT-based methods [2–13]. When applied to uplink MU-MIMO in LTE and LTE-A, however, the bandlimited structure of RS leads to spreading (alternatively known as energy leakage) of CIRs in the time domain, which results in mutual interference between the estimated CIRs of the different antenna channels. Such "inter-channel interference (ICI)" in channel estimation is especially troublesome when the UEs are allocated narrow uplink bands, and the severity of the problem also increases with number of co-scheduled UEs [9–12]. A tradeoff thus exists between the amount of "CIR truncation error" and the amount of retained ICI. One way to deal with the above problem is to make use of quasi-pseudo-inverse in DCT-based least-squares (LS) estimation [13]. But numerical property nevertheless poses some issue in matrix inversion.

Besides the DFT and DCT approach, an approach that easily comes to mind is linear minimum mean-square error (LMMSE) estimation. However, it requires some second-order statistics of the channel as well as matrix inversion, giving rise to practical and numerical issues.

In this paper, we propose a technique that seeks to minimize an L_1 norm of channel estimation error. It does not require matrix inversion and thus has good numerical stability, and it yields a performance between the LS, DFT, and LMMSE techniques. The rest of this paper is organized as follows. Section 2 formulates the problem in a way that helps the derivation MU-MIMO channel estimators. Section 3 presents several channel estimators based on the formulation given in Sec. 2. They include the LS, LMMSE, and the proposed method based on L_1-norm minimization. Section 4 presents some simulation results. And Section 5 is the conclusion.

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2. PROBLEM FORMULATION

Let the system employ size-N FFT/IFFT. Consider a set of U single-antenna UEs that are co-scheduled for uplink MU-MIMO. The UEs are indexed $u = 0, 1, \ldots, U-1$. Let them be allocated B subcarriers. Let $\mathbf{x}$ be the B-vector of the RS sequence assigned to these UEs for a given RS symbol. Then the transmitted RS vector of UE u, in the frequency domain, can be written as

$$\mathbf{g}_u = \mathbf{W}_u \mathbf{x} \tag{1}$$

where $\mathbf{W}_u = \text{diag}(e^{j\frac{2\pi N_u \cdot 0}{N}}, e^{j\frac{2\pi N_u \cdot 1}{N}}, \ldots, e^{j\frac{2\pi N_u \cdot (B-1)}{N}})$ with N_u being the delay (in number of time-domain samples) assigned to the CIR of UE u.

Let $\mathbf{F}$ be the $N \times N$ normalized DFT matrix whose (p, q)th element is given by $[\mathbf{F}]_{p,q} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi qp}{N}}$. Let the B subcarriers assigned to the U UEs begin at index k_0. Assume perfect synchronization. Then the received RS vector of length N, in the time domain, at an eNB antenna after removal of CP is given by

$$\mathbf{r} = \sum_{u=0}^{U-1} \mathbf{H}_u^t \mathbf{F}_B^H \mathbf{W}_u \mathbf{x} + \mathbf{r}_o + \mathbf{n}^t \tag{2}$$

where $\mathbf{H}_u^t$ is the $N \times N$ circulant matrix with its first column given by the CIR associated with UE u, $\mathbf{F}_B$ is a "bandlimited DFT matrix" consisting of the k_0th to the (k_0+B-1)th rows of $\mathbf{F}$, superscript H denotes Hermitian transpose, $\mathbf{r}_o$ is the contribution from other UEs (which are allocated other subcarriers than the B of concern), and $\mathbf{n}^t$ is a vector of additive white Gaussian noise (AWGN) of variance σ_n^2. After N-point DFT, the recieved RS vector in the frequency domain on the B subcarriers of concern is given by

$$\begin{aligned}\mathbf{y} = \mathbf{F}_B \mathbf{r} &= \sum_{u=0}^{U-1} \mathbf{F}_B \mathbf{H}_u^t \mathbf{F}_B^H \mathbf{W}_u \mathbf{x} + \mathbf{n}^f \\ &\triangleq \sum_{u=0}^{U-1} \mathbf{H}_u^f \mathbf{W}_u \mathbf{x} + \mathbf{n}^f = \sum_{u=0}^{U-1} \mathbf{X} \mathbf{W}_u \mathbf{h}_u^f + \mathbf{n}^f \end{aligned} \tag{3}$$

where $\mathbf{H}_u^f = \mathbf{F}_B \mathbf{H}_u^t \mathbf{F}_B^H$ is the diagonal matrix of channel frequency response (CFR) for the B subcarriers associated with UE u, $\mathbf{h}_u^f$ is the vector of the diagonal terms of $\mathbf{H}_u^f$, $\mathbf{X} = \text{diag}(\mathbf{x})$, and $\mathbf{n}^f = \mathbf{F}_B \mathbf{n}^t$ is an AWGN vector in the frequency domain with the same element variance σ_n^2 as $\mathbf{n}^t$.

