# Sparsity-Aware Channel Estimation with Contaminated Pilot Sequence

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Abstract—In this paper the up-link channel estimation problem in a multi-cell network with a Single-Input-Multiple-Output (SIMO) fading correlated channel and finite-length, nonorthogonal and contaminated pilots is considered. Our contribution is a novel channel estimation technique, referred to as the Sparse Channel Estimator (SCE), which exploits the Karhunen-Loéve Transform (KLT) sparse-representation of the channel. We show that, by simply utilizing an interferencecontaminated channel sample-covariance matrix, the proposed approach outperforms Least Squares (LS)-based and Minimum Mean Squared Error (MMSE) techniques relying on the same type of assumptions.

# I. INTRODUCTION

Multi-cell cooperation, also referred to as Coordinated Multi-Point (CoMP) system, is one of the key features in Long Term Evolution (LTE)-advanced technology and its future enhancements [1]. CoMP provides an efficient way to increase the network throughput by coordinating data transmissions from multiple base-stations while managing intercell interference [2]. Theoretical research has provided some relevant, although optimistic, indications about the potential of this scheme [3], especially when applied together with cooperative beam-forming. In this paper we focus on the problem of pilot-based uplink channel estimation with the assumptions of finite-length non-orthogonal pilot sequences, limited spectrum, and crucially, in the presence of inter-cell interference [4], [5]. Specifically, we consider the scenario in which the simultaneous transmission of non-orthogonal pilots sequences from mobile nodes in different cells occurs and causes the well-known pilot contamination problem.

Differently than classical Least Squares (LS)-based and Bayesian methods, *e.g.* Minimum Mean Squared Error (MMSE), that respectively rely on orthogonal pilots and the knowledge of the exact channel covariance matrix per user and interferers, we propose a sparsity-inducing algorithm, hereafter referred to as Sparse Channel Estimator (SCE). Unlike the algorithms [6] in [7] and that describe two different solutions to the estimation of an Orthogonal Frequency-Division Multiplexing (OFDM) Multiple-Input-Multiple-Output (MIMO) channel mainly exploiting the assumption of a sparse Channel Impulse Response (CIR), the proposed method tackles the problem of channel estimation with interferred pilots. Specifically we exploit the Karhunen-Loéve Transform (KLT) to obtain a sparse representation of the channel and use the eigenspace of different channel covariance matrixes to circumvent the non-orthogonality of the pilot sequences. Indeed, we show that with a contaminated covariance matrix, namely the sample covariance, the proposed SCE method outperforms the LS-based and MMSE utilizing the same type of information.

The reminder of this paper is organized as follows. Section II describes the signal and channel models. Section III reviews the stat-of-the-art methods and presents our contributions. Section IV shows comparisons to Least Squares (LS)-based and MMSE estimators.

### II. SYSTEM AND CHANNEL MODEL

Consider a cellular network with  $N_{\rm BS}$  cells severed by one base-station and  $N_{\rm u}$  users per cell. All base-stations are equipped with an array of M antennas, whereas users' terminals have a single omnidirectional antenna.

Let  $\mathbf{h}^{i,p,j} \in \mathbb{C}^M$  denote the SIMO channel between the *i*-th base-station and the *p*-th user of the *j*-th base-station. Using the fading correlation model proposed in [8], the channel vector  $\mathbf{h}^{i,p,j}$  can be obtained as

$$\mathbf{h}^{i,p,j} = \gamma_{i,p,j} \mathbf{R}^{1/2}_{i,p,j} \mathbf{z},\tag{1}$$

where  $\gamma_{i,p,j}$  is the pathloss,  $\mathbf{A}^{1/2}$  indicates the squared-root of a positive semi-definite (psd) matrix  $\mathbf{A}, \mathbf{z} \in \mathbb{C}^M$  is a vector of independent identically distributed (iid) random variables obtained from the zero-mean multivariate complex Gaussian distribution, *i.e.*  $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I}_M)$  with  $\mathbf{I}_M \in \mathbb{R}^{M \times M}$  denoting the identity matrix and, the matrix  $\mathbf{R}_{i,p,j} \in \mathbb{C}^{M \times M}$  is the channel correlation matrix with the *nm*-the element given by

