

BLIND QUBIT STATE DISENTANGLEMENT WITH QUANTUM PROCESSING: PRINCIPLE, CRITERION AND ALGORITHM USING MEASUREMENTS ALONG TWO DIRECTIONS

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ABSTRACT

In the framework of Blind Quantum Source Separation, we investigate the adaptation of a separating system which receives coupled quantum bit (qubit) states and processes them with quantum means in its feedforward path, to uncouple them. We propose the first separation principle which ensures that the output qubit states of this system are disentangled and that they restore the non-entangled source qubit states up to limited indeterminacies. This separation principle exploits measurements of output spin components along two directions and has some links with the non-quantum Independent Component Analysis (ICA) principle. It opens the way to various practical separation criteria and algorithms, some of which are described here.

Index Terms— blind source separation, quantum bit (qubit), entanglement, unsupervised learning, independent component analysis

1. PRIOR WORK AND PROBLEM STATEMENT

Within the information processing (IP) domain, various fields developed very rapidly during the last decades. One of these fields is Source Separation (SS) and especially *blind* SS (BSS), which was particularly introduced in [5],[16],[18] and is now well-established in the signal processing community [2],[3],[4],[6],[10],[17],[19]. Until recently, all (B)SS investigations were performed in a “classical”, i.e. non-quantum, framework. Independently from (B)SS, another growing field within the overall IP domain is Quantum Information Processing (QIP) [1],[15],[20],[21],[23]. QIP is closely related to Quantum Physics (QP). It uses abstract representations of systems whose behavior is requested to obey the laws of QP. This already made it possible to develop new and powerful IP methods, which manipulate the states of so-called quantum bits, or qubits.

We recently bridged the gap between classical (B)SS and QIP/QP, by introducing a new field, Quantum Source (or Signal) Separation (QSS), first proposed in [7] and then especially detailed in [13] (see also [13] for QSS applications). The QSS problem consists in restoring the information contained in individual *quantum* source signals, eventually only using the mixtures (in SS terms [13]) of states of these qubits which result from their undesired coupling. The QSS problem gives rise to different configurations, especially depending whether the processing means used to restore the source information from the observed mixtures are of classical and/or quantum nature, and whether this separation is performed non-blindly or blindly. Hereafter, as in most of our previous papers, we study the blind configuration (i.e. when the parameter values of the mixing operator are initially unknown), which is more difficult to

handle. Moreover, we here again consider the case when all qubits physically consist of spins $1/2$, described in the standard basis.

In almost all our previous works, we solved the blind QSS (BQSS) problem by first converting the observed quantum data into classical-form signals and then processing these signals with classical means. We thus established various properties and derived corresponding processing methods in [7],[8],[9],[11],[12],[13]. These methods allow one to efficiently process the data derived from quantum/classical conversion. However, this initial conversion yields significant limitations, as detailed in [14] (in particular, this approach requires many qubit initializations). To reduce them, a very different approach consists in using *quantum* processing, especially in the “feedforward path” of the separating system, i.e. in its part which receives the observed mixed data (coupled qubits) and which outputs the estimated source information. We only reported a preliminary version of such a system in [14]. More precisely, a complete BQSS investigation consists in defining the same items as in classical BSS, namely: (1) considered mixing model, (2) proposed separating system structure, (3) proposed separation principle (see e.g. forcing output independence in classical ICA) preferably with an analysis of resulting indeterminacies, (4) proposed separation criterion (see e.g. output mutual information minimization in classical ICA), (5) proposed separation algorithm (e.g. gradient-based minimization of cost function). In [14], we only addressed the above first three items. Moreover, we showed that the separation principle proposed in [14] yields significant separation indeterminacies. This occurs because that principle only uses measurements of the spin components of the output qubits of the separating system along a single axis, O_z .

In the current paper, we consider the same mixing model as in [14] (see Section 2), and we keep the same feedforward path in the separating system (see Section 3). We then present two major types of extensions as compared with the investigation that we reported in [14]: (i) we first propose a new feedback path for the separating system and associated separation principle, based on output measurements in *two* directions, which strongly reduces separation indeterminacies (see Section 4), (ii) using the above satisfactory separation principle, we then propose corresponding separation criterion and algorithm (see Section 5). We then conclude in Section 6.

