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ROBUST SPARSE CHANNEL ESTIMATION FOR OFDM SYSTEM USING AN ITERATIVE ALGORITHM BASED ON COMPLEX MEDIAN

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ABSTRACT

In this paper, we present a robust approach to estimate communications channel in OFDM systems exploiting the sparsity of the channel impulse response (CIR) commonly found in multi-path channels. The CIR is found as the solution of a regularized optimization problem where we minimize the least absolute deviation of a residual signal while, at the same time, encourage sparsity in the solution by an ℓ_0 pseudo-norm regularization term. The proposed approach reduces to estimate iteratively each tap of the CIR using a complex median based operator followed by a relevance test that forces sparsity in the solution. A blanking filter at the front end of the receiver is used to further mitigate the impact of the impulsive noise. Extensive simulations show that the proposed approach performs better than conventional approaches for AWGN channels and for situations where the additive noise follows a heavier-than-Gaussian tail distribution.

Index Terms— Sparse Channel Estimation, Impulsive Noise, Complex Weighted Median, OFDM, Robust Estimation.

1. INTRODUCTION

In recent years, OFDM has been chosen as the preferable modulation technique adopted for communications standards in 4G mobile communications, wireless networks, terrestrial digital TV, among others. One of the benefits of using OFDM modulation is that the equalization process can be made using a single-tap frequency domain equalizer where the transmitted symbol at the k-th subcarrier is estimated by just taking the ratio of the received signal and the channel frequency response at that particular subcarrier. To exploit this advantage, it is required an accurate knowledge of the OFDM communication channel to achieve the demodulation process with a minimal error, in particular for frequency selective channels. Several methods have been proposed to perform OFDM channel estimation (CE) [1, 2, 3, 4, 5]. An approach that is of particular interest is the data-aided framework, where pilot signals are uniformly interleaved among information symbols and used to estimate channel frequency response at pilot subcarriers. Estimation methods based on least square (LS) and Minimum Mean Square Error (MMSE) [1] are commonly used to estimate the channel at pilot frequencies. All these methods, however, assume that the noise is AWGN, in which case a quadratic loss function is considered optimal under the maximum likelihood (ML) principle to address the CE problem. There exist several communications channels [6] where the noise is better modeled by heavier-than-Gaussian tail distributions, i.e. impulsive kind of noise, on those cases the above-mentioned approaches have poor performance. Furthermore, conventional CE approaches are not able to exploit the inherent sparsity of the transmission channel [3, 7]. These call for the development of robust CE algorithms for OFDM transmission system that at the same time exploit the sparsity of CIR.

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In this paper, we propose a robust approach for OFDM CE that, on one hand, exploits the sparsity observed in multi-path CIR [3, 7] and, on the other, adds the desired robustness to impulsive noise in the estimation stage. Firstly, we mitigate the impulsive noise effects by a blanking filter at the front end that cancels out strong impulse noise forcing it to zero [8, 9]. Next, we address the channel estimation problem in the framework of a linear regression problem with a sparse parameter vector [10] where the CIR plays the role of a sparse vector, the prefiltered-received signal at pilot subcarrier as the observed data and a Fourier submatrix as the regressors. This linear regression problem is solved by minimizing a robust loss function subject to the constraint that the CIR is sparse. More precisely, our approach adds robustness to the estimation stage using an absolute value function (l_1-norm) as a loss term and, to force sparsity in the solution, a regularization term based on the l_0 pseudo-norm is incorporated in the optimization problem. This optimization problem is solved by using a coordinate descend framework together with a continuation approach for the selection of the regularization parameter[11], reducing the solution to an iterative algorithm that estimates each tap of the channel impulse response using complex median operator followed by a hard threshold operator that decides whether the tap is relevant or not.

The proposed approach is extensively tested under different conditions for impulsiveness of the channel contamination, number of pilot signals, number of subcarriers and number of relevant taps of the channel. Furthermore, the performance achieved by the proposed approach is compared to those yielded by conventional channel estimation approaches like, LS and MMSE, and to that yielded by a ℓ_1-regularized LS based CE approach[2, 3, 4].