$$[\mathbf{R}_{i,p,j}]_{mk} = \frac{1}{L} \sum_{\ell=1}^{L} \exp\left(-j2\pi \frac{1}{\lambda} (\bar{\boldsymbol{\psi}}_{\ell}^{i,p,j})^{\mathrm{T}} (\mathbf{r}_{k} - \mathbf{r}_{m})\right), \quad (2)$$

where <sup>T</sup> is transpose,  $\lambda$  is the wavelength of the carrier frequency, L is the total number of arriving multipaths,  $\mathbf{r}_m \in \mathbb{R}^3$  is the vector coordinate for the *m*-th antenna and  $\bar{\psi}_{\ell}^{i,p,j} \triangleq [\cos \theta_{\ell}^{i,p,j} \cos \phi_{\ell}^{i,p,j}, \sin \theta_{\ell}^{i,p,j} \cos \phi_{\ell}^{i,p,j}]^{\mathrm{T}}$  with  $\theta_{\ell}^{i,p,j}$  and  $\varphi_{\ell}^{i,p,j}$  indicating the azimuth and elevation angle of arrival of the  $\ell$ -th path, respectively.

We focus on the up-link channel estimation problem and consider a scenario in which all users simultaneously transmit a pilot-sequence to their serving base-station under the assumption of full-spectrum reuse. Without loss of generality, consider the signal at the base-station of the cell-1 and assume that  $\mathbf{h}^{1,1,1}$  is the channel of interests. For notational convenience, let us also omit the index *i* from the triplet "*i*, *p*, *j*". Therefore, the received signal at the base-station of the cell-1, denoted by  $\mathbf{Y} \in \mathbb{C}^{M \times N_z}$ , is given by

$$\mathbf{Y} = \underbrace{\mathbf{h}^{1,1}\mathbf{s}_{1,1}^{\mathrm{T}}}_{\substack{\text{user}\\\text{signal}}} + \underbrace{\sum_{p=2}^{N_{u}}\mathbf{h}^{p,1}\mathbf{s}_{p,1}^{\mathrm{T}}}_{\substack{\text{intra-cell}\\\text{interference}}} + \underbrace{\sum_{j=2}^{N_{u}}\sum_{p=1}^{N_{u}}\mathbf{h}^{p,j}\mathbf{s}_{p,j}^{\mathrm{T}}}_{\substack{\text{inter-cell}\\\text{interference}}} + \mathbf{N}, \quad (3)$$

 $\mathbf{s}_{p,j} \in \mathbb{C}^{N_z}$  is the pilot-sequence of length  $N_z$  used by the *p*-th terminal of the *j*-th base-station whose energy is  $\|\mathbf{s}_{p,j}\|_2^2 = N_z$  and  $\mathbf{N} \in \mathbb{C}^{M \times N_z}$  is the spatially and temporally white additive zero-mean Gaussian noise with variance  $\sigma^2$ .

Equation (3) shows that the terminal-signal is affected by the interference due to intra base station cell and inter base-station cells pilot transmissions. If pilot sequences are not orthogonal, the estimation of  $h^{1,1}$  is affected by the interference and the intensity is proportional to the correlation amongst pilots. In the literature, this error is referred to as *pilot contamination*.

#### **III. CHANNEL ESTIMATION ALGORITHMS**

In this section, a briefly review of classical channel estimation techniques as well as our contribution are provided. For the sake of clarity, only one user per cell is considered, thus also the index "p" is removed from all pairs "p, j".

#### A. Classical Estimation Approaches

Conventional channel estimation techniques can be formulated, for instance, as a LS or a Bayesian estimation problem. Assuming that only  $s_1$  is known at the base-station, a classic estimator is the LS given by

$$\hat{\mathbf{h}}_{\mathrm{LS}}^{1} = \mathbf{Y}(\mathbf{s}_{1}^{\mathrm{T}})^{\dagger},\tag{4}$$

where <sup>†</sup> is the Moore-Pernouse pseudo-inverse.

If all transmitted pilot-sequences are known at the basestation of cell-1, the Global Least Squares (G-LS) estimator of the total channel matrix  $\mathbf{H} \triangleq [\mathbf{h}^1, \cdots, \mathbf{h}^{N_{BS}}] \in \mathbb{C}^{M \times N_{BS}}$  is obtained as

$$\hat{\mathbf{H}}_{\text{G-LS}} = \mathbf{Y} \left( \mathbf{S}^{\text{T}} \right)^{\dagger}, \tag{5}$$

where  $\mathbf{S} \in \mathbb{C}^{Nz \times N_{BS}}$  and  $\mathbf{S} \triangleq [\mathbf{s}_1, \cdots, \mathbf{s}_{N_{BS}}]$ .