Consider $\tilde{\mathbf{h}}^f = (\mathbf{X}\mathbf{W}_0)^{-1}\mathbf{y}$. We get

$$\begin{aligned}\tilde{\mathbf{h}}^f &= \sum_{u=0}^{U-1} \mathbf{W}_0^{-1} \mathbf{W}_u \mathbf{h}_u^f + \mathbf{W}_0^{-1} \mathbf{X}^{-1} \mathbf{n}^f \\ &= \sum_{u=0}^{U-1} \mathbf{W}_0^{-1} \mathbf{W}_u \mathbf{F}_B \mathbf{h}_u^t + \tilde{\mathbf{n}}^f \end{aligned} \tag{4}$$

where $\mathbf{h}_u^t$ is the first column of $\mathbf{H}_u^t$ and $\tilde{\mathbf{n}}^f \triangleq \mathbf{W}_0^{-1} \mathbf{X}^{-1} \mathbf{n}^f$. By elementary signals and systems theory, multiplication by a complex exponential in the frequency domain corresponds to a shift in the time domain [14]. Thus

$$\tilde{\mathbf{h}}^f = \sum_{u=0}^{U-1} \mathbf{F}_B \mathbf{P}^{N_u - N_0} \mathbf{h}_u^t + \tilde{\mathbf{n}}^f \tag{5}$$

where $\mathbf{P}$ is a permutation matrix obtained by a cyclic left shift of the $N \times N$ identity matrix by one column. Premultiplication of $\mathbf{h}_u^t$ by $\mathbf{P}^{N_u - N_0}$ thus shifts the elements of $\mathbf{h}_u^t$ downwards cyclically by $N_u - N_0$ positions.

Let L denote CP length and assume that the lengths of the CIRs associated with the U UEs do not exceed L. Let $\mathbf{h}_{u,L}^t$ denote the leading L-subvector of $\mathbf{h}_u^t$ and let $\mathbf{F}_{B,u}$ be the $B \times L$ submatrix of $\mathbf{F}_B$ that retains only the elements that may be multiplied with nonzero elements of $\mathbf{P}^{N_0 - N_u} \mathbf{h}_u^t$ in (5), i.e., $[\mathbf{F}_{B,u}]_{p,q} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi(q+N_u-N_0)(p+k_0)}{N}}$ where $p = 0, \ldots, B-1$, $q = 0, \ldots, L-1$. Then

$$\tilde{\mathbf{h}}^f = \sum_{u=0}^{U-1} \mathbf{F}_{B,u} \mathbf{h}_{u,L}^t + \tilde{\mathbf{n}}^f \triangleq \mathbf{F}_{B,L'} \mathbf{h}^t + \tilde{\mathbf{n}}^f \tag{6}$$

where $\mathbf{F}_{B,L'} = [\mathbf{F}_{B,0} \ \cdots \ \mathbf{F}_{B,U-1}]$ and $\mathbf{h}^t = [(\mathbf{h}_{0,L}^t)^T \ \cdots \ (\mathbf{h}_{U-1,L}^t)^T]^T$, with superscript T denoting transpose. With the formulation given in (6), the CIRs of the UEs participating in MU-MIMO can be estimated, in principle, in a way similar to the single-user case.

In summary, upon a received RS symbol $\mathbf{r}$, the receiver prepares the data for channel estimation by first computing $\mathbf{y} = \mathbf{F}_B \mathbf{r}$ via FFT and then calculates $\tilde{\mathbf{h}}^f = (\mathbf{X}\mathbf{W}_0)^{-1}\mathbf{y}$. The channel estimator will make use of the signal model (6). In the present work, we only consider channel estimation at the RS symbols. A complete channel estimator will need to obtain channel estimates for the data symbols also, e.g., by time-domain interpolation. But this issue is outside the main focus of the present work.