2. OFDM SYSTEM MODEL

Let $\mathbf{X}^{(n)} \in \mathcal{C}^N$ be a vector denoting the n-th OFDM symbol comprising of M pilot signals at location $\Omega = \{p_1, p_2, \ldots, p_M\}$ and $N-M$ complex information symbols located at Ω^c where $\Omega \cup \Omega^c = \{0, 1, \ldots, N-1\}$, with $\Omega \cap \Omega^c = \emptyset$. Once the OFDM symbol is formed, it is mapped to the time domain using Inverse Discrete Fourier Transform (IDFT), i.e. $\mathbf{x}^{(n)} = \mathbf{F_N}^{-1}\,\mathbf{X}^{(n)}$, where $\mathbf{F_N}^{-1} \in \mathcal{C}^{N\times N}$ is the N-point DFT matrix. In the time domain, a cyclic prefix (CP) defined as $\mathbf{x}_{cp}^{(n)} = [x^{(n)}(N-L), x^{(n)}(N-L+1), \ldots, x^{(n)}(N-1)]^T$ is appended at the front of $\mathbf{x}^{(n)}$, where L is the length of the CP which is assumed to be longer than the channel delay spread avoiding thus Inter Symbol Interference (ISI) between adjacent OFDM symbols.

The extended time domain symbol $\mathbf{x}_t^{(n)} = [\mathbf{x}_{cp}^{(n)^T}, \mathbf{x}^{(n)^T}]^T$ is then transmitted through a multi-path communications channel modeled as $h[k] = \sum_{\ell=1}^{J} h_\ell \delta[k-d_\ell]$ where J is the number of paths, h_ℓ is the ℓ-th complex path gain and d_ℓ is the corresponding path delay. We adopt a sparse channel model where the nonzero gain path h_ℓ fol-

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lows a zero-mean complex random process and the path delays, d_ℓ, are assumed to be sampled-spaced. Both path delays and path gains are considered fixed during the transmission of the n-th ODFM symbol but they may vary for subsequent OFDM symbols. Thus, fast fading channel is assumed. In vector form, $h[k]$ can be represented as $\mathbf{h} \in \mathcal{C}^J$, with each entry equals to h_ℓ for $\ell = 1, \ldots, J$. Assuming perfect synchronism between transmitter and receiver and since $J \leq L$, i.e. ISI free, the received signal for the n-th transmitted OFDM symbol is given by $\mathbf{y}_t^{(n)} = \mathbf{x}_t^{(n)} * \mathbf{h}^{(n)} + \mathbf{w}^{(n)}$, where $*$ denotes the convolution operator, $\mathbf{y}_t^{(n)}$ is the received signal in the time domain with length $N + L + J - 1$, $\mathbf{w}^{(n)} \in \mathcal{C}^{N+L+J-1}$ denotes the time domain noise and $\mathbf{h}^{(n)}$ denotes the J-length channel impulse response. Once the signal $\mathbf{y}_t^{(n)}$ is received, the L-length CP is removed and an N-point FFT is applied on the following N samples of the resultant signal leading to a frequency domain representation of the received OFDM symbol, denoted as $\mathbf{Y}^{(n)}$. Having the received signal in the frequency domain, data symbols and pilot signals are separated to perform CE using the information contained at pilot subcarriers. After that, the transmitted symbol at the k-th subcarrier is estimated through a one-tap frequency domain equalizer, i.e. $\hat{X}(k)^{(n)} = Y(k)^{(n)}/\hat{H}(k)^{(n)}$ for $k \in \Omega^c$ where $\hat{H}(k)$ is the estimated channel frequency response.

2.1. OFDM channel estimation

The simplest approach to achieve CE is to estimate the channel frequency response at pilot subcarrier as $\hat{H}^{ls}(k) = Y(k)/X(k)$ for $k \in \Omega$ where $X(k)$ is the pilot symbol known at the receiver. From this channel information at pilot subcarriers, channel at data subcarriers can be estimated by linear interpolation or second order interpolation [1]. This CE approach at pilot subcarriers is known as *Least Square (LS)* CE. A second approach commonly used for OFDM CE is the one based on minimum mean square error (MMSE) which assumes that the CIR $\mathbf{h}$ is Gaussian and uncorrelated with the additive channel noise $\mathbf{w}$ [1].