Finally, if in addition to the transmitted pilot sequences, the second order channel statics are also known, then it is possible to use the following Maximum A Posteriori (MAP) probability estimator [9]

$$\hat{\mathbf{h}}_{\mathrm{MAP}} = (\bar{\mathbf{R}}\bar{\mathbf{S}}^{\mathrm{H}}\bar{\mathbf{S}} + \sigma^{2}\mathbf{I}_{MN_{\mathrm{BS}}})^{-1}\bar{\mathbf{R}}\bar{\mathbf{S}}^{\mathrm{H}}\mathbf{y}, \tag{6}$$

where  $\otimes$  is the Kronecker product,  $\mathbf{y} \in \mathbb{C}^{N_{\text{BS}}N_z}$  with  $\mathbf{y} \triangleq$ vec( $\mathbf{Y}$ ), vec( $\cdot$ ) is a matrix-to-vector function,  $\mathbf{\bar{S}} \in \mathbb{C}^{N_z \times N_{\text{BS}}M}$ with  $\mathbf{\bar{S}} \triangleq \mathbf{S} \otimes \mathbf{I}_M$  and  $\mathbf{\bar{R}} \in \mathbb{R}^{N_{\text{BS}}M \times N_{\text{BS}}M}$  is a block-diagonal matrix with blocks { $\mathbf{R}_1, \cdots, \mathbf{R}_{N_{\text{BS}}}$ }. Under the assumptions of iid Gaussian channel coefficients, equation (6) is equivalent to the MMSE estimator (also linear-MMSE without Gaussian assumptions) [9]

$$\hat{\mathbf{h}}_{\text{MMSE}} = \bar{\mathbf{R}}\bar{\mathbf{S}}^{\text{H}}(\bar{\mathbf{S}}\bar{\mathbf{R}}\bar{\mathbf{S}}^{\text{H}} + \sigma^{2}\mathbf{I}_{N_{z}M})^{-1}\mathbf{y}.$$
 (7)

It is well-known that the performance for the aforementioned techniques depend on the scenario and channel model assumptions. For instance, the LS estimator (4) suffers from lack of orthogonality between desired and interfering pilots. The G-LS, which is equivalent to a Maximum Likelihood (ML) estimator, exploits only the Gaussian assumption on the noise. Finally, Bayesian estimators are sensitive to the errors affecting the estimates of the covariance matrices and, in some cases, impractical since the acquisition of the second order statistics can be *inefficient* [9, Sec. VI].

## B. Sparsity-Aware Regularized Least Squares

In the sequel, we provide our main contribution, namely the SCE algorithm. Differently from the aforementioned methods, our approach relies on a sparse representation of the channel vector  $\mathbf{h} \triangleq \text{vec}(\mathbf{H})$  obtained via the KLT. Specifically, we consider the transformation

$$\mathbf{c} = \bar{\mathbf{U}}^{\mathrm{H}} \mathbf{h},\tag{8}$$

where  $\bar{\mathbf{U}} \in \mathbb{R}^{N_{\text{BS}}M \times N_{\text{BS}}M}$  is a block-diagonal matrix with blocks  $\{\mathbf{U}^1, \cdots, \mathbf{U}^{N_{\text{BS}}}\}$ , where  $\mathbf{U}^i$  is obtained from the eigen-decomposition of  $\mathbf{R}_i$ , *i.e.*  $\mathbf{R}_i = \mathbf{U}^i \boldsymbol{\Sigma}_i (\mathbf{U}^i)^{\text{H}}$  with  $\boldsymbol{\Sigma}_i \in \mathbb{R}^{M \times M}$  denoting the eigenvalue matrix of  $\mathbf{R}_i$ .

Notice that the above channel reprentation can also be derived from (1) as follows

$$\mathbf{c} = \bar{\mathbf{U}}^{\mathrm{H}} \mathbf{h} = \boldsymbol{\Sigma}_{i}^{1/2} \mathbf{z},\tag{9}$$

from which it is evident that the sparsity of c relates to the non-zero eigenvalues of the channel covariance matrix.

