# FAST TIME-VARYING CHANNEL ESTIMATION METHOD FOR LTE SC-FDMA SYSTEMS

*Dan Li*[1], *Feng Ke*[2]

[1]School of Computer Science, Shaoguan University, Shaoguan, China
[2]Department of Electronic and Information Engineering, South China Univ. of Tech., Guangzhou, China
Email: lidan_1121@163.com, fengke@scut.edu.cn

**ABSTRACT**

In this paper, a fast time-varying channel estimation method for Single Carrier Frequency division Multiple Access (SC-FDMA) communications on the Long Term Evolution (LTE) uplink is proposed. In uplink SC-FDMA, fast variations of the channel result in inter-carrier interference (ICI) which cannot be estimated properly with conventional estimation method. The proposed estimation method is performed all in the frequency domain and only part of channel frequency response for each uplink user is needed to estimate. The time variations of the frequency domain transmission function are modeled by the Basis expansion model (BEM). Simulation results demonstrate the performance and effectiveness of the proposed method over fast time-varying channels.

***Index Terms—*** Fast time-varying channels, Channel estimation, LTE, SC-FDMA

## 1. INTRODUCTION

Long Term Evolution (LTE) developed by Third Generation Partnership Project (3GPP) is to improve the Universal Mobile Telecommunications System (UMTS) standard for high-speed data cellular services. Its physical layer air interface employs Orthogonal Frequency-Division Multiplexing based multi-user scheme due to its higher spectral efficiency, flexibility on subcarrier allocation and multi-user diversity utilization over multi-path fading channels. OFDMA is used for LTE downlink and single-carrier frequency-division multiple access (SC-FDMA) for uplink [1].

SC-FDMA is introduced for low Peak to Average Power Ratio (PAPR) in uplink, which has similar performance and essentially the same overall complexity as OFDMA and can be viewed as DFT-spread OFDMA, where time domain data symbols are transformed to frequency domain by Discrete Fourier Transform before going through OFDMA modulation [2,3]. Similar to OFDMA, one of the most challenging problems of SC-FDMA system is the channel estimation in fast time-varying channels. According to [1], user mobility supported by LTE is up to 350km/h (or perhaps up to 500km/h depending on the frequency band). In such a high mobility environment, the channel is not static during a symbol period, which destroys the orthogonality between subcarriers and introduces Inter-Carrier-Interference (ICI).

Previous works [4-9] specialize in channel estimation methods for LTE SC-FDMA. Methods in [4-6] consider channels varying from symbol to symbol and the ICI in fast time-varying channels was ignored or regarded as noise. In [7, 8], ICI and channel Gaussian noise are minimized by a transformed domain filter but cannot be removed completely especially in fast time-varying channels. Channel variations over a symbol period are estimated in [9] by a time domain Kalman filter and ICI can be mitigated in high mobility environments. But, it assumes only a single user in uplink SC-FDMA and all subcarriers in pilot symbols are available for the frequency to time domain transformation. This assumption is not the case in practice. In uplink SC-FDMA, subcarriers are divided into subsets and allocated to different users [1]. Transforming to the time domain is not always applicable with partial subcarriers of the whole bandwidth especially for non-sample spaced channels. Time domain method [9] estimates the whole channel taps in time domain, while for OFDMA or SC-FDMA systems only a part of channel frequency response (in the user subset) is need.

Based on the analysis above, current estimators are not suitable for practical uplink LTE SC-FDMA systems over fast time-varying channels. In this paper, a frequency domain fast time-varying channel estimation method for LTE SC-FDMA systems is proposed. Estimation is all performed in the frequency domain and only part of channel frequency response for each uplink user is needed to estimate. Channel variations in a symbol period are models by the Basis expansion model (BEM), which has recently taken a lot of attention for fast time-varying channel estimations [12, 13]. Compared to Kalman filter, BEM is easy to use and no need of prior channel statics.

The rest of this paper is organized as follows: the system model is introduced in Section II. In Section III, the proposed method is described. In Section IV, simulation results are presented and conclusions are given in Section V.

## 2. SYSTEM MODEL

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

### 2.1. General System Model

System diagram of SC-FDMA is depicted on figure1.