Note that neither LS nor MMSE CE takes into account the sparsity characteristics of the CIR [3, 7]. To exploit the sparsity structure of the channel, it is convenient to restate the CE problem under the framework of a linear regression problem with a sparse vector parameter. To this end, note that the received signal at pilot subcarriers can be stated as:

$$\mathbf{Y}_\Omega = \mathbf{X_p}[\mathbf{F}_N]_{\Omega,L}\mathbf{h} + \mathbf{W}_\Omega \tag{1}$$

where $\mathbf{h}$ is a L-dimensional vector assuming, for simplicity, that the channel length is equal to the CP length, i.e. $L = J$. $[\mathbf{F}_N]_{\Omega,L}$ is a $M \times L$ matrix built by keeping the rows of $\mathbf{F}_N$ at pilot positions and retaining the first L columns of $\mathbf{F}_N$ [3]. Finally, $\mathbf{X_p} = \text{diag}[X_{p_1}, X_{p_2} \ldots X_{p_M}]$.

Note that (1) can be thought of as an inverse problem where we are interested in finding the sparse vector $\mathbf{h}$ from the received signal at pilot tones such that the residual error $\epsilon = \mathbf{Y}_\Omega - \mathbf{X_p}[\mathbf{F}_N]_{\Omega,L}\mathbf{h}$ becomes as small as possible under certain loss function.

Following this line of thought, several methods have been recently proposed [2] that minimize a quadratic loss function (ℓ_2-norm) of the residual error ϵ while promoting, at the same time, sparsity in the solution by an ℓ_1-norm.

Like LS and MMSE CE approaches, these ℓ_1-LS based CE approaches are suitable when the channel contamination follows a Gaussian model. Although optimal under the ML criterion for this kind of noise, the ℓ_2-norm based approaches tend to be very

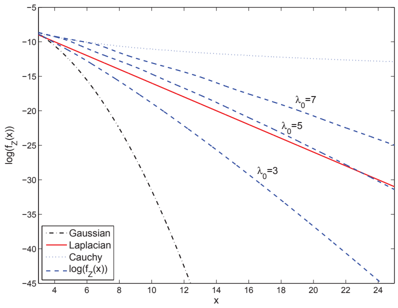


Fig. 1. Tails of the distribution at the output of the blanking filter for various values of the nonlinearity parameter λ_0.

sensitive to impulsive noise and their performances, in general, degrade notably for noisy environments that are better characterized by an heavier-than-Gaussian-tail distributions. Furthermore, using ℓ_1-norm as sparsity-inducing term might not be optimal since, as an approximation to the pseudo-norm ℓ_0, it may not induce enough sparsity in the solution.

3. ROBUST CHANNEL ESTIMATION APPROACH

To overcome the above mentioned drawbacks, we propose an approach to estimate OFDM channel coefficients that, on one hand, adds robustness against impulsive noise to the CE stage and, on the other hand, induces the required sparsity in the solution.

Let's consider that the underlying contamination, $\mathbf{w}$, of our OFDM transmission system is modeled by a Symmetric Alpha-Stable (SαS) distribution [12] defined by its characteristic function $\Phi(\omega) = e^{-\gamma|\omega|^\alpha}$ where $\alpha \in (0, 2]$ indicates the thickness of the tails of the distribution ranging from extremely impulsive noise ($\alpha \to 0$), to not impulsive at all for $\alpha = 2$ Gaussian distributed noise. γ is the dispersion parameter of the SαS distribution. Since the received signal is mapped to the frequency domain, one may be tempted to think that due to the mapping operation of the OFDM system that spreads the energy of each impulse evenly across all the subcarrier of the transmitted symbol, the impulsive noise do not harm at all the receiver performance. However, when there are enough impulses during the transmission of the same OFDM symbol their effects combine linearly in each subcarrier leading to a performance degradation in the channel estimation stages. Furthermore, the generalized central limit theorem [12] states that the noise at the output of the DFT operator becomes α-stable, and, hence, with heavier-than-Gaussian tail distribution, the ℓ_2-norm based postprocessing is no longer optimum.