Utilizing recent results in sparse-signal estimation theory [10], the vector c can be estimated from y as

$$\hat{\mathbf{c}}_{\text{CSE}} = \arg \min_{\mathbf{c} \in \mathbb{C}^{MN_{\text{BS}}}} \mu \| \boldsymbol{\omega}^{\text{T}} \mathbf{c} \|_{1} + \frac{1}{2} \| \mathbf{y} - \bar{\mathbf{S}} \bar{\mathbf{U}} \mathbf{c} \|_{\text{F}}, (10)$$

where  $\boldsymbol{\omega} \in \mathbb{R}^{MN_{\text{BS}}}_+$  is a weighing vector with higher values to those coefficients that must be set to zero<sup>1</sup>,  $\mu \propto \sigma^2$ , *e.g.*,  $\mu = \frac{1}{2}\sigma^2$  [11], [12] and  $\|\cdot\|_{\text{F}}$  is the Frobenius norm.

Before proceeding with the performance comparison, a few considerations are in order. Firstly, notice that this optimization problem is well-known in the literature as Least Absolute Shrinkage and Selection Operator (LASSO), where the  $\|\cdot\|_1$  acts as a regularization term and sparsity inducing operator. Therefore, the proposed algorithm becomes more and more effective when the vector c is *sparse*, *e.g.* when *M* is large [13] or the angular spread of the arrival paths is small [9].

Secondly, notice that the optimization is formulated only with the eigenvectors of  $\mathbf{R}_i$ . On the one hand, this is drawback with respect to a Bayesian approach since the information

 $<sup>^{1}</sup>$ If the channel covariance matrix  $\mathbf{R}_{i}$  is known, then the coefficients weighing the corresponding eivenvectors are inversely proportional to the its eigenvalues.

on the eigenvalues, thus the relevance of each eigenvector, is neglected. On the other hand, this is an advantage, because the optimization seeks the optimal weights for the eigenvectors. This becomes important when an imperfect estimate of  $\mathbf{R}_i$ , denoted by  $\hat{\mathbf{R}}_i$ , is available and, the Bayesian estimators fail.

Specifically, let  $\mathbf{R}_i$  be the sample covariance<sup>2</sup> given by

$$\hat{\mathbf{R}}_{i} = \frac{1}{K} \sum_{k=1}^{K} (\hat{\mathbf{h}}_{\mathrm{LS},k}^{i}) (\hat{\mathbf{h}}_{\mathrm{LS},k}^{i})^{\mathrm{H}},$$
(11)

where the index k refers to the k-th out K pilot transmissions. Rewrite  $\hat{\mathbf{R}}_i$  as  $\hat{\mathbf{R}}_i = \mathbf{R}_i + \boldsymbol{\Delta}_i$ , with  $\boldsymbol{\Delta}_i$  given by

$$\boldsymbol{\Delta}_{i} = \frac{1}{K} \sum_{k=1}^{K} \left( \sum_{\substack{j\neq i\\j=1}}^{N_{\text{BS}}} \sum_{\substack{t\neq i\\t=1}}^{N_{\text{BS}}} \alpha_{ji} \alpha_{ti}^{*} \mathbf{h}_{k}^{j} (\mathbf{h}_{k}^{t})^{\text{H}} + 2 \sum_{\substack{j\neq i\\j=1}}^{N_{\text{BS}}} \alpha_{ji}^{*} \mathbf{h}_{k}^{i} (\mathbf{h}_{k}^{j})^{\text{H}} + 2 \sum_{j=1}^{N_{\text{BS}}} \alpha_{ji}^{*} \mathbf{h}_{k}^{i} (\mathbf{s}_{i}^{\dagger}) \mathbf{N}_{k}^{\text{H}} + \mathbf{N}_{k} (\mathbf{s}_{i}^{\text{T}})^{\dagger} (\mathbf{s}_{i}^{\dagger}) \mathbf{N}_{k}^{\text{H}} \right), \quad (12)$$

where  $\alpha_{ji} = \mathbf{s}_{j}^{\mathrm{T}}(\mathbf{s}_{i}^{\mathrm{T}})^{\mathrm{H}}$  and  $\mathbf{a}^{*}$  indicates conjugate.