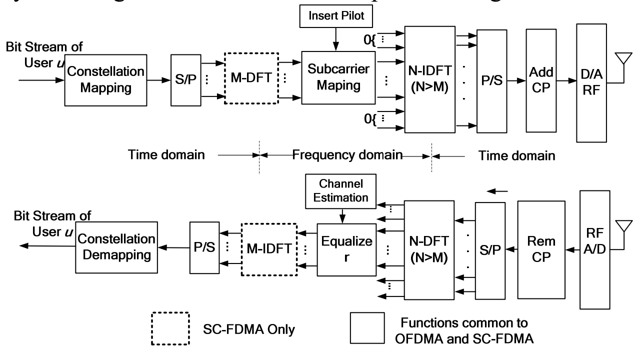


Fig. 1. Block Diagram of SC-FDMA transmitter and receiver

We assume that there are $U$ users in the uplink. Input data stream of user $u$ ( $0 \le u \le U-1$ ) is mapped to quadrature-amplitude modulation (QAM) symbols. Block of $M$ symbols is then converted to the frequency domain using $M$-point DFT.

$$D^{p}(\kappa)=\frac{1}{\sqrt{M}}\sum_{m=0}^{M-1} d^{p}(m)e^{-j2\pi\kappa m/N} \quad (1)$$

$M$ output samples are mapped to the user's subcarriers by distributed subcarrier mapping or localized subcarrier mapping [2]. Distributed mapping occupy a comb-shaped spectrum. Localized mapping occupy a continuous spectrum. After setting other $N$-$M$ subcarriers zeros as (2), the $N$-point IDFT is operated as (3). Where, $\Phi^{u}$ is the subcarrier mapping set of user $u$.

$$X^{u}(k)=\begin{cases} D^{u}(\kappa), & k\in\Phi^{u} \\ 0, & otherwise \end{cases} \quad (2)$$

$$x^{u}(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X^{u}(k)e^{j2\pi nk/N} \quad (3)$$

After sending over the channel, removing CP, the received signal at base station can be expressed as

$$y(n)=\sum_{u=0}^{U-1}\sum_{l=0}^{L-1} h^{u}(n,l)x^{u}(n-l)+w^{u}(n) \quad (4)$$

where, $w^{u}(n)$ represents the additive white Gaussian noise (AWGN) and $h^{u}(n,l)$ denotes the time varying channel impulse response of user $u$. $l$ is the "path" index, $n$ is the "time" index, $L$ is the maximum channel length ($0\le l\le L-1, 0\le n\le N-1$).

The DFT output at $p^{\text{th}}$ subcarrier for user $u$ can be expressed as

$$Y^{u}(p)=G^{u}(p,p)X^{u}(p)+\underbrace{\sum_{f=0,f\neq p}^{f\in\Phi^{u}} G^{u}(p,f)X^{u}(f)}_{ICI}+W^{u}(p). \quad (5)$$

$$G^{u}(p,f)=\frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h^{u}(n,l)e^{(-j\frac{2\pi fl}{N})}e^{(j\frac{2\pi n(f-p)}{N})}. \quad (6)$$

where $p,f\in\Phi^{u}$ ,$0\le f\le N-1$. $W^{u}(p)$ is the DFT of AWGN. $G^{u}(p,f)$ is defined as the $(p,f)^{\text{th}}$ element of $\boldsymbol{G}$, which is the frequency domain channel matrix of user $u$. When the channel is time-varying during a DFT symbol period, $\boldsymbol{G}$ becomes a nondiagonal matrix. Its nondiagonal elements $G^{u}(p,f)$ introduce ICI from subcarrier $f$ to subcarrier $p$.

### 2.2. Demodulation Reference Signal

For LTE uplink SC-FDMA system, the transmitted signal in each slot is described by one or several resource grids. A resource grid is defined as 7 (for normal cyclic prefix) or 6 (for extended cyclic prefix) consecutive SC-FDMA symbols in the time domain and 12 consecutive subcarriers in the frequency domain. Each element in the resource grid called resource element is uniquely defined by the index pair $(a,b)$ in a slot $(0\le a\le 6, 0\le b\le 11)$ , where "$a$" and "$b$" are the indices in the time and frequency domains, respectively. Demodulation reference signals (DMRS) for channel estimation are transmitted in the fourth SC-FDMA symbol ($a$=3) of the slot [1]. To multiplex DMRS from different users, frequency-division multiplexing is used and DMRS occupy the same subcarriers as data. Localized mapping for DMRS illustrated in Fig.2 is recommended for LTE due to its less susceptible to frequency shift from adjacent users [2].