One should note that, whereas we are here concerned with configurations where one aims at extracting information about quantum states after *undesired* coupling (with Heisenberg’s model), on the contrary a two-qubit gate using liquid NMR *takes advantage* [22] of the scalar coupling. Besides, as detailed in [13], quantum state tomography and quantum process tomography techniques [20], which were e.g. used in [24] for two-qubit systems, cannot achieve BQSS.

2. MIXING MODEL

As stated above, qubits are used instead of classical bits for performing computations in the field of QIP [20]. In [14], we first detailed the required concepts for a single qubit and then presented the type of coupling between two qubits that we consider and that defines the “mixing model”, in (B)SS terms, of our investigation. We hereafter summarize the major aspects of that discussion, which are required in the current paper.

A qubit with index i considered at a given time t_0 has a quantum state. If this state is pure, it belongs to a two-dimensional space \mathcal{E}_i and may be expressed as

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \quad (1)$$

in the basis of \mathcal{E}_i defined by the two orthonormal vectors that we hereafter denote $|+\rangle$ and $|-\rangle$, whereas α_i and β_i are two complex-valued coefficients constrained to meet the condition

$$|\alpha_i|^2 + |\beta_i|^2 = 1 \quad (2)$$

which expresses that the state $|\psi_i(t_0)\rangle$ is normalized.

In the BQSS configuration studied in this paper, we first consider a system composed of two qubits, called “qubit 1” and “qubit 2” hereafter, at a given time t_0 . This system has a quantum state. If this state is pure, it belongs to the four-dimensional space \mathcal{E} defined as the tensor product (denoted \otimes) of the spaces \mathcal{E}_1 and \mathcal{E}_2 respectively associated with qubits 1 and 2, i.e. $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. We hereafter denote \mathcal{B}_+ the basis of \mathcal{E} composed of the four orthonormal vectors $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$, where e.g. $|+-\rangle$ is an abbreviation for $|+\rangle \otimes |-\rangle$, with $|+\rangle$ corresponding to qubit 1 and $|-\rangle$ corresponding to qubit 2. Any pure state of this two-qubit system may then be expressed as

$$|\psi(t_0)\rangle = c_1(t_0)|++\rangle + c_2(t_0)|+-\rangle + c_3(t_0)|-+\rangle + c_4(t_0)|--\rangle \quad (3)$$

and has unit norm. It may also be represented by the corresponding vector of complex-valued components in basis \mathcal{B}_+ , which reads

$$C_+(t_0) = [c_1(t_0), c_2(t_0), c_3(t_0), c_4(t_0)]^T \quad (4)$$

where T stands for transpose. In particular, we study the case when the two qubits are independently initialized, with states defined by (1) respectively with $i = 1$ and $i = 2$. We then have

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle \quad (5)$$

$$= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle. \quad (6)$$

Besides, we consider the case when the two qubits, which correspond to two spins 1/2, have undesired coupling after they have been initialized according to (5). The considered coupling is based on the Heisenberg model with a cylindrical-symmetry axis collinear to Oz , the direction common to the applied magnetic field and to our first chosen quantization axis. This coupling may be represented as

$$C_+(t) = MC_+(t_0) \quad (7)$$

where $C_+(t)$ is the counterpart of (4) at time t and defines the coupled state $|\psi(t)\rangle$ of the two-qubit system at that time. In basis \mathcal{B}_+ , the evolution of the system’s quantum state from t_0 to t is thus represented by the matrix M of (7). Our previous calculations show that, for the considered type of coupling

$$M = QDQ^{-1} = QDQ \quad (8)$$

with

$$Q = Q^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

and D equal to

$$\begin{bmatrix} e^{-i\omega_{1,1}(t-t_0)} & 0 & 0 & 0 \\ 0 & e^{-i\omega_{1,0}(t-t_0)} & 0 & 0 \\ 0 & 0 & e^{-i\omega_{0,0}(t-t_0)} & 0 \\ 0 & 0 & 0 & e^{-i\omega_{1,-1}(t-t_0)} \end{bmatrix} \quad (10)$$

where the imaginary unit i , present e.g. in $e^{-i\omega_{1,1}(t-t_0)}$, should not be confused with the qubit *index* i . The four real (angular) frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (10) depend on the physical setup and their values are unknown in practice.

