

SET-MEMBERSHIP ADAPTIVE CONSTRAINED CONSTANT MODULUS REDUCED-RANK ALGORITHM FOR BEAMFORMING

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ABSTRACT

In this work, we propose an adaptive set-membership (SM) reduced-rank filtering algorithm using the constrained constant modulus (CCM) criterion for beamforming. We develop a stochastic gradient (SG) type algorithm based on the concept of SM technique for adaptive implementation. The filter weights are updated only if the bounded constraint cannot be satisfied. In addition, we also propose a scheme of time-varying bound and incorporate parameter dependence to characterize the environment for improving the tracking performance of the proposed algorithm. Simulation results show that the proposed adaptive SM reduced-rank beamforming algorithm with dynamic bounds achieves superior performance to previously reported methods at a reduced update rate.

Index Terms— Adaptive filtering, beamforming, interference suppression, reduced-rank algorithm, set-membership filtering.

1. INTRODUCTION

The constrained constant modulus (CCM) criterion is considered as one of the most promising design criteria for adaptive beamforming that has been widely developed for different applications such as radar, sonar and wireless communications [1]. The CCM technique is based on a method that penalizes deviations of the modulus of the received signal away from a fixed value and forced to satisfy one or a set of linear constraints such that signals from the desired user are detected [2]-[5]. The CCM-based algorithms operate without the need for training sequences, and lead to a solution comparable to that obtained from the minimization of the mean squared error (MSE).

Recursive least squares (RLS) and stochastic gradient (SG) algorithms (e.g., least mean squares) are considered as the most commonly used adaptive implementation algorithms for beamforming [6]. Despite the fast convergence of RLS algorithms, however, it is preferable to implement adaptive beamformers with SG algorithms due to complexity and cost issues. For this reason the improvement of blind SG techniques is an important research and development topic. One problem for the adaptive SG algorithms is that the convergence depends on the eigenvalue spread of the received data covariance matrix. This condition could be worse when the number of filter elements is large since it requires a large amount of snapshots to reach the steady-state. In this context, reduced-rank signal processing has received significant attention in the past years. The reduced-rank technique projects the received vector onto a lower dimensional subspace and performs the filter optimization within this subspace. A number of reduced-rank algorithms have been developed to design the subspace projection matrix and the reduced-rank filter [7]-[16]. Compared to the full-rank algorithms operating with a large number of parameters, they provide faster convergence speed,

better tracking performance and an increased robustness against interference.

However, a common problem with standard adaptive algorithms is the computational complexity associated with the adaptation for every time instant. Set-membership (SM) filtering techniques have been proposed to address this issue [17]-[21]. They specify a bound on the magnitude of the estimation error or the array output, and can reduce the complexity due to data-selective updates. SM filtering techniques usually rely on two steps: 1) information evaluation and 2) parameter update. If step 2) does not occur frequently, and step 1) does not require much complexity, the overall complexity can be reduced significantly. From [17]-[21], we can see that the SM filtering techniques are able to achieve a reduction in computation without performance degradation compared to conventional algorithms due to the use of an adaptive step size for each update. The work in [19] appears to be the first approach to combine the SM filtering algorithm with the CCM criterion. To the best of our knowledge, there is a very small number of works combining the reduced-rank algorithm with the SM technique, and there has been no work with SM reduced-rank filtering using the CCM criterion.

In this paper, we present extensions of the reduced-rank approach using joint interpolation, decimation and filtering (JIDF) reported in [15] to the adaptive set-membership filtering using CCM criterion for beamforming. We develop a SG-type algorithm based on the concept of SM technique for adaptive implementation. The filter weights are updated only if the bounded constraint cannot be satisfied. In addition, we also propose a scheme of time-varying bound for beamforming and incorporate parameter dependence to characterize the environment for improving the tracking performance of the proposed algorithm. Simulation results show that the proposed adaptive SM JIDF-CCM beamforming algorithm with dynamic bounds achieves superior performance to previously reported methods at a reduced update rate.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