It is easy to see that with  $K \to \infty$ ,  $\Delta_i$  is not vanishing due to the presence of noise as well as the non-orthogonality amongst pilot sequences. Therefore, the eigenvector and eigenvalue matrices of  $\hat{\mathbf{R}}_i$ , respectively denoted by  $\hat{\mathbf{U}}_i$  and  $\hat{\boldsymbol{\Sigma}}_i$ , will be also affected by an error and, when we compute the KLT with  $\hat{\mathbf{U}} \in \mathbb{R}^{N_{\text{BS}}M \times N_{\text{BS}}M}$  replacing  $\bar{\mathbf{U}}$ , a new transformation of  $\mathbf{h}$  is obtained, *i.e.*,

$$\hat{\mathbf{c}} = (\hat{\mathbf{U}})^{\mathrm{H}} \mathbf{h} = (\hat{\mathbf{U}})^{\mathrm{H}} \bar{\mathbf{U}} \mathbf{c}.$$
 (13)

Indeed  $\hat{\mathbf{c}}$  is different from  $\mathbf{c}$  since  $(\hat{\mathbf{U}})^{\mathrm{H}} \hat{\mathbf{U}} \neq \mathbf{I}$  due to the imperfect estimate of  $\mathbf{R}_i$ . However,  $\hat{\mathbf{c}}$  can preserve a sparse representation as long as the eigenvector estimates and the true ones are sufficiently aligned. In order to verify the sparsity of  $\hat{\mathbf{c}}$ , we compute the average compression-rate of the vector, which refers to the exponent of the compressible signal model  $p_c(n) = An^{-\nu}$  [10], and show that is not close to 0, *i.e.* no compression. Specifically, given a fixed set of pilot sequences, and distribution of the angular spread, we estimate the average compression-rate of  $\mathbf{c}$  and  $\hat{c}$ , respectively denoted by  $\nu$  and  $\hat{\nu}$ , as a function of K. The result shown in Figure 1 validates our claim since  $\hat{\nu} > 0$  and increases with K. Nevertheless, the vector  $\hat{\mathbf{c}}$  does not converges to  $\nu$  due to the interference.

# **IV. SIMULATION RESULTS**

The simulation scenario consists of a classical multi-cell network with  $N_{\rm BS} = 7$  hexagonal-cell-shape and, for simplicity,  $N_u = 1$  user per cell. Without loss of generality, the base-station of cell-1 is performing channel estimation.

We assume an Uniform Rectangular Array (URA) with  $8 \times 3$ antenna elements placed on the plane (x, 0, z). Let  $\mathbf{a}_i \triangleq [a_{i,1}, a_{i,2}, a_{i,3}]^T$  and  $\mathbf{z}_i \triangleq [z_{i,1}, z_{i,2}, z_{i,3}]^T$  be the antenna's coordinate vectors for the *i*-th base-station and the mobile served by the *i*-th base-station, respectively. Each channel

<sup>2</sup>Alternative methods for sample covariance estimators can be applied, *e.g.*, form MMSE. The better the estimate is, the faster is the convergence to the ideal case of exact knowledge of  $\mathbf{R}_i$ .



Figure 1. Effects of the sample covariance error on the average compressionrate of **h**.

realization is generated by equation (1), where the set of angles  $\theta_{\ell}^i$  and  $\phi_{\ell}^i$  are random variables with uniform distributions<sup>3</sup>  $p_{\theta_i} = \mathcal{U}(\bar{\theta}^i, \pm \delta_{\theta_i})$  and  $p_{\phi_i} = \mathcal{U}(\bar{\phi}^i, \pm \delta_{\phi_i})$ , where  $\bar{\theta}^i$  and  $\bar{\phi}^i$  be the *direct* azimuth and elevation angle measured between  $\mathbf{a}_1$  and  $\mathbf{z}_i$ . The sample channel covariance is obtained by equation (11) using K consecutive pilot transmissions.

The pilot sequence  $\mathbf{s}_i \in \mathbb{C}^{N_z}$  is a Zadoff-Chu (ZC)-sequence with the *i*-th symbol given by [14]

$$s_i = \exp\left(\frac{-j\pi u n(n+2q)}{N_z}\right),\tag{14}$$

where  $N_z$  is an even number,  $q \in \mathbb{Z}$ , n = (i - 1),  $0 < u < N_z$  is the ZC-sequence root such that  $gdc(N_z, u) = 1$  where gdc(x, y) is the great-common-divisor between x and y. Other simulation parameters are provided in the Table I.