3. CHANNEL ESTIMATION METHODS

3.1. LS Estimation

Applying the LS approach to the signal model (6), we get the (cascaded) CIR estimate as

$$\hat{\mathbf{h}}_{LS}^t = (\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})^{-1} \mathbf{F}_{B,L'}^H \tilde{\mathbf{h}}^f. \tag{7}$$

The corresponding MSE is given by

$$\begin{aligned}\text{MSE}_{LS} &= \frac{1}{UL} E[\|\hat{\mathbf{h}}_{LS}^t - \mathbf{h}^t\|^2] \\ &= \frac{\sigma_n^2/\sigma_x^2}{UL} \text{tr}((\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})^{-1}) = \frac{\sigma_n^2/\sigma_x^2}{UL} \sum_{i=0}^{UL-1} \frac{1}{\lambda_i} \end{aligned} \tag{8}$$

where σ_x^2 is the power of RS data and λ_i is the ith largest eigenvalue of $(\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})$. Numerical computation shows

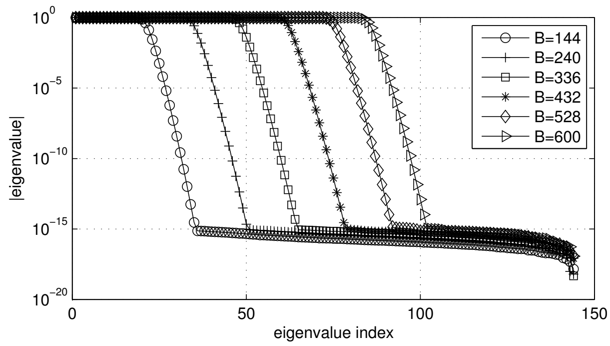

Fig. 1. Eigenvalues of $(\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})$ for different values of B at $N = 1024$, $L = 72$ and $U = 2$.

that $(\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})$ can be highly ill-conditioned, especially when B is relatively small. For example, Fig. 1 shows the eigenvalues for $N = 1024$, $L = 72$, $U = 2$, and various values of B. Many eigenvalues are close to zero even for $B = 600$. Hence, in practical finite-precision implementations, the smaller eigenvalues may cause significant numerical problem and noise enhancement in low signal-to-noise ratio (SNR) conditions.

3.2. LMMSE Estimation

Applying the LMMSE approach to the signal model (6), we get the (cascaded) CIR estimate as

$$\hat{\mathbf{h}}_{LMMSE}^t = \mathbf{R}_h^t [\mathbf{R}_h^t + \frac{\sigma_n^2}{\sigma_x^2} (\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})^{-1}]^{-1} \hat{\mathbf{h}}_{LS}^t \quad (9)$$

where $\mathbf{R}_h^t = E[\mathbf{h}^t (\mathbf{h}^t)^H]$. The resulting MSE is

$$\begin{aligned} \text{MSE}_{LMMSE} &= \frac{1}{UL} E[\|\hat{\mathbf{h}}_{LMMSE}^t - \mathbf{h}^t\|^2] \\ &= \frac{1}{UL} \text{tr}((\mathbf{R}_h^t + \frac{\sigma_x^2}{\sigma_n^2} \mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})^{-1}). \end{aligned} \quad (10)$$

The eigenvalues of $(\mathbf{F}_{B,L'}^H \mathbf{F}_{B,L'})$ also influence the MSE, but the effect is mitigated by noise as well as CIR characteristics. But it needs second-order statistics of the channel and noise. Complexity and practicality aside, estimation errors in these second-order statistics may also degrade the performance.

3.3. Proposed Method That Minimizes L_1 Norm of Error

To attain better numerical stability at reasonable complexity, we propose to minimize an L_1 norm of error as

$$\hat{\mathbf{h}}_{L_1}^t = \arg\min_{\hat{\mathbf{h}}^t} \sum_{k=0}^{B-1} \mid \tilde{h}^f[k] - \mathbf{F}_{B,L'}^{(k)} \hat{\mathbf{h}}^t \mid \quad (11)$$

where $\tilde{h}^f[k]$ is the kth element in $\tilde{\mathbf{h}}^f$ and $\mathbf{F}_{B,L'}^{(k)}$ is the kth row vector in $\mathbf{F}_{B,L'}$. It is known that the L_1-norm error measure is more robust against outliers than the L_2-norm measure and thus less prone to noise enhancement compared to LS estimation [15, 16]. But its property is such that the optimal solution is not easily characterizable as the LS and LMMSE approaches. We consider iterative search for a suitable solution using the so-called subgradient method.