Let us suppose that q narrowband signals impinge on a uniform linear array (ULA) of M ($M \geq q$) sensor elements. The sources are assumed to be in the far field with direction of arrivals (DOAs) $\theta_0, \dots, \theta_{q-1}$. The i th snapshot's received vector $\mathbf{r} \in \mathcal{C}^{M \times 1}$ can be modeled as

$$\mathbf{r}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{b}(i) + \mathbf{n}(i), \quad (1)$$

where $\boldsymbol{\theta} = [\theta_0, \dots, \theta_{q-1}]^T \in \mathcal{R}^{q \times 1}$ is the vector with the DOAs of the signals, $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_0), \dots, \mathbf{a}(\theta_{q-1})] \in \mathcal{C}^{M \times q}$ comprises the normalized signal steering vectors $\mathbf{a}(\theta_k) \in \mathcal{C}^{M \times 1}$

$$\mathbf{a}(\theta_k) = [1, e^{-2\pi j \frac{u}{\lambda_c} \cos(\theta_k)}, \dots, e^{-2\pi j (M-1) \frac{u}{\lambda_c} \cos(\theta_k)}]^T, \quad (2)$$

where $k = 0, \dots, q-1$, λ_c is the wavelength, u ($u = \frac{\lambda_c}{2}$ in general) is the inter-element distance of the ULA, and $(\cdot)^T$ stands for transpose. To avoid mathematical ambiguities, the steering vectors $\mathbf{a}(\theta_k)$ are assumed to be linearly independent, $\mathbf{b}(i) = [b_0(i), b_1(i), \dots, b_{q-1}(i)]^T$ is the source data vector, where we assume that the symbols are independent and identically distributed (i.i.d) random variables with equal probability from the set $\{\pm 1\}$. The vector $\mathbf{n} \in \mathcal{C}^{M \times 1}$ is a Gaussian noise with $E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}$, where σ_n^2 denotes the noise variance, \mathbf{I} denotes an identity matrix of appropriate dimension, $E[\cdot]$ stands for expected value, and $(\cdot)^H$ stands for the Hermitian transpose. The output of a narrowband beamformer is given by

$$y(i) = \mathbf{w}^H(i) \mathbf{r}(i), \quad (3)$$

where $\mathbf{w}(i) = [w_1, \dots, w_M]^T \in \mathcal{C}^{M \times 1}$ is the complex weight vector of the filter.

2.2. Problem Statement

The full-rank adaptive filtering algorithms usually provide poor convergence performance for the beamformer design in the dynamic scenario with large M . The reduced-rank schemes which process the received vector $\mathbf{r}(i)$ in two stages have been proposed to solve these problems [7]-[16]. The first stage performs a dimensionality reduction by projecting the large dimension data vector $\mathbf{r}(i)$ into a lower dimensional subspace. The second stage is carried out by a reduced-rank filter. The output of a reduced-rank scheme is given by

$$y(i) = \bar{\mathbf{w}}^H(i) \mathbf{S}_D^H(i) \mathbf{r}(i) = \bar{\mathbf{w}}^H(i) \bar{\mathbf{r}}(i), \quad (4)$$

where $\mathbf{S}_D(i)$ denotes an $M \times D$ projection matrix which performs dimensionality reduction and $\bar{\mathbf{w}}(i) = [\bar{w}_1, \bar{w}_2, \dots, \bar{w}_D]^T$ denotes the $D \times 1$ reduced-rank filter. The basic problem of CCM reduced-rank algorithms is how to effectively devise the projection matrix \mathbf{S}_D and reduced-rank filter $\bar{\mathbf{w}}$ using the CCM criterion:

$$\text{minimize } E[e^2(i)]. \quad (5)$$

$$\text{s.t. } \bar{\mathbf{w}}^H(i) \mathbf{S}_D^H(i) \mathbf{a}(\theta_0) = \nu, \quad (6)$$

where $e(i) = |y(i)|^2 - 1$, and ν is a constant to ensure the convexity of the optimization problem as discussed in [22].