3. FEEDFORWARD PATH OF SEPARATING SYSTEM

The part of the separating system of [14] kept here consists of its feedforward path, which uses quantum means for deriving its quantum output state from its quantum input state, which is the above coupled state $|\psi(t)\rangle$. Although this part of the system is intrinsically of quantum nature, this paper does not require detailed knowledge about quantum physics, because we here build upon the principles and results detailed in [14] and thus only use linear algebra tools.

The output quantum state of our separating system is denoted as

$$|\Phi\rangle = c_1|++\rangle + c_2|+-\rangle + c_3|-+\rangle + c_4|--\rangle. \quad (11)$$

It may also be represented by the corresponding vector of components of $|\Phi\rangle$ in basis \mathcal{B}_+ , denoted as

$$C = [c_1, c_2, c_3, c_4]^T. \quad (12)$$

We then have

$$C = UC_+(t) \quad (13)$$

where U defines the unitary quantum-processing operator applied by our separating system to its input $C_+(t)$. As justified further in this paper, we choose this operator U to belong to the class defined by

$$U = Q\tilde{D}Q \quad (14)$$

$$\text{with } \tilde{D} = \begin{bmatrix} e^{i\gamma_1} & 0 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 & 0 \\ 0 & 0 & e^{i\gamma_3} & 0 \\ 0 & 0 & 0 & e^{i\gamma_4} \end{bmatrix} \quad (15)$$

where γ_1 to γ_4 are free real-valued parameters.

4. FEEDBACK PATH AND SEPARATION PRINCIPLE

The above configuration yields a BQSS problem which may be defined as follows. The overall quantum “source state” $|\psi(t_0)\rangle$ involved in this problem is created by independently initializing the two qubit states $|\psi_1(t_0)\rangle$ and $|\psi_2(t_0)\rangle$ at a given time t_0 . The corresponding “observed mixture” (in SS terms) is the quantum state $|\psi(t)\rangle$ of this two-qubit system available at a later time t . It has a much more complex form than $|\psi(t_0)\rangle$, due to undesired coupling: it takes the form (3) with t_0 replaced by t and with components defined by (7). Thus, in general, it cannot be expressed as a tensor

product, unlike the initial state (5). In QP terms, the general pure state (3) with t_0 replaced by t is entangled, whereas the pure state (5) is not entangled. The information, which was originally separately available for each qubit in each of the two initial states $|\psi_i(t_0)\rangle$, is thus “mixed” (in the QSS sense¹) in the final state $|\psi(t)\rangle$ of the two-qubit system. Only using (one or several values of) this final state $|\psi(t)\rangle$, we aim at restoring the information contained in the initial states $|\psi_i(t_0)\rangle$ of the two qubits, possibly up to some indeterminacies as in classical BSS. To this end, we use the feedforward path of the separating system defined in Section 3. For suitable values of its free parameters γ_1 to γ_4 , this system is indeed able to restore the source state: it may easily be shown that setting these parameters so that $\tilde{D} = D^{-1}$ yields $U = M^{-1}$, which results in $C = C_+(t_0)$ and $|\Phi\rangle = |\psi(t_0)\rangle$. However, the condition $\tilde{D} = D^{-1}$ cannot be used as a *practical* procedure for directly assigning \tilde{D} , because D is unknown. Instead, a procedure for adapting the parameters γ_1 to γ_4 of \tilde{D} by only using the observed mixture(s) is therefore required, which corresponds to a *blind* (quantum) source separation problem.

The general type of adaptation procedure that we started to consider in [14] for adapting γ_1 to γ_4 consists in first converting the quantum output $|\Phi\rangle$ of the separating system² into classical-form data by means of measurements, and then processing the latter data with classical processing means. We thus used a classical-processing feedback path, from the output of the structure associated with (14), to its adaptive block \tilde{D} . The measurements that we used in [14], consist in simultaneously measuring the components along Oz axis of the spins of the two qubits which define the output of our separating system. This couple of measurements has four possible values, namely $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units. Their respective probabilities are

$$P_{1z} = |c_1|^2, P_{2z} = |c_2|^2, P_{3z} = |c_3|^2, P_{4z} = |c_4|^2. \quad (16)$$

