PILOT-ASSISTED ERGODIC INTERFERENCE ALIGNMENT FOR WIRELESS NETWORKS

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ABSTRACT

This paper considers the ergodic block fading multi-user Gaussian interference channel (IC) in which each source desires to communicate to an intended destination. We assume that there is no CSI a priori available at terminals. We develop achievable rate results and compute the associated degrees of freedom by using a pilot-assisted interference alignment scheme. In this scheme, each source first sends known pilot symbols via which the destinations estimate channel gains, and the destinations then broadcast the estimated channel gains via orthogonal feedback channels. The estimated channel gains are used to perform interference alignment for data transmission. The pilot transmission power can be different from the data transmission power. By allocating more power to pilot transmission, channel gains can be estimated more accurately which implies less power left for data transmission. We find the optimum power allocation to pilot symbols and data symbols. Our study recommends, in large networks, to allocate more power to channel training instead of data transmission. In addition, our results reveal that for a K-user ergodic IC with a coherence time T, the total degrees of freedom $\frac{1}{2}K_{\text{opt}}\left(1-\frac{K_{\text{opt}}}{T}\right)$ is achievable, where $K_{\text{opt}}=\min\left\{K,\frac{T}{2}\right\}$ is the optimum number of users selected to be active in the network. This recommends to perform a user selection in large networks $(K > \frac{T}{2})$, and apply channel training and interference alignment within the set of selected users.

Index Terms— Interference alignment, channel training, user selection.

1. INTRODUCTION

Characterizing the fundamental performance limits of wireless interference channel (IC) has been the subject of extensive research. Recently, via a novel interference management technique referred to as *interference alignment* [1, 2], it has been shown that ICs are fundamentally *not* interference limited in the high signal-to-noise ratio (SNR) regime. Through properly designing the transmitted signals, the received interference signals at each destination can be aligned such that they occupy only a sub-space of the received signal space. Consequently, a *K*-user time-varying (or frequency-selective) IC can achieve the total degrees of freedom $d_{\Sigma} = \frac{1}{2}K$, where $d_{\Sigma} = \lim_{SNR\to\infty} R_{\Sigma}/\log(SNR)$ in which R_{Σ} is the achievable sum-rate [2]. This achievable degrees of freedom is substantially higher than that of the time-division multiple access (TDMA), which is only $d_{\Sigma} = 1$. Furthermore, when the channel gains are ergodic time-varying and symmetrically distributed (e.g. Rayleigh fading channels), the *ergodic interference alignment* scheme has been developed in [3] which achieves the sum-rate of $R_{\Sigma} = \frac{1}{2} K \mathbb{E}_h [\log(1+2|h|^2 \text{SNR})]$, where h is the channel gain. This implies that IC under time-varying channel environments is *not* interference limited at any SNR.

To achieve the performance promised by the aforementioned schemes, however, global channel state information (CSI) is assumed to be perfectly known at all terminals. Acquiring such perfect CSI is a challenging problem. Therefore, references [3–7] have investigated cases in which each destination knows perfect CSI of its incoming channel gains, and it provides either the quantized or the uncoded version of the channel gains to the other terminals through digital or analog feedback, respectively. It has been shown that if the rate of the digital feedback signals [3–6], or the power of the analog feedback signals [7] properly scale with transmit power, the outstanding performance of interference alignment is still achievable.

In practice, no CSI is a priori available at destinations. However, the CSI can be acquired through a pilot-based channel training scheme in which each source allocates a portion of the total transmission time for transmitting pilot symbols and the rest for data transmission. The impact of the allocated time for channel training on the performance of interference alignment for multiple antenna systems has been addressed in [8]. It has been assumed that transmission power for pilot transmission is the same as the one for data transmission. In general, the power for pilot transmission can be different from the one for data transmission. A more accurate channel estimation can be obtain by allocating more power for pilot transmission which implies a lower power is left for data transmission. The interesting problem of optimum power allocation to pilot symbols and data symbols in point to point communication scenarios has been investigated in [9]. In multi user IC, finding the optimum power allocation to pilot symbols and data symbols is of an even more importance because of the fact that the quality of CSI estimation not only affects the performance of each decoder, but also determines how accurately the interference alignment is performed.

In this paper, we investigate the performance limits of a pilotassisted *ergodic* interference alignment scheme for single-antenna users, and find the *optimum* power allocation for pilot transmission and data transmission. We first derive an achievable rate region of the ergodic IC with a pilot-assisted interference alignment scheme. Next, we optimize power allocation to maximize the achievable rates. To gain insight into the performance of the system at high SNR regime, we derive the achievable degrees of freedom region. Finally, we suggest to perform a user selection, in large networks, prior to the transmission of pilot symbols and performing interference alignment.