To describe the method, let $f_{(k)}(\hat{\mathbf{h}}^t) = |\tilde{h}^f[k] - \mathbf{F}_{B,L'}^{(k)} \hat{\mathbf{h}}^t|$. Then

$$\nabla_{\hat{\mathbf{h}}^t} f_{(k)}(\hat{\mathbf{h}}^t) = -\text{sgn}(\tilde{h}^f[k] - \mathbf{F}_{B,L'}^{(k)} \hat{\mathbf{h}}^t)(\mathbf{F}_{B,L'}^{(k)})^T \quad (12)$$

where $\text{sgn}(\cdot)$ is the signum function. The update equation for the mth iteration is then given by

$$\hat{\mathbf{h}}^t(m+1) = \hat{\mathbf{h}}^t(m) - \mu \sum_{k=0}^{B-1} \nabla_{\hat{\mathbf{h}}^t(m)} f_{(k)}(\hat{\mathbf{h}}^t(m)) \quad (13)$$

where μ is the step size. The convergence condition can be set to $|\hat{\mathbf{h}}^t(m+1) - \hat{\mathbf{h}}^t(m)| < \epsilon$ where ϵ is a threshold chosen to meet the desired estimation error, or set to a maximum allowed number of iterations. The step size can also be variable to hasten the convergence in the earlier stages of iteration and yield low errors towards the end. For example, we have considered a three-stage design using, respectively, the three step-size values 0.1, 0.01 and 0.001.

Concerning complexity, the gradient computation in (13) requires the most computation, which amounts to BUL complex multiplications per iteration and totals to $BULI$ complex multiplications where I is the number of iterations. On the other hand, the LMMSE method requires inversion of a $UL \times UL$ matrix, which has a complexity of $O(U^3 L^3)$ complex operations. The relative complexity of the proposed method and LMMSE is, therefore, not obvious. If we assume that B is of a similar order of magnitude as UL, then the relative complexity will depend on how I compares to UL. However, one thing in favor of the proposed method is that it does not need to estimate the second-order statistics of the channel as does the LMMSE method. Moreover, the matrix inversion required in LMMSE estimation can pose a significant numerical issue for large UL values. This is avoided in the proposed method.

4. SIMULATIONS

Consider a 10-MHz-bandwidth system, where the sample rate is 15.36 MHz and DFT size $N = 1024$. Let it employ normal CP so that $L = 72$. Let there be $U = 2$ UEs assigned to the same $B = 144$ subcarriers. This makes $B = UL$ so that the channel estimation formulation (6) does not suffer from underdetermination. We simulate two channel models, namely, the ITU Pedestrian B (PB) and Vehicular A (VA), whose maximum delay spreads are 3.7 μs and 2.51 μs, respectively [17].

The performance results presented below only deal with RS symbols but not data symbols. They are averages over 5000 runs (i.e., 5000 RS symbols) over all UEs. Since the

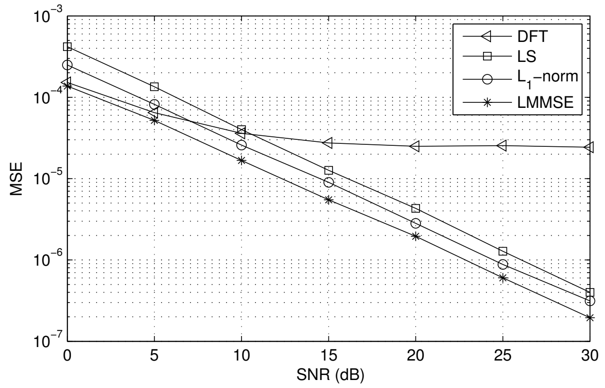

Fig. 2. MSE over two UEs in PB channel at speed 3 km/h.

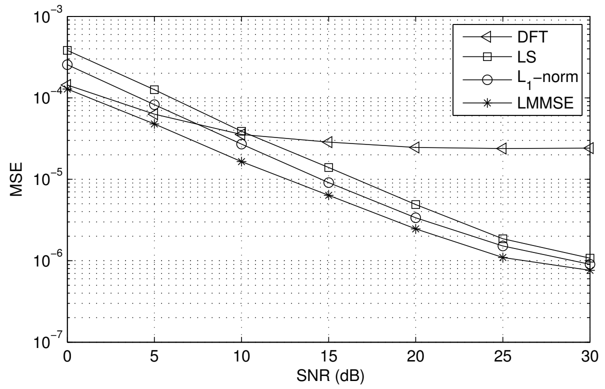

Fig. 3. MSE over two UEs in VA channel at speed 120 km/h.