3. THE JIDF REDUCED-RANK SCHEME

In this work, we design the subspace projection matrix \mathbf{S}_D by considering interpolation and decimation. In this case, the receive filter length is substantially reduced, which results in significantly reduced computational complexity and very fast convergence speed. The $M \times 1$ received vector $\mathbf{r}(i)$ is processed by a framework contains an interpolator and a decimation unit, followed by a reduced-rank receive filter. The received vector is operated by the interpolator $\mathbf{p}(i) = [p_1(i), \dots, p_I(i)]^T$ with filter length I , $I < M$, the output of the interpolator is expressed by

$$\tilde{\mathbf{r}}(i) = \mathbf{P}^H(i) \mathbf{r}(i) \quad (7)$$

where the $M \times M$ Toeplitz convolution matrix $\mathbf{P}(i)$ is given by

$$\mathbf{P}(i) = \begin{pmatrix} p_1(i) & 0 & \dots & 0 \\ \vdots & p_1(i) & \dots & 0 \\ p_I(i) & \vdots & \dots & 0 \\ 0 & p_I(i) & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_1(i) \end{pmatrix}.$$

In order to facilitate the description of the scheme, we introduce an alternative way to present the vector $\tilde{\mathbf{r}}(i)$,

$$\tilde{\mathbf{r}}(i) = \mathbf{P}^H(i) \mathbf{r}(i) = \mathbf{R}'(i) \mathbf{p}^*(i) \quad (8)$$

where the $M \times I$ matrix $\mathbf{R}'(i)$ with the samples of $\mathbf{r}(i) = [r_0(i), \dots, r_{M-1}(i)]^T$ has a Hankel structure [24] given by

$$\mathbf{R}'(i) = \begin{pmatrix} r_0(i) & r_1(i) & \dots & r_{I-1}(i) \\ \vdots & \vdots & \dots & \vdots \\ r_{M-1}(i) & r_{M-I+1}(i) & \dots & r_{M-1}(i) \\ r_{M-I+1}(i) & r_{M-I+2}(i) & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-2}(i) & r_{M-1}(i) & 0 & 0 \\ r_{M-1}(i) & 0 & 0 & 0 \end{pmatrix}.$$

The dimensionality reduction is performed by a decimation unit with $D \times M$ decimation matrix \mathbf{T} that projects $\tilde{\mathbf{r}}(i)$ onto $D \times 1$ vector $\bar{\mathbf{r}}(i)$, where D is the rank. The $D \times 1$ vector $\bar{\mathbf{r}}(i)$ is given by

$$\bar{\mathbf{r}}(i) = \underbrace{\mathbf{T} \mathbf{P}^H(i)}_{\mathbf{S}_D^H(i)} \mathbf{r}(i) = \mathbf{T} \tilde{\mathbf{r}}(i) = \mathbf{T} \mathbf{R}'(i) \mathbf{p}^*(i) \quad (9)$$

where $\mathbf{S}_D(i)$ denotes the equivalent subspace projection matrix. The output of the reduced-rank receive filter $\bar{\mathbf{w}}(i)$ is given by $y(i) = \bar{\mathbf{w}}^H(i) \bar{\mathbf{r}}(i)$.

The elements of the decimation matrix only take the value 0 or 1. This corresponds to the decimation unit simply keeping or discarding the samples. We introduce the structure of the decimation matrix as follows,

$$\mathbf{T} = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \dots \quad \mathbf{t}_D]^T$$

where the $M \times 1$ vector \mathbf{t}_d denotes the d -th basis vector of the decimation unit, $d = 1, \dots, D$, and its structure is given by

$$\mathbf{t}_d = \underbrace{[0, \dots, 0]_{q_d}}_{q_d} \underbrace{[1, 0, \dots, 0]_{M-q_d-1}}_{M-q_d-1}^T \quad (10)$$

where q_d is the number of zeros before nonzero element. Note that it is composed of a single 1 and $M-1$ 0s. We set the value of q_d in a deterministic way which can be expressed as $q_d = \lfloor \frac{M}{D} \rfloor \times (d-1)$. The simulation results will show that the proposed reduced-rank scheme with the decimation unit design method works very well.

4. PROPOSED SM REDUCED-RANK ALGORITHM

In this section, we introduce a novel adaptive CCM reduced-rank algorithm by combining the JIDF scheme with the SM technique to realize the data selective updates for beamforming.