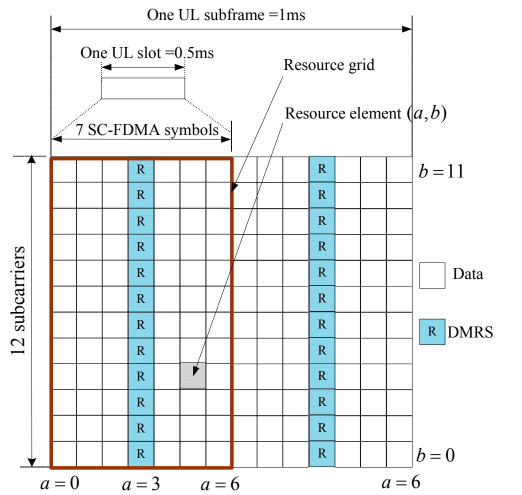


Fig.2 DMRS in Uplink (UL) resource grid for normal cyclic prefix

### 3. Channel Estimation Method

In order to compensate for the ICI, the time-varying channel impulse response or the frequency domain channel matrix is required in the receiver. Most of the current estimator [4-8] neglect the ICI or regard it as noise, which will encounter performance decline in fast time-varying channels. The time domain Kalman filter based estimator [9] is not always practical because the pilot number and distribution of each uplink user are limited in the allocated subset, which may not support the frequency domain to time domain transformation. For example, when the number of pilot subcarriers is less than channel taps (e.g. in non-sample-spaced channel, time domain impulse response leaks to all taps), the initial values for Kalman filtering in time domain cannot be obtained by DFT.

Actually, we just need to estimate a part of channel frequency response in the assigned subcarrier set, not the whole channel impulse in time domain. In this section, we present an all-in frequency domain channel estimation method seeks to estimate the needed part of the channel matrix without any time domain processing.

As distinct from time domain method, a frequency domain time-varying transmission function $H^u(n,f)$ defined as (7) in [10,11] is employed in this paper to track the channel variations and estimate the channel matrix **G**. During a symbol period, $0 \le n \le N-1,\ f \in \Phi^u$. Then, (6) can be written as (8). From (7), we know that $H^u(n,f)$ has the same time-varying property as $h^u(n,l)$. We just need to track the time variations of $H^u(n,f)$ and the channel matrix is obtained from (8).

$$H^u(n,f)=\sum_{l=0}^{L-1} h^u(n,l)e^{-j\frac{2\pi fl}{N}} \tag{7}$$

$$G^u(p,f)=\frac{1}{N}\sum_{n=0}^{N-1} H^u(n,f)e^{j\frac{2\pi n(f-p)}{N}} \tag{8}$$

Basis Expansion Model (BEM) is used to track the time variations of $H^u(n,f)$.The idea behind BEM is to express the channel as a function of ($Q$+1) basis functions approximating the variations of the channel during a certain time window. In our case, the complex-exponential basis expansion model (CE-BEM) described in [12.13] is used as the basis function. Other basis functions will do likewise. In this modeling, for the $f^{\text{th}}$ subcarrier of user $u$, we write

$$H^u(n,f)=\sum_{q=0}^{Q} H_q^u(f)b_{q,n} \tag{9}$$

where, $H_q^u(f)$ represents the $q$ th basis coefficient, which is constant during the time window. $b_{q,n}=e^{j\frac{2\pi f_{\mathrm{D}}(q-Q/2)n}{\lambda N}}$ is the complex-exponential basis function. $Q=2\lceil f_{\max}\lambda NT_s\rceil$, $\lceil . \rceil$ stands for integer ceiling. $f_{\max}$ is the maximum Doppler spread, $T_s$ denotes the sample period, $NT_s$ is the symbol period containing cyclic prefix, $\lambda$ is the number of symbols in our time window and $f_{\mathrm{D}}=f_{\max}\lambda NT_s$ is the normalized Doppler spread.