3.1. Impulsive Noise Mitigation

Several techniques for impulsive noise mitigation in OFDM system have been proposed in the literature [8]. An approach that is particularly appealing is to pre-filter the received signal by a blanking filter placed at the receiver front end. This pre-filter operation identifies the large received samples and reduce their effect by forcing them to zero. This memoryless nonlinearity operation is applied on the real part and imaginary part of the received signal leading to an output signal given by:

$$z_F(i) = \begin{cases} F(y(i)) & \text{if} \quad |F(y(i))| \leq \lambda_0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $F \in \{\Re(\cdot), \Im(\cdot)\}$ denotes, either the real or the imaginary part of the argument and λ_0 is a threshold parameter that depends on the SNR and the impulsiveness of the communication channel.

The output signals of these nonlinear operators are then combined as $\mathbf{z} = \mathbf{z}_R + j\mathbf{z}_I$ and feed to the CP removing block and, subsequently, to DFT block to obtain $\mathbf{Z}$. Although this filtering operation reduces the undesired effect of the impulse noise, it modifies the statistical of the contamination in the frequency domain tending to a distribution with heavier-than-Gaussian tail distribution as λ_0 increases. To see this, in Fig. 1, it is shown the tails of the distribution of the real part of $\mathbf{Z}$ for the k-th subcarrier found by N successive convolutions of the pdf of $\mathbf{Z}$ ($f_Z(x)$) for several values of λ_0. For comparative purpose, we also show the tails of Gaussian, Laplacian and Cauchy distributions with unitary dispersion and zero location parameter. Note that, as expected the tail heaviness of the distribution tends to increase as the threshold parameter becomes higher. Furthermore, $f_Z(x)$ has heavier tails than Gaussian's and going closer to the Laplacian's ones leading us to think that the use of the absolute value loss function ($\rho(x) = |x|$) for channel estimation may be a good choice.

3.2. Robust channel estimation by ℓ_0-LAD

Based on this observation, we propose to estimate the channel impulsive response as a solution of an ℓ_0-regularized least absolute deviation (ℓ_0-LAD) optimization problem, this is

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \|\mathbf{X_p}^{-1}\mathbf{Z}_\Omega - \mathbf{A}\mathbf{h}\|_{\ell_1} + \tau\|\mathbf{h}\|_{\ell_0} \qquad (3)$$

where $\mathbf{Z}_\Omega$ is the DFT of $\mathbf{z}$ at pilot subcarriers and $\mathbf{A} = [\mathbf{F}_N]_{\Omega,L}$. Note that we use an l_1-norm in the residual error term to add robustness to the estimation problem. Furthermore, as sparsity-promoting term we use the pseudo-norm ℓ_0 to encourage sparsity in the solution. In (3), τ is the regularization parameter that controls the influence of the sparsity term in the solution. Solving the optimization problem (3) is computationally expensive since this is a NP-hard problem due to the use of the pseudo-norm as sparsity inducing term.

To overcome this apparent limitation, we modify an optimization algorithm recently proposed in [11] that solves an optimization problem similar to that formulated in (3). Extending these ideas to our optimization problem, first note that (3) can be reduced, after some manipulations, to a scalar minimization problem

$$\tilde{\mathrm{h}}_n = \arg\min_{\mathrm{h}_n} \sum_{i=1}^{M} |A_{i,n}| \left| \frac{Z'_i - \sum_{j=1,j\neq n}^{L} A_{i,j}h_j}{A_{i,n}} - h_n \right| + \tau\|h_n\|_{\ell_0} \qquad (4)$$

assuming that the n-th component of the unknown vector, $\mathbf{h}$, is allowed to vary while the others are fixed to some pre-estimated values where $Z'_i = Z(p_i)/X(p_i)$. Thus, the N-dimensional inverse problem (3) with complex coefficients is splitted into N one-dimensional optimization subproblems, one for each element of the unknown vector. This weighted least absolute deviation problem regularized by the ℓ_0-norm can be solved by computing the weighted median operator on a complex data [11, Theorem 1]:

$$\tilde{\mathrm{h}}_n = \text{median}\left(|A_{in}| \diamond \frac{Z'_i - \sum_{j=1,j\neq n}^{L} A_{i,j}\mathrm{h}_j}{A_{i,n}} \Bigg|_{i=1}^{M} \right) \qquad (5)$$

followed by a hard thresholding operator

$$\hat{\mathrm{h}}_n = \begin{cases} \tilde{\mathrm{h}}_n & \text{if} \quad \|\mathbf{r}_n\|_{\ell_1} - \|\mathbf{r}_n - \mathbf{A}_n\tilde{h}_n\|_{\ell_1} > \tau \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