4.1. Proposed Reduced-Rank SM Scheme

In the conventional full-rank SM filtering scheme [18]-[20], the filter \mathbf{w} is designed to achieve a predetermined or time-varying bound on the magnitude of the estimation error. This bound can be regarded as a constraint on the filter design, which performs the data-selective updates. In this work, we develop the SM adaptive JIDF-CCM reduced-rank algorithm. We take both the interpolator \mathbf{p} and the reduced-rank weight vector $\bar{\mathbf{w}}$ into consideration due to the feature of their joint iterative exchange of information. Let \mathcal{H}_i denote the set containing all the pairs of $\{\mathbf{p}(i), \bar{\mathbf{w}}(i)\}$ for which the associated error at time instant “ i ” is upper bounded in magnitude by γ , which is given by

$$\mathcal{H}_i = \{\bar{\mathbf{w}} \in \mathcal{C}^{M \times 1}, \mathbf{p} \in \mathcal{C}^{I \times 1} : e^2(i) \leq \gamma^2\}, \quad (11)$$

where the set \mathcal{H}_i is referred to as the *constraint set*. We then define the exact *feasibility set* \mathcal{T}_i as the intersection of the constraint sets over the time instants $l = 1, \dots, i$, which is described by

$$\mathcal{T}_i = \bigcap_{l=1, (\mathbf{b}_0, \mathbf{r}) \in \mathbf{Q}}^i \mathcal{H}_l, \quad (12)$$

where \mathbf{b}_0 denotes the desired signal and \mathbf{Q} denotes the set including all possible data pairs $\{\mathbf{b}_0, \mathbf{r}\}$. The aim of (12) is to develop adaptive algorithms that update the parameters such that they will always remain within the feasibility set. In practice, it is impossible to traverse all possible data pairs. Under this condition, we define the *membership set* constructed from the observed data pairs, which is given by $\psi_i = \bigcap_{l=1}^i \mathcal{H}_l$. Note that the two sets are equal only if all possible data pairs are traversed up to time instant i .

The proposed adaptive scheme introduces the principle of the SM filtering technique into the JIDF-CCM reduced-rank algorithm. Therefore, it operates with respect to certain snapshots and provides data-selective updates. Compared to the existing reduced-rank algorithms that updates for all the snapshots the proposed scheme reduces the computational complexity. Furthermore, the data-selective updates will lead to highly effective variable step size for the SG-based reduced-rank beamforming algorithm. In the following, we will describe the proposed algorithm in detail.

4.2. Proposed SM JIDF-CCM Reduced-Rank Algorithm

We devise a gradient descent strategy to compute the reduced-rank filter weight vector $\bar{\mathbf{w}}$ and the interpolator \mathbf{p} that minimize the instantaneous CCM cost function, the adaptation is required when the square of the error $e^2(i)$ exceeds a specified error bound $\gamma^2(i)$. The bound here can be assumed to be time-varying and based on the estimated parameters of the filter. The problem is formulated as follows,

$$\text{minimize} \quad J_{CM} = e^2(i) = (|\bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)|^2 - 1)^2, \quad (13)$$

$$\text{s.t.} \quad \bar{\mathbf{w}}^H(i)\mathbf{TD}(\theta_0)\mathbf{p}^*(i) = \nu, \quad \text{whenever} \quad e^2(i) \geq \gamma^2(i), \quad (14)$$

where the $M \times I$ matrix $\mathbf{D}(\theta_0)$ is a Hankel matrix with the elements of $\mathbf{a}(\theta_0)$. This problem can be solved based on the method of Lagrange multipliers using the equality constraint $e^2(i) = \gamma^2(i)$ [23]:

$$\mathcal{L} = (|\bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)|^2 - 1)^2 + 2\lambda\mathcal{R}\{\bar{\mathbf{w}}^H(i)\mathbf{TD}(\theta_0)\mathbf{p}^*(i) - \nu\} \quad (15)$$

where λ is a Lagrange multiplier and $\mathcal{R}\{\cdot\}$ selects the real part of the quantity. In order to update the reduced-rank filter, we consider

the following gradient search procedure:

$$\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) - \mu_w \frac{\partial \mathcal{L}}{\partial \bar{\mathbf{w}}^*} \quad (16)$$

where μ_w is the effective step size for (16). By taking the gradient of (15) with respect to $\bar{\mathbf{w}}^*$ and using the constraint $\bar{\mathbf{w}}^H\mathbf{TD}(\theta_0)\mathbf{p}^* = \bar{\mathbf{w}}^H\mathbf{TP}^H\mathbf{a}(\theta_0) = \nu$, we obtain the following SG algorithm