These probabilities may be estimated in practice, by using our RWR procedure [14], which consists in first Repeatedly Writing (i.e. preparing) the same source state and Reading (i.e. performing the above type of measurements for) the corresponding output of our separating system, and then computing the sample frequencies of all four possible measurement outcomes. The method that we proposed in [14] for adapting γ_1 to γ_4 consists in constraining them to take values such that

$$P_{1z}P_{4z} = P_{2z}P_{3z}. \quad (17)$$

Briefly, we selected this adaptation principle because our calculations show that the output of our separating system is non-entangled if and only if

$$c_1c_4 = c_2c_3 \quad (18)$$

and this condition (18) implies (but is not equivalent to)

$$|c_1c_4| = |c_2c_3| \quad (19)$$

which is equivalent to condition (17), due to (16)³. Enforcing (17) is therefore an attractive first step towards output disentanglement and hence BQSS but, as shown by our calculations in [14], it only restores the source state up to significant indeterminacies and, especially, it yields a still entangled output quantum state. Our first goal in the current paper is therefore to improve this separation principle.

¹Here, we are not talking about “mixtures” in the QP sense, i.e. we are not considering quantum states which are statistical mixtures.

²As detailed in [14], the overall proposed approach does not use the same instance of $|\Phi\rangle$ both as the output of the complete system and in its internal feedback path, because this would not be compatible with the so-called “no-cloning theorem” for quantum states.

³This approach also has a link with classical ICA, as explained in [14].

To this end, we still use a classical-processing feedback path, but we enrich the information that it processes, by also measuring the components along Ox axis (orthogonal to Oz) of the spins of the two qubits which define the output of our separating system⁴. This is the very first time we use measurements along Ox in all our QSS investigations, i.e. since [7]. The couple of measurements along Ox axis also has four possible values, namely $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units. Their probabilities are respectively denoted as P_{1x} , P_{2x} , P_{3x} , P_{4x} . We still consider a quantum state which is defined by (11), and which is therefore expressed with respect to vectors associated with the Oz axis. Quantum calculations not detailed here show that the above probabilities read

$$P_{1x} = \frac{1}{4}|c_1 + c_2 + c_3 + c_4|^2 \quad (20)$$

$$P_{2x} = \frac{1}{4}|c_1 - c_2 + c_3 - c_4|^2 \quad (21)$$

$$P_{3x} = \frac{1}{4}|c_1 + c_2 - c_3 - c_4|^2 \quad (22)$$

$$P_{4x} = \frac{1}{4}|c_1 - c_2 - c_3 + c_4|^2. \quad (23)$$

These probabilities are used in the extended separation principle that we propose in this paper, which consists in adapting γ_1 to γ_4 so that they meet two constraints, namely (17) and

$$P_{1x}P_{4x} = P_{2x}P_{3x}. \quad (24)$$

The latter condition is selected due to its symmetry with (17). Moreover, it has a link with classical ICA: considering two classical binary-valued random variables (RVs), with outcomes equal to $\pm\frac{1}{2}$ for both RVs, and denoting as P_{1x} , P_{2x} , P_{3x} , P_{4x} the probabilities of the outcomes $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$, $(-\frac{1}{2}, -\frac{1}{2})$ of this couple of RVs, it can be shown (one may use the general results about RVs provided in [14]) that these RVs are statistically independent if and only if (24) is met.

By combining (24) and (20)-(23), one can express condition (24) as

$$\Re[(c_1^2 + c_4^2 - c_2^2 - c_3^2)(c_1c_4 - c_2c_3)^*] = 0 \quad (25)$$

where $\Re[\cdot]$ and $*$ respectively stand for real part and conjugation.

The separation conditions that we set are expressed as (19) and (25) for *any* state defined by (11). We now apply them to a specific class of states, namely the states that may be taken by the output of our separating system. To this end, the expressions of the components c_1 to c_4 of that output state $|\Phi\rangle$ are first derived from the expression (12) of C defined by (13), (14), (9), (15) and by the expression of $C_+(t)$ derived from previous equations in this paper. The resulting expressions of c_1 to c_4 are complicated and therefore skipped here, due to space limitations.