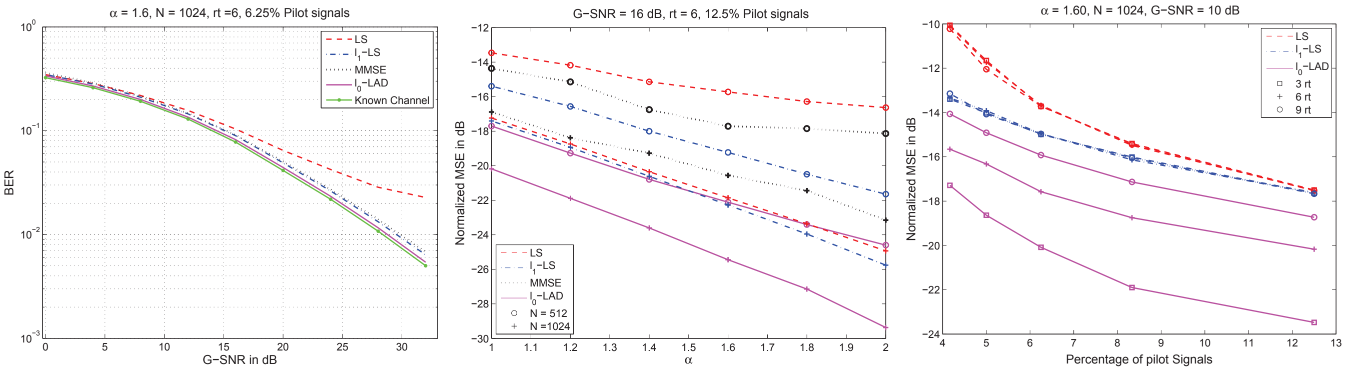
where $\diamond$ denotes the replication operator defined as $w_i \diamond z_i = \overbrace{z_i, \ldots, z_i}^{w_i \text{ times}}$, $\mathbf{A}_k$ denotes the k-th column-vector of the matrix $\mathbf{A}$, and $\mathbf{r}_n = \mathbf{Z}' - \sum_{j=1,j\neq n}^{L} \mathbf{A}_j \mathrm{h}_j$ is the residual term that remains after subtracting from the frequency response vector at pilot subcarriers the influences of the multi-path terms but the n-th one. Equations (5) and (6) reveal that the estimation of $\tilde{\mathrm{h}}_n$ can be thought of as a two-stage operation. First, compute the weighted median operator on the data samples that result from shifting and scaling $\mathbf{Z}'$, using the column-vector $|\mathbf{A}_n|$ of the Fourier sub-matrix as weights. Secondly, the WM output is then passed through a hard threshold operator to decide whether the estimated value is relevant or not. Thus, the n-th channel tap is considered relevant if it leads to a variation on the ℓ_1-norm of the residual vector $\mathbf{r}_n$ larger than the regularization parameter τ, otherwise its effect is considered negligible and, hence, it is forced to zero inducing sparsity in the CIR.