$$\bar{\mathbf{w}}(i+1) = \Omega(i)(\bar{\mathbf{w}}(i) - \mu_w e(i)y^*(i)\bar{\mathbf{r}}(i)) + \nu \bar{\mathbf{a}}(\theta_0)(\bar{\mathbf{a}}^H(\theta_0)\bar{\mathbf{a}}(\theta_0))^{-1}, \quad (17)$$

where $\bar{\mathbf{a}}(\theta_0) = \mathbf{TP}^H\mathbf{a}(\theta_0)$ and $\Omega(i) = \mathbf{I} - \frac{\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0)}{\bar{\mathbf{a}}^H(\theta_0)\bar{\mathbf{a}}(\theta_0)}$. Then, we consider the following gradient search procedure to update the interpolator:

$$\mathbf{p}(i+1) = \mathbf{p}(i) - \mu_p \frac{\partial \mathcal{L}}{\partial \mathbf{p}^*}, \quad (18)$$

where μ_p is the effective step size for (18) and

$$\frac{\partial \mathcal{L}}{\partial \mathbf{p}^*} = e(i)\mathbf{R}^{T'}(i)\mathbf{T}^T\bar{\mathbf{w}}^*\bar{\mathbf{w}}^T\mathbf{TR}'^*(i)\mathbf{p}(i) + \lambda\mathbf{D}^T(\theta_0)\mathbf{T}^T\bar{\mathbf{w}}^*. \quad (19)$$

By multiplying $\bar{\mathbf{w}}^H\mathbf{TD}(\theta_0)$ on both sides of (18) and using the constraint $\bar{\mathbf{w}}^H\mathbf{TD}(\theta_0)\mathbf{p}^* = \nu$, we obtain

$$\lambda = -\frac{e(i)\bar{\mathbf{w}}^T\mathbf{TD}^*(\theta_0)\mathbf{R}'^T(i)\mathbf{T}^T\bar{\mathbf{w}}^*\bar{\mathbf{w}}^T\mathbf{TR}'^*(i)\mathbf{p}(i)}{\bar{\mathbf{w}}^H\mathbf{TD}(\theta_0)\mathbf{D}^H(\theta_0)\mathbf{T}^T\bar{\mathbf{w}}} \quad (20)$$

and finally we have

$$\mathbf{p}(i+1) = \mathbf{p}(i) - \mu_p e(i)y^*(i) \left(\mathbf{I} - \frac{\mathbf{D}^T(\theta_0)\mathbf{T}^T\bar{\mathbf{w}}^*\bar{\mathbf{w}}^T\mathbf{TD}^*(\theta_0)}{\bar{\mathbf{w}}^H\mathbf{TD}(\theta_0)\mathbf{D}^H(\theta_0)\mathbf{T}^T\bar{\mathbf{w}}} \right) \times \mathbf{R}^{T'}(i)\mathbf{T}^T\bar{\mathbf{w}}^*. \quad (21)$$

The update is performed only if the constraint $e^2(i) = \gamma^2(i)$ cannot be satisfied. Specifically, by substituting (17) and (21) into the constraint on the time-varying bound, respectively, we obtain (22) and (23):

$$\mu_w(i) = \begin{cases} \frac{1 - \frac{\sqrt{1+\gamma(i)}}{|y(i)|}}{e(i)\bar{\mathbf{r}}^H(i) \left(\mathbf{I} - \frac{\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0)}{\bar{\mathbf{a}}^H(\theta_0)\bar{\mathbf{a}}(\theta_0)} \right) \bar{\mathbf{r}}(i)} & \text{if } |y(i)| \geq \sqrt{1+\gamma(i)} \\ \frac{1 - \frac{\sqrt{1-\gamma(i)}}{|y(i)|}}{e(i)\bar{\mathbf{r}}^H(i) \left(\mathbf{I} - \frac{\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0)}{\bar{\mathbf{a}}^H(\theta_0)\bar{\mathbf{a}}(\theta_0)} \right) \bar{\mathbf{r}}(i)} & \text{if } |y(i)| \leq \sqrt{1-\gamma(i)} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

The proposed SM JIDF-CCM adaptive algorithm consists of (17), (21), (22) and (23), and it is employed to determine a set of estimates $\{\bar{\mathbf{w}}(i), \mathbf{p}(i)\}$ that satisfy the bounded constraint.