These expressions are first applied to separation condition (19). Assuming that c_1 to c_4 are non-zero, tedious calculations show that (19) yields two solutions for γ_1 to γ_4 . The first one is suitable, since the resulting condition on γ_1 to γ_4 does not depend on the considered source state. This condition reads

$$\delta_3 - \delta_2 = m\pi \quad (26)$$

where m is an arbitrary integer and

$$\delta_2 = \gamma_2 - \omega_{1,0}(t - t_0) \quad (27)$$

$$\delta_3 = \gamma_3 - \omega_{0,0}(t - t_0). \quad (28)$$

⁴These measurements are performed for other instances of the same states $|\Phi\rangle$ than measurements along Oz axis, since s_z and s_x do not commute and hence cannot be both measured for the same arbitrary state instance.

The second solution is a spurious one, as the resulting condition on γ_1 to γ_4 depends on the considered source state. It need not be detailed here, as it may be avoided (e.g. using the cost function F_z proposed below in Section 5), by requesting separation condition (19) to be met for at least two source states which do not all yield the same spurious solution (the corresponding condition for these source states is provided in [14]).

Additional calculations then show that the output of our separating system corresponding to the above first solution is defined by

$$c_1 = \alpha_1 \alpha_2 e^{i\delta_1} \quad (29)$$

$$c_2 = \alpha_1 \beta_2 e^{i\delta_2} \quad \text{if } m \text{ even} \quad (30)$$

$$c_2 = \beta_1 \alpha_2 e^{i\delta_2} \quad \text{if } m \text{ odd} \quad (31)$$

$$c_3 = \beta_1 \alpha_2 e^{i\delta_2} \quad \text{if } m \text{ even} \quad (32)$$

$$c_3 = \alpha_1 \beta_2 e^{i\delta_2} \quad \text{if } m \text{ odd} \quad (33)$$

$$c_4 = \beta_1 \beta_2 e^{i\delta_4} \quad (34)$$

with

$$\delta_1 = \gamma_1 - \omega_{1,1}(t - t_0) \quad (35)$$

$$\delta_4 = \gamma_4 - \omega_{1,-1}(t - t_0). \quad (36)$$

We then derive in which case the above state meets (25). To this end, we insert (29)-(34) into (25), which thus becomes, whatever m

$$\begin{aligned} & \Re[(\alpha_1^2 \alpha_2^2 e^{i2\delta_1} + \beta_1^2 \beta_2^2 e^{i2\delta_4} - \alpha_1^2 \beta_2^2 e^{i2\delta_2} - \beta_1^2 \alpha_2^2 e^{i2\delta_2}) \\ & \quad \times (\alpha_1 \alpha_2 \beta_1 \beta_2)^* (e^{i(\delta_1 + \delta_4)} - e^{i2\delta_2})^*] = 0. \end{aligned} \quad (37)$$

We constrain our separating system to meet this condition for several source states, indexed by n with $n \in \{1, \dots, N_x\}$ and $N_x \geq 2$. These states are defined by the values of the corresponding parameters $\alpha_1(n), \beta_1(n), \alpha_2(n), \beta_2(n)$. Inserting these parameters into (37) yields a set of N_x equations. The solutions of these equations are determined by introducing the following property, which may be proved by considering the polar representation of the considered complex numbers.

Property 1. *Let d be a non-zero complex number. Let $d(n)$, with $n \in \{1, \dots, N\}$ and $N \geq 2$, be a set of non-zero complex numbers such that*

$$\Re[d(n)d^*] = 0 \quad \forall n \in \{1, \dots, N\}. \quad (38)$$

Then the phases of the complex numbers $d(n)$ are all equal, up to multiples of π .