Table 1. Proposed algorithm for OFDM sparse channel estimation

Inputs	$\mathbf{Z}' = \mathbf{X_p}^{-1}\mathbf{Z}_\Omega$ Fourier sub-matrix $\mathbf{A} = [\mathbf{F}_N]_{\Omega,L}$ Number of iterations I_0
Init	Initial Regularization parameter $\tau_0 = \|\mathbf{A}^T\mathbf{Z}'\|_\infty$ Iteration index $k = 1$ Channel impulse response $\hat{\mathbf{h}}^{(1)} = \mathbf{0_N}$
Iteration	
Step A	For each entry of $\hat{\mathbf{h}}$, compute $\Re(\tilde{h}_n) = \text{median}\left(\Re\left\{\frac{Z'_i - \sum_{j=1,j\neq n}^{L} A_{i,j}\hat{h}_j^{(k)}}{A_{i,n}}\right\}\Big|_{i=1}^{M}\right)$ $\Im(\tilde{h}_n) = \text{median}\left(\Im\left\{\frac{Z'_i - \sum_{j=1,j\neq n}^{L} A_{i,j}\hat{h}_j^{(k)}}{A_{i,n}}\right\}\Big|_{i=1}^{M}\right)$ $\tilde{h}_n = \Re(\tilde{h}_n) + j\Im(\tilde{h}_n) \quad \mathbf{r}_n = \mathbf{Z}' - \sum_{j=1,j\neq n}^{L} \mathbf{A}_j\tilde{h}_j$ $\hat{h}_n^{(k)} = \begin{cases} \tilde{\mathrm{h}}_n & \text{if} \quad \|\mathbf{r}_n\|_{\ell_1} - \|\mathbf{r}_n - \mathbf{A}_n\tilde{\mathrm{h}}_n\|_{\ell_1} > \tau_k \\ 0 & \text{otherwide.} \end{cases}$
Step B	Update the regularization parameter and the estimation of $\mathbf{h}$ $\tau_k = \tau_0\beta^k, \quad \hat{\mathbf{h}}^{(k+1)} = \hat{\mathbf{h}}^{(k)}$
Step C	Check stopping criterion If $k \leq I_0$ then k = k+1, go to step A; else end
Output	Estimated Channel Impulse Response $\hat{\mathbf{h}}$

Upon closer look to the optimization problem formulated in (4), it can be seen that all quantities involved are complex numbers (Fourier sub-matrix, CIR and prefiltered-received signal at pilot tones), therefore solving this optimization problem leads to computing a median operation on complex data. It turns out that the sorting operation involved in computing the weighted median is not well established for complex samples. To overcome this apparent limitation, we extend the definition of the weighted median operator acting on complex data with complex weights [13]. Thus, the weighted median operation is computed independently on the real and imaginary parts of the complex sample data to yield estimated values for the real and imaginary parts of n-th component of the CIR.

The n-th estimated component of complex channel impulsive response is then obtained as $\tilde{\mathrm{h}}_n = \Re(\tilde{h}_n) + j\Im(\tilde{h}_n)$ which is tested for relevance by the thresholding operator (6). By doing this marginal median approach, the loss function that is minimized reduces to $\rho(\epsilon) = |\Re(\epsilon)| + j|\Im(\epsilon)|$ while the sparsity is jointly forced according to (6). Note that in solving the scalar optimization problem (4), we have assume that h_k for $k \neq n$ has been fixed to a previously estimated value. Once $\tilde{h}_n$ is found, it is fixed and the estimation process continues with the entry $n+1$ of $\mathbf{h}$ considering the other entries as fix values. This process continues iteratively until all the components of the CIR are estimated.



(a) (b) (c)

Fig. 2. (a) BER curve for the channel estimation approaches. (b) Performances of the various channel estimation approaches as the channel noise impulsiveness varies. (c) Performance where the number of pilot symbols increases for different number of relevant taps (rt).

Table 1 shows the proposed iterative algorithm to estimate OFDM channel impulse response. Note that the algorithm starts assuming that all taps of the channel impulse response take zero values. As the algorithm progresses, the entries of $\mathbf{h}$ are modified as soon as the estimated value $\check{h}_n$ produces a variation on the ℓ_1-norm of $\mathbf{r}_n$ greater than the regularization parameter τ. This parameter, in turn, decreases its value as the algorithm progresses, i.e. ($\tau = \tau_0 \beta^k$) for $0 < \beta < 1$. Thus, it is expected that, as the algorithm progresses, new non-zero entries being incorporated to $\hat{\mathbf{h}}$. β controls the decaying rate of the regularization parameter τ and its value defines the convergence rate and the minimal error achieved by the proposed algorithm.