UEs are only assigned B subcarriers, only the channel estimation errors at these subcarriers are of concern. Hence the MSEs shown in the following figures are defined as

$$\mathrm{MSE} = \frac{1}{QUL} \sum_{q=0}^{Q-1} \sum_{u=0}^{U-1} \|\mathbf{h}_u^f(q) - \mathbf{F}_{B,u}\hat{\mathbf{h}}_{u,L}^t(q)\|^2 \qquad (14)$$

where $Q = 5000$ is the number of runs, $\mathbf{h}_u^f(q)$ is the B-vector of average CFR over the useful time of symbol q associated with UE u for the B subcarriers of concern, and $\hat{\mathbf{h}}_{u,L}^t(q)$ is the estimate of CIR $\mathbf{h}_{u,L}^t$ for UE u in symbol q of a channel estimation method. We arbitrarily let the number of iterations in the L_1-based channel estimation be $I = 1120$.

Fig. 2 shows the MSE performance of different channel estimation methods where both UEs experience PB channel at 3 km/h speed. The method designated "DFT" in the figure does conventional DFT-based channel estimation, which uses a window of length L to extract the CIR for each UE from the inverse DFT of $\tilde{\mathbf{h}}^f$. For LMMSE, we assume perfect knowledge of $\mathbf{R}_h^t$ and channel noise variance σ_n^2. Its performance can thus be viewed as a kind of performance upper bound. Under the conditions of Fig. 2, the DFT method yields performance close to perfect LMMSE in low SNR, because

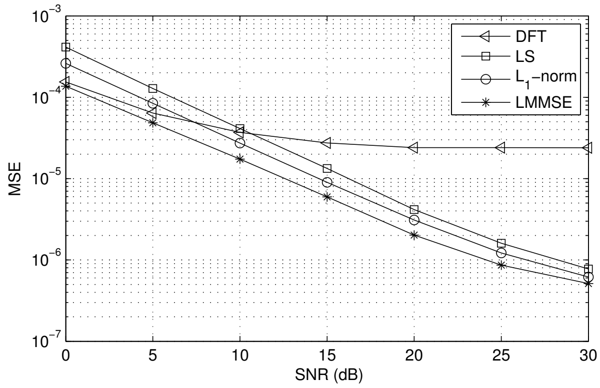

Fig. 4. MSE over two UEs, where UE 0 experiences PB channel at 3 km/h and UE 1 VA channel at speed 120 km/h.

the time-domain windowing is effective in cutting out noise. But the truncation error arising from CIR spreading results in a significant error floor in high SNR. There is roughly a 4 dB gap between the performance of LS and LMMSE estimations over the range of simulated SNR values, which can be atrributed to the better condition number of the inverted matrix in LMMSE estimation versus that in LS. The proposed method has a performance about half way in the gap.

Fig. 3 shows the performance of different methods where both UEs experience VA channel at 120 km/h speed. Except for the error floors in high SNR, induced by channel time-variation associated with the high motion speed, the relative performance of different methods is similar to that in PB at 3 km/h. It is also of interest to note that, in the high SNR region where error floors begin to appear, the LS, L_1-norm, and LMMSE methods have maintained the same performance order as in lower SNR values.

Fig. 4 considers the case where the two UEs are subject to different channel statistics, namely, PB at 3 km/h and VA at 120 km/h. The primary difference in performance, compared to that shown in Figs. 2 and 3, lies in the high SNR region, where the phenomenon of error floors is (perhaps expectedly) less prominent than in VA at 120 km/h. The relative performance of different methods is otherwise similar to the earlier conditions.

5. CONCLUSIONS

We considered the uplink channel estimation problem in MU-MIMO for LTE and LTE-A. And we proposed a method based on minimizing an L_1 norm of error. The method does not suffer from the CIR spreading problem as in conventional DFT-based channel estimation. Moreover, unlike LMMSE, it requires no matrix inversion or second-order statistics of the channel, thus implying better numerical property. Simulation shows that the proposed method yields a performance between the DFT, LS, and LMMSE methods.

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