We apply this property to (37) by selecting

$$\begin{aligned} d(n) = & [\alpha_1^2(n) \alpha_2^2(n) e^{i2\delta_1} + \beta_1^2(n) \beta_2^2(n) e^{i2\delta_4} - \alpha_1^2(n) \beta_2^2(n) e^{i2\delta_2} \\ & - \beta_1^2(n) \alpha_2^2(n) e^{i2\delta_2}] [\alpha_1(n) \alpha_2(n) \beta_1(n) \beta_2(n)]^* \end{aligned} \quad (39)$$

$$d = e^{i(\delta_1 + \delta_4)} - e^{i2\delta_2} \quad (40)$$

and by considering the case when (38) is met with $N = N_x \geq 2$, the complex numbers $d(n)$ are non-zero and do *not* all have the same phase up to multiples of π . Then, Property 1 guarantees that $d = 0$, i.e.

$$\delta_1 + \delta_4 = 2\delta_2 + 2\pi k \quad (41)$$

where k is an integer. Inserting (41) into (29)-(34) then makes it possible to express the components of the output state of our separating system only with respect to δ_1 and δ_4 . This state is defined by (11). Denoting

$$\delta_5 = \frac{\delta_4 - \delta_1}{2} - \pi k, \quad (42)$$

additional manipulations then show that this state may be expressed as follows if m is even:

$$|\Phi\rangle = e^{i\delta_1} (\alpha_1|+\rangle + \beta_1 e^{i\delta_5}|-\rangle) \otimes (\alpha_2|+\rangle + \beta_2 e^{i\delta_5}|-\rangle) \quad (43)$$

whereas, if m is odd

$$|\Phi\rangle = e^{i\delta_1} (\alpha_2|+\rangle + \beta_2 e^{i\delta_5}|-\rangle) \otimes (\alpha_1|+\rangle + \beta_1 e^{i\delta_5}|-\rangle). \quad (44)$$

The proposed separation principle thus guarantees that, for the resulting values of γ_1 to γ_4 , the output state $|\Phi\rangle$ obtained for any non-entangled source state (5) is a tensor product, i.e. is also non-entangled. Moreover, the factors of this tensor product (43) or (44) are respectively equal to each of the source qubit states (1), possibly up to a permutation and a phase $e^{i\delta_5}$ for one qubit state component with respect to the other (the phase $e^{i\delta_1}$ should be ignored, since it applies to all qubit state components and therefore has no physical consequence). Output permutations and scale factors are also very common in classical BSS, where they are called “indeterminacies”.

5. SEPARATION CRITERION AND ALGORITHM

A practical blind adaptation procedure may e.g. be derived as follows from the above separation principle. The first step of this procedure uses a set of source states indexed by n with $n \in \{1, \dots, N_z\}$ and $N_z \geq 2$. The corresponding probabilities (16) are denoted as $P_{1z}(n)$ to $P_{4z}(n)$ and estimates of them are used in practice. This step of the procedure aims at ensuring (26). Due to (27)-(28), this is achieved by adapting one of the parameters γ_2 and γ_3 , while the other one, as well as γ_1 and γ_4 , are constant. The separation criterion used in this adaptation consists in looking for the global minimum of the cost function

$$F_z = \sum_{n=1}^{N_z} |P_{1z}(n)P_{4z}(n) - P_{2z}(n)P_{3z}(n)|^p \quad (45)$$

e.g. with $p = 1$ or 2 . This minimum is ideally equal to zero and reached for the first solution of (19) but not for its spurious solution, under the above-mentioned conditions. A simple algorithm derived from the above criterion consists in performing a sweep on γ_2 (or γ_3 ; besides, in practice, the sweep is performed e.g. on the voltage which controls γ_2 or γ_3), computing the corresponding estimated values of F_z and keeping the value of γ_2 which minimizes F_z . One then freezes γ_2 (and γ_3).

Similarly, the second step of our procedure consists of a sweep on γ_1 or γ_4 , which aims at minimizing the cost function

$$F_x = \sum_{n=1}^{N_x} |P_{1x}(n)P_{4x}(n) - P_{2x}(n)P_{3x}(n)|^p \quad (46)$$

in order to ensure (41).

6. CONCLUSION

In this paper, we explored the separation of coupled qubits by quantum processing means (in the feedforward path). We derived the first separation principle which ensures that the output qubit states of our separating system are disentangled and restore the source qubit states up to limited indeterminacies. This separation principle opens the way to a variety of separation criteria and algorithms. We here started to describe them and we will report on more advanced versions in future, longer, papers. We also plan to analyze their numerical performance by developing a software emulation of Heisenberg-coupled qubits.

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Erratum: replace two terms $E\{r_i\}E\{q_i\}$ in (33) of [7] by $E\{r_i q_i\}$, since q_i depends on r_i .
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