Here, we consider $\partial$ time consecutive SC-FDMA slots as the CE-BEM time window ($\lambda=\partial\times 7$). Slots in our time window are numbered as $1,\cdots,i,\cdots,\partial-1$. Using (9) in (8), we obtain

$$G_{i,a}^u(p,f)=\frac{1}{N}\sum_{n=0}^{N-1}\sum_{q=0}^{Q} H_q^u(f)e^{j\frac{2\pi f_{\mathrm{D}}(q-Q/2)(n+(7i+a)N)}{\lambda N}}e^{j\frac{2\pi n(f-p)}{N}} \tag{10}$$

According to (5), the received DMRS symbol ($a$=3) at $p^{\text{th}}$ ($0\le p\le 11$) subcarrier in slot $i$ of user $u$ is written as (11).

$$Y_{i,a=3}^u(p)=G_{i,a=3}^u(p,p)X_{i,a=3}^u(p)+\sum_{f=0,f\ne p}^{f\in\Phi^u} G_{i,a=3}^u(p,f)X_{i,a=3}^u(f)$$
$$+W_{i,a=3}^u(p).\quad (p,f\in\Phi^u) \tag{11}$$

Combining (10) and (11), we can get a linear equation in matrix form as (12)

$$\boldsymbol{Y}_i=\boldsymbol{a}\boldsymbol{X}_i\boldsymbol{H}+\boldsymbol{W}_i \tag{12}$$

where, $\boldsymbol{Y}_i$ is the received $12\times 1$ vector, $\boldsymbol{a}$ is a $12\times 12(Q+1)$ matrix, $\boldsymbol{X}_i$ is the $12(Q+1)\times 12(Q+1)$ diagonal matrix with transmitted signal on its diagonal. $\boldsymbol{H}$ is the unknown $12(Q+1)\times 1$ basis coefficient vector, $\boldsymbol{W}_i$ is $12\times 1$ noise vector. They are described as follows: $\boldsymbol{Y}_i=[Y_{i,a=3}^u(0),\cdots,Y_{i,a=3}^u(p),\cdots,Y_{i,a=3}^u(11)]^T$, $\boldsymbol{a}=[\boldsymbol{a}^0,\cdots,\boldsymbol{a}^p,\cdots,\boldsymbol{a}^{11}]^T$, $\boldsymbol{a}^p=[\boldsymbol{a}_0^p,\cdots,\boldsymbol{a}_q^p,\cdots,\boldsymbol{a}_Q^p]^T$, $\boldsymbol{a}_q^p=[a_q^{p,0},\cdots,a_q^{p,f},\cdots,a_q^{p,11}]$, $a_q^{p,f}=\gamma\boldsymbol{F}\boldsymbol{b}_q$, $\gamma=e^{j\frac{2\pi f_{\mathrm{D}}(q-Q/2)(7i+a)N}{\lambda N}}$ is a constant coefficient. $\boldsymbol{F}=\frac{1}{N}[e^{j\frac{2\pi 0(f-p)}{N}},\cdots,e^{j\frac{2\pi n(f-p)}{N}},\cdots,e^{j\frac{2\pi(N-1)(f-p)}{N}}]$, $\boldsymbol{b}_q=[b_q^0,\cdots,b_q^n,\cdots,b_q^{N-1}]^T$, $\boldsymbol{X}_i=diag\{\boldsymbol{X}_0,\cdots,\boldsymbol{X}_q,\cdots,\boldsymbol{X}_Q\}$, $\boldsymbol{X}_q=[X_{i,a=3}^u(0),\cdots,X_{i,a=3}^u(f),\cdots,X_{i,a=3}^u(11)]$. $diag\{\boldsymbol{X}\}$ is a diagonal matrix with vector $\boldsymbol{X}$ on its diagonal. $\boldsymbol{H}=[\boldsymbol{H}(0),\cdots,\boldsymbol{H}(f),\cdots,\boldsymbol{H}(11)]^T$, $\boldsymbol{H}(f)=[H_0^u(f),\cdots,H_q^u(f),\cdots,H_Q^u(f)]$ and $\boldsymbol{W}_i=[W_{i,a=3}^u(0),\cdots,W_{i,a=3}^u(p),\cdots,W_{i,a=3}^u(11)]^T$.