To evaluate the performance of the channel estimation schemes two metrics are used. The first one is the *relative* Mean Square Error (MSE) in dB-scale, defined by

rMSE(
$$\mathbf{h}_1$$
) = 10log<sub>10</sub>  $\left( \frac{\mathbb{E}\{\|\mathbf{h}_1 - \hat{\mathbf{h}}_1\|_F^2\}}{\mathbb{E}\{\|\mathbf{h}_1\|_F^2\}} \right)$ , (15)

where  $\mathbb{E}\{\cdot\}$  denotes the expectation.

The second metric is the Bit Error Rate (BER) measured in the up-link and for an uncoded data with QAM modulation.

Table I	
OTHER SIMULATION PARAMETERS	
Network Parameter	Value
Cell radius, $R_c$	1 km
Number of user per cell, $N_u$	1
Base-station antenna elevation, $z_a$	100 m
User antenna elevation, $z_m$	[1-20] m
Pilot length, $N_z$ ,	11
Antenna x-spacing, $\Delta_x$	$\lambda/2$
Antenna z-spacing, $\Delta_z$	$\lambda/2$

<sup>3</sup>Utilizing this simulation strategy, the absolute location of the users with respect to the base station is not relevant.

We assume a Zero-Forcing (ZF) equalizer to remove the interference from the data transmitted by the user in cell-1, *i.e.*,  $\hat{\mathbf{b}}_1^{\mathrm{T}} = \mathbf{w}\mathbf{Y}$ , where  $\mathbf{w}$  is the first row-vector of  $\hat{\mathbf{H}}^{\dagger}$ ,  $\mathbf{Y} = \mathbf{B} + \mathbf{N}$ , with  $\mathbf{B} \triangleq [\mathbf{b}_1, \cdots, \mathbf{b}_{N_{\mathrm{BS}}}]^{\mathrm{T}}$  and  $\mathbf{b}_i$  is the vector of QAM symbols transmitted by the *i*-th user.

We compare the proposed SCE algorithm with uniform weights to the LS, G-LS and the MMSE using the sample covariance and illustrate the performance of an ideal MMSE and SCE using the information on the eigenvectors and eigenvalues of  $\mathbf{R}_i$ . These algorithms are labelled with \*, *e.g.* MMSE\*.

Figure 2(a) shows the relative-MSE as a function of the number of pilot sequences K utilized in the learning phase of the covariance matrix. We assume a fixed Signal-to-Interference-Noise Ratio (SINR) = 0dB and angular spreads  $\delta_{\theta} = \frac{\pi}{18}$  and  $\delta_{\phi} = \frac{\pi}{36}$ . The result shows that while the performance of the LS, the G-LS and the MMSE\* are invariant with K, the estimation error obtained with the MMSE and SCE decrease with the increase of pilot transmissions used in the learning phase of the covariance matrix. Interestingly, for very large K, the difference between the MMSE and SCE is small. Moreover, the ideal SCE\*, that uses the exact covariance matrix, reaches the optimal performance of the MMSE\*.

Figure 2(b) shows the relative-MSE as a function of the SINR, with K = 5 and angular spread as above. Also in this study the results demonstrate that the proposed SCE outperforms the alternative techniques using the same assumptions. Approximately a gap of  $\approx 10$  dB is separating the SCE from the MMSE<sup>\*</sup> due to the error in the covariance matrix. This gap reduces with the increase of K.

Figure 2(c) shows the comparison of the BER achieved in the uplink for increasing values of the SINR. The result clearly indicates a large leap from the performance of the MMSE to those of the SCE proving that approach is more robust to the imperfect estimation of the covariance matrix. The G-LS, however, shows relative good performance but inferior to those of the SCE due to a lack of structure in the problem formulation solved by the G-LS. Specifically, the G-LS provides the optimal solution in the the LS-sense when h does not have a sparse representation. Finally, notice that the SCE\* performs as good as the MMSE\*.

## V. CONCLUSIONS

In this paper we addressed the problem of pilot-based uplink SIMO channel estimation with the assumption of inter-cell interference, *i.e.* pilot contamination. By exploiting the spatial channel correlation, we proposed an sparsity-aware channel estimation algorithm, referred to as SCE. This transformation is effective also in the presence of imperfect estimation of the channel covariance matrix. Simulation results demonstrated the advantages with respect to classical algorithms, namely, the LS, the G-LS and the MMSE. Future work focuses on the effects of doppler and multipaths based on the model in [6].

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Figure 2. Comparison of different channel estimation schemes in a multi-cell scenario with  $N_i=6$  interferers.

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