Furthermore, the regularization parameter τ which turns out to be the thresholding parameter of the threshold operator controls whether the estimated value, $\hat{h}_n$, is relevant or not. Its value changes as the algorithm progresses. Thus, for a fixed value of τ_k at the k-th iteration, a median based estimator is run over all entries of $\mathbf{h}$ yielding $\tilde{\mathbf{h}}^{(k)}$. This estimated valued is the initial condition for the next iteration with a new τ_{k+1} smaller than the one used at the k-th iteration.

4. SIMULATIONS

To test the proposed OFDM channel estimation algorithm, we carry out a series of simulations where the performance of the proposed approach is compared to those yielded by conventional channel estimation methods, (LS and MMSE [1]) and an l_1-LS based algorithm [2, 3, 4] that solves an ℓ_1-regularized least square optimization problem to estimate the OFDM channel, where the regularization parameter is set to $0.01\|\mathbf{A}^T\mathbf{Z}'\|_\infty$ while for the proposed approach $\tau_0 = \|\mathbf{A}^T\mathbf{Z}'\|_\infty$, $\beta = 0.65$ and 10 iterations. The ODFM system used for testing is a 16 QAM/OFDM with N_p subcarriers out of N subcarriers used as pilot signals uniformly distributed on the entire OFDM frequency domain symbol. The cyclic prefix is set to 32. The relevant taps (rt) of $\mathbf{h}$ are randomly located and their complex path gains are randomly generated according to a zero-mean complex Gaussian. The threshold parameter for the blanking filter is set to $\lambda_0 = 0.5$, the optimum value found in [8]. We assume a fast fading channel, hence a new realization of the channel is used for each OFDM symbol. As performance criterion, we use the Normalized Mean Square Error (NMSE) defined by $\frac{1}{T}\sum_{k=1}^{T}\frac{|\mathbf{H}_k-\tilde{\mathbf{H}}_k|^2}{|\mathbf{H}_k|^2}$ where T is the number of trails and $\mathbf{H}_k$ denotes the true channel at the k-trail. In all the simulations, we use the Geometrical Signal to Noise Ratio (G-SNR) as a measure of signal-to-noise strength [14]. Furthermore, since we found beneficial for all channel estimation approaches to mitigate the effect of impulsive noise by the nonlinear prefiltering operation, we used it for the LS, MMSE and ℓ_0-LS channel estimation algorithms.

Figure 2 shows the performances yielded by the various OFDM CE schemes under additive SαS noise ($\alpha = 1.6$) using 6.25% of subcarriers as pilot signals. Note in Fig. 2(a) that the proposed approach achieves a BER curve closest to the ideal BER curve obtained assuming that the channel frequency response is known at the receiver. Fig. 2(b) shows the NMSE achieved by the various algorithms as the noise impulsiveness changes. Note that the proposed approach offers robustness through the all range of impulsiveness level. Furthermore, as expected, increasing the number of subcarriers from $N = 512$ to $N = 1024$ improves the channel estimation performances.

Fig. 2(c) depicts the NMSE as the number of pilot symbols changes for several relevant tap numbers. As expected, as the number of pilot signals increases, the performances achieved by the various CE approaches improve at expensive of sacrificing data throughput. Note that the performance of LS CE remains practically unchangeable as the number of rt changes insomuch as this approach does not exploit the sparsity of the communications channel. Our approach, on the other hand, tends to loss performance as the number of relevant taps increases, indeed, if the number of nonzero taps is around 40% the channel length, the communications channel is no longer considered as sparse so little is gain by exploiting sparsity. Nevertheless, since the robustness of the loss function used in our approach, it outperforms the other approaches.

5. CONCLUSIONS

In this paper, we have shown that by exploiting the sparsity structure exhibited by a multi-path channel in the CE stage of an OFDM system significant improvement in performance can be achieved. Furthermore, the use of a robust loss function in the CE stage makes this approach suitable when the underlying channel contamination follows a heavy tail distribution function. The proposed approach combines a memoryless nonlinearity operation to mitigate the effect of impulsive noise followed by an estimation stage that solves a ℓ_0-regularized LAD minimization problem to estimate the channel impulse response, exploiting thus the sparsity characteristic of the channel while, at the same time, adds robustness to impulse noise.

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