It is worth to mention that the bound should be selected appropriately to describe the characteristics of the environment. Therefore, it leads to improved convergence and tracking performance. In the following, we introduce a parameter dependent bound (PDB) scheme that was reported in [20] to update the reduced-rank weight vector for detecting the desired user and mitigating the interference and noise, the scheme is given by

$$\gamma(i+1) = (1-\rho)\gamma(i) + \rho\sqrt{g\|\mathbf{P}(i)\mathbf{T}^T\bar{\mathbf{w}}(i)\|^2\hat{\sigma}_n^2(i)}, \quad (24)$$

where ρ is a forgetting factor parameter that should be set to guarantee a proper time-averaged estimate of the evolution of the power of weight vector $\mathbf{w}(i)$, which is given by $\mathbf{w}(i) = \mathbf{P}(i)\mathbf{T}^T\bar{\mathbf{w}}(i)$, where

$$\mu_p(i) = \begin{cases} \left(1 - \frac{\sqrt{1+\gamma(i)}}{|y(i)|}\right) \frac{1}{e(i)\bar{\mathbf{w}}^H \mathbf{T} \mathbf{R}'(i) \left(\mathbf{I} - \frac{\mathbf{D}^H(\theta_0) \mathbf{T}^T \bar{\mathbf{w}} \bar{\mathbf{w}}^H \mathbf{T} \mathbf{D}(\theta_0)}{\bar{\mathbf{w}}^H \mathbf{T} \mathbf{D}(\theta_0) \mathbf{D}^H(\theta_0) \mathbf{T}^T \bar{\mathbf{w}}}\right) \mathbf{R}'^H(i) \mathbf{T}^T \bar{\mathbf{w}}} & \text{if } |y(i)| \geq \sqrt{1+\gamma(i)} \\ \left(1 - \frac{\sqrt{1-\gamma(i)}}{|y(i)|}\right) \frac{1}{e(i)\bar{\mathbf{w}}^H \mathbf{T} \mathbf{R}'(i) \left(\mathbf{I} - \frac{\mathbf{D}^H(\theta_0) \mathbf{T}^T \bar{\mathbf{w}} \bar{\mathbf{w}}^H \mathbf{T} \mathbf{D}(\theta_0)}{\bar{\mathbf{w}}^H \mathbf{T} \mathbf{D}(\theta_0) \mathbf{D}^H(\theta_0) \mathbf{T}^T \bar{\mathbf{w}}}\right) \mathbf{R}'^H(i) \mathbf{T}^T \bar{\mathbf{w}}} & \text{if } |y(i)| \leq \sqrt{1-\gamma(i)} \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

g ($g > 1$) is a tuning coefficient and $\hat{\sigma}_n^2(i)$ is an estimate of the noise power. We assume that the noise power is known beforehand at the receiver. The time-varying bound provides a smoother evolution of the weight vector trajectory and thus avoids too high or low values of the squared norm of the weight vector. It establishes a relation between the estimated parameters and the environmental coefficients. The proposed SM reduced-rank algorithm is summarized in Table 1.

The proposed algorithm requires $DM + D + \eta((D+2)I + 3D - 3)$ additions and $(D+1)M + D + 2 + \eta((D+3)I + 4D + 3)$ multiplications for each snapshot, where η denotes the update rate. In the simulation, we will show that the proposed algorithm achieves a better performance while operates with a low update rate.

Table 1. The Proposed SM JIDF-CCM Algorithm

1	Initialization:
2	$\mathbf{p}(0) = [1, 0, \dots, 0]^T$, $\bar{\mathbf{w}}(0) = \frac{\mathbf{T} \mathbf{P}^H(0) \mathbf{a}(\theta_0)}{\ \mathbf{T} \mathbf{P}^H(0) \mathbf{a}(\theta_0)\ ^2}$.
3	$\mu_w(0)$ and $\mu_p(0)$ are small positive values.
4	For each time instant, $i = 1, \dots, N$
5	Compute the bound $\gamma(i)$ by using (24).
6	if $e^2(i) \geq \gamma^2(i)$
7	Compute the step size \mathbf{u}_p by using (23).
8	Compute the interpolator $\mathbf{p}(i)$ by using (21).
9	Compute the step size \mathbf{u}_w by using (22).
10	Compute the filter $\bar{\mathbf{w}}(i)$ by using (17).
11	else
12	$\mathbf{p}(i) = \mathbf{p}(i-1)$ and $\bar{\mathbf{w}}(i) = \bar{\mathbf{w}}(i-1)$.
13	end