Consider $\partial$ time consecutive SC-FDMA slots in the time window, (12) yields (13).

$$\begin{bmatrix}\boldsymbol{Y}_i\\ \vdots\\ \boldsymbol{Y}_{i+\partial}\end{bmatrix}=\begin{bmatrix}\boldsymbol{a}\boldsymbol{X}_i\\ \vdots\\ \boldsymbol{a}\boldsymbol{X}_{i+\partial}\end{bmatrix}\boldsymbol{H}+\begin{bmatrix}\boldsymbol{W}_i\\ \vdots\\ \boldsymbol{W}_{i+\partial}\end{bmatrix}$$

or
$$\boldsymbol{Y}=\boldsymbol{A}\boldsymbol{H}+\boldsymbol{W} \tag{13}$$

The left side of equation is the received $12\partial$ DMRS vector. On the right side of the equation, the $12\partial\times 12(Q+1)$ matrix $\boldsymbol{A}$ is known and $\boldsymbol{H}$ is the unknown $12(Q+1)\times 1$ basis coefficient vector, which needs to be estimated. When $Q+1\le\partial$, the least square (LS) solution of the (13) is given by (14), where $\boldsymbol{A}^{\dagger}$ is the pseudoinverse of $\boldsymbol{A}$.

$$\boldsymbol{H}=\boldsymbol{A}^{\dagger}\boldsymbol{Y} \tag{14}$$

From (14), basis coefficients $H_q^u(f)$ ($f\in\Phi^u$, $0\le q\le Q$) of user $u$ can be estimated and the channel matrix of every data symbol in the time window (i.e. $G_{i,a}^u(p,f)$ with $0\le i\le \partial$, $0\le a\le 6$ and $p,f\in\Phi^u$ ) can be obtained by (10).

## 4. SIMULATION RESULTS

In this section, simulations have been performed to evaluate the performance of the proposed method. System parameters for LTE uplink are summarized in Table 1.

Table 1. Parameters of LTe uplink[2]

| ***Parameters*** | ***Value*** |
|---|---|
| Bandwidth | $B$=20M |
| DFT size | $N$=2048 |
| Subcarrier spacing | $\Delta f$ = 15KHz |
| Carrier frequency | $f_c$ = 2.0GHz |
| Cyclic prefix length | Normal CP |
| $Q$ | 2 |
| Channel model | LTE-EVA |
| Modulation | QPSK |
| User number | 4 |

Mean Square Error (MSE) performances are shown in Fig.3 and 4. Fig.3 plots the MSE performances versus SNR when $V$=300km/h ( $f_D$=0.8). Performance improvements are clearly seen when compared to methods in [4] and [7]. It was because that methods in [4] and [7] regard the channel is constant during a symbol period, and our proposed method estimate the channel variations in each symbol period. Fig.4 shows that when the user mobility decrease MSE performance will get better.

Symbol Error Rate (SER) performances are given in Fig.5. Methods in [4] and [7] suffer error floor in high SNR due to ICI caused by channel variation during one symbol period. And the error floor of the proposed method in high SNR are mainly caused by BEM model errors.

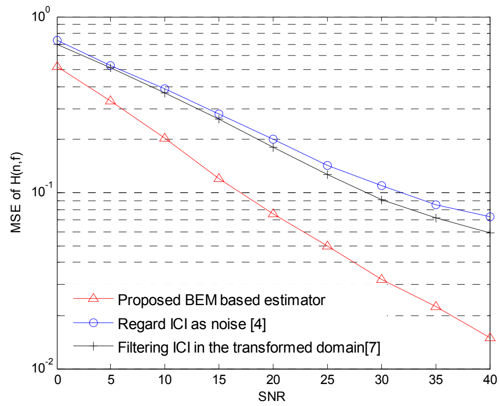


Fig. 3 MSE vs SNR ($V$=300km/h $f_D$=0.8)

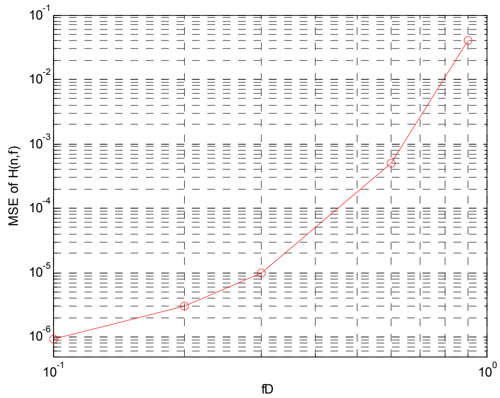


Fig.4 MSE vs the normalized Doppler spread $f_D$

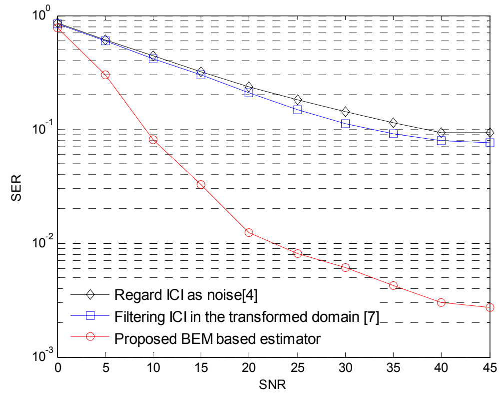


Fig. 5 SER vs SNR (V=300km/h $f_D$=0.8)