5. SIMULATIONS

In this section, we evaluate the performance of the proposed set-membership adaptive reduced-rank beamforming algorithm and compare it with the existing adaptive blind full-rank and reduced-rank beamforming algorithms. In the simulations, we assume that there is one desired user in the system and the related DOA is known by the receiver. Simulations are performed with a ULA containing $M = 40$ sensor elements with half-wavelength inter-element spacing. The DOAs are randomly generated with uniform random variables between 0 and 180 degrees for each experiment. The results are averaged by 1000 runs. We consider the binary phase shift keying (BPSK) modulation and set $\nu = 1$.

Fig. 1 (a) indicates the SINR convergence performance versus the number of snapshots for the proposed SM JIDF-CCM adaptive reduced-rank algorithm and the conventional adaptive beamforming algorithms, namely, the CCM full-rank algorithm, the CMV full-rank algorithm, the MWF reduced-rank algorithm and the SM CCM full-rank algorithm. The input SNR is 15 dB and the number of users is $q = 6$. The coefficients for the PDB scheme is given by $\rho = 0.98$, $g = 10$ and $\gamma(0) = 0$. We note that all the parameters for the analyzed algorithms are optimized based on simulations.

From the results, we can see that the proposed SM JIDF-CCM adaptive beamforming algorithm with the PDB scheme achieves the best convergence performance. While it only requires around 4.38% of the time for filter parameter updates and can save significant computational resources.

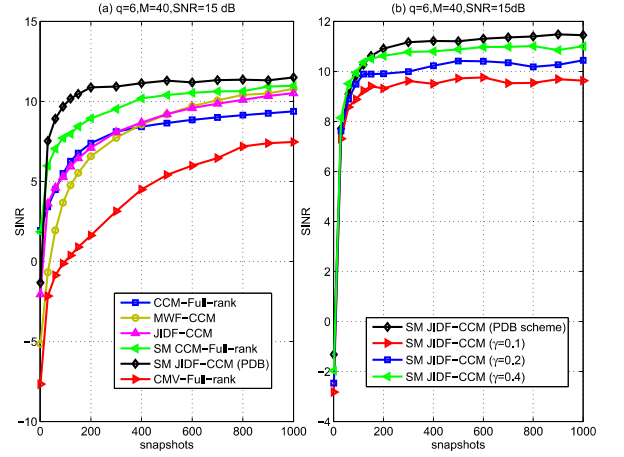


Fig. 1. SINR performance versus number of snapshots. $D = I = 8$, $q = 6$, $M = 40$, $\text{SNR} = 15\text{dB}$.

In Fig. 1 (b), we compare the proposed SM reduced-rank adaptive beamforming algorithm with fixed bounds and that with time-varying bounds in the same system configuration. The coefficients of the proposed adaptive beamforming algorithm with time-varying bound scheme are well tuned as the simulations of Fig. 1 (a). For the fixed bound scheme, we set $\gamma = 0.1$, $\gamma = 0.2$ and $\gamma = 0.4$ to test the performance. The simulation results illustrate that the proposed adaptive algorithm with time-varying bound scheme outperforms the adaptive algorithms with fixed bound schemes. Due to the data-selective update feature the SM JIDF-CCM reduced-rank algorithm with $\gamma = 0.1$, $\gamma = 0.2$ and $\gamma = 0.4$ can provide 35.01%, 27.32% and 15.22% update rates, respectively.

6. CONCLUSION

In this paper, we proposed a novel set-membership reduced-rank algorithm for CCM beamforming. We have developed a SG-type algorithm based on the concept of SM filtering for adaptive implementation. We updated the filter weights only if the bounded constraint cannot be satisfied. Moreover, we also proposed a scheme of time-varying bound and incorporated parameter dependence to characterize the environment for improving the tracking performance of the proposed algorithm. Simulation results have shown that the proposed SM CCM reduced-rank algorithm achieves superior performance to previously reported algorithms at a reduced update rate.

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