## 5. CONCLUSION

Our study shows that the proposed frequency domain fast time-varying channel estimation is practical for LTE SC-FDMA system at the user mobility lower than 350km/h. Estimation is performed in the frequency domain by using the frequency domain time-varying transmission function without any need of estimating all the channel taps in the time domain. Channel variations are model by BEM, which is easily to use and no need of prior statics. Simulation results demonstrate the effectiveness of the proposed method. When user mobility is higher than 350km/h， $Q+1>\partial$ , (13) has no LS solution. Estimation in such environments will be considered in our future works.

## 6. ACKNOWLEDGMENT


The author would like to thank all the reviewers and editors. The work presented in this paper was supported in part by NSFC (No.61101134), NSFC of Guangdong (No. S2013010011947) and Foundation for Distinguished Young Talents in Higher Education of Guangdong, China ( No. LYM11116).


## 7. REFERENCES

[1] 3GPP TR 36.211. "Requirements for Evolved UTRA (E-UTRA) and Evolved UTRAN (E-UTRAN)." 2008,6.

[2] 3GPP TR 25.814. "Physical Layer Aspects for Evolved UTRA." 2006, 9.

[3] H.G Myung, J.Lim, and D.J. Goodman, "Single carrier FDMA for uplink wireless transmission." *IEEE Vehicular Technology Magazine*, IEEE, 2006, 1(3), pp: 30-38.

[4] Ma YK, Tan P, Wang XY, et al. "Research on Channel Estimation Based on Adaptive Filtering for LTE Uplink." IEEE, *2nd International Conference on Consumer Electronics, Communications and Networks*, Yichang, China: IEEE, 2012.4, pp: 2010-2014.

[5] Zhou ML, Jiang B, Zhong W, et al. "Efficient Channel Estimaiton for LTE Uplink. " *IEEE, International Conference on Wireless Communications & Signal Processing*, Nanjing, China: IEEE, 2009.11, pp:1-5.

[6] Yang L,Ren G, Yang B, etc. "Fast Time-Varying Channel Estimation Technique for LTE Uplink in HST Environment." *IEEE Transactions on Vehicular Technology*, IEEE, 2012,61(9), pp: 4009-4019.

[7] Huang SC, Lin JC and Chou KP, "Novel Channel Estimation Techniques on SC-FDMA Uplink Transmission." IEEE, *71st Vehicular Technology Conference*, Ottawa, Canada: IEEE, 2010.9, pp:1-5.

[8] Yameogo S, Palicot J and Cariou L., "Mobile Radio Channels' Estimation for SC-FDMA Systems by Means of Adequate Noise and Inter-carrier Interference Filtering in a Transformed domain." IEEE, *Globecom Workshop*, Miami, USA: IEEE, 2010, pp:1302-1306.

[9] Karakaya B, Arslan H., "Channel Estimation for LTE Uplink in High Doppler Spread." IEEE, *Wireless Communications and Networking Conference*, Las Vegas, USA: IEEE, 2008: 1126-1130.

[10] Li D., Feng S., and Wu Y., "Pilot-Assisted Channel Estimation Method for OFDMA Systems over Time-Varying Channels." *IEEE Communications Letters*, IEEE, 2009, 13(11), pp:826-828.

[11] El Hajj Shehadeh Y and Sezginer S., "Fast Varying Channel Estimation in Downlink LTE Systems." IEEE, *21st International Symposium on Personal Indoor and Mobile Radio Communications*, Paris, France：IEEE, 2010：608-613.

[12] Qi J, Speidel J. And Zhao C., "Pilot-Assisted OFDM Channel Estimation and ICI Cancellation for Double Selective Channels." IEEE, *Global Telecommunications Conference*,Washington D.C, USA: IEEE, 2007: 150-4154.

[13] Tang Z, Cannizzaro R.C, Leus G, etc, "Pilot-Assisted Time-Varying Channel Estimation for OFDM Systems." *IEEE Transactions on Signal Processing,* IEEE, pp: 2226-2238, 2007,55(5).