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Fig. 1: Transmitted symbols within one block in a *K*-user interference channel. The crosshatched red slot, the plain green slot, and the blue angle lined slots denote no transmission, pilot symbols, and data symbols, respectively.

2. SYSTEM MODEL

We consider a time-varying IC composed of K single-antenna source-destination pairs, as shown in **Fig. 1**. The sources and the destinations are denoted by S_l and D_k $(k, l \in \{1, 2, ..., K\})$, respectively. The channel gain from S_l to D_k at time t is denoted as h_{kl}^t . The channels follow *block fading model* in which the channel gains are constant over one fading block. At fading block n, we have $h_{kl}^{nT} = h_{kl}^{nT+i}$ (i = 1, ..., T - 1), where T is channel coherence time. The channel gains are ergodic time-varying and have independent and identical distribution across different fading blocks. The channel gains are independently drawn from a zero-mean unit variance complex Gaussian distribution, i.e. $h_{kl}^{nT} \sim C\mathcal{N}(0, 1)$.

As shown in **Fig. 1**, transmission within each block is performed in two phases: *pilot transmission phase* and *data transmission phase* which have the duration of αT and $(1 - \alpha)T$, respectively. The parameter α ($K/T \le \alpha \le 1$) is a design parameter. In the following, we will explain these phases in more details.

2.1. Pilot Transmission Phase

Channel training is performed in an orthogonal fashion in which the training period is divided into K equal time slots (each has the duration of $T_{\tau} = \alpha T/K$), as shown in **Fig. 1**. Each destination estimates the gain of the corresponding direct link and interference links. Let source S_l ($l \in \{1, ..., K\}$), at fading block n, sends T_{τ} known pilot symbols with power P_{τ} as follows

$$X_{\tau,l}^{i} = \sqrt{P_{\tau}}$$
 $i = nT + (l-1)T_{\tau} + 1, ..., nT + lT_{\tau},$

then the received symbols at D_k are $Y_{\tau,k}^i = \sqrt{P_{\tau}} h_{kl}^{nT} + Z_k^i$, and the MMSE estimate of the channel gain between S_l and D_k is obtained as

$$\tilde{h}_{kl}^{nT} = \frac{P_{\tau}}{N_0 + T_{\tau} P_{\tau}} \sum_{i=nT+(l-1)T_{\tau}+1}^{nT+lT_{\tau}} Y_{\tau,k}^i.$$
(1)

The following equation holds

$$h_{kl}^{nT} = \tilde{h}_{kl}^{nT} + \varepsilon_{kl}^{nT}, \qquad (2)$$

where ε_{kl}^{nT} is the channel estimation error. The random variables \tilde{h}_{kl}^{nT} and ε_{kl}^{nT} are independent zero mean Gaussian distributed with

variances $1 - \sigma_{\varepsilon}^2$ and σ_{ε}^2 , respectively, where

$$\sigma_{\varepsilon}^2 = \frac{1}{1 + T_{\tau} P_{\tau} / N_0}.$$
(3)

All destinations broadcast their estimations to the other terminals through orthogonal feedback channels. Since the aim of this paper is to investigate the impact of imperfect channel estimation, we assume that feedback channels are error free.

2.2. Data Transmission Phase

For data transmission, we consider a multiplexed coding scheme similar to the one proposed in [10], where there are multiple codebooks each associated with a specific channel state. For a given channel state, S_k encodes message m_k to a length $N'T_d$ codeword $\{X_k^i\}_{i=1}^{N'T_d}$, where N' is the number of the corresponding fading blocks and $T_d = (1-\alpha)T$ is the duration of data transmission within each block. There is a power constraint $\mathbb{E}\left[|X_k|^2\right] < P_d$. In fading block n, S_k sends $\{X_k^i\}_{i=nT+KT_r+1}^{(n+1)T}$ during T_d data transmission time slots. The channel output at D_k is

$$Y_{d,k}^{i} = h_{kk}^{nT} X_{k}^{i} + \sum_{l=1, l \neq k}^{nT} h_{kl}^{nT} X_{l}^{i} + Z_{k}^{i},$$

$$i = nT + KT_{\tau} + 1, ..., (n+1)T$$
(4)

where $Z_k^i \sim \mathcal{CN}(0, 1)$. We apply the ergodic interference alignment scheme proposed in [3], but we assume that only the estimated channel gains are available at terminals. Thus, if the estimated channel gains at fading blocks n and n_p ($n_p > n$) satisfy

$$\tilde{h}_{kk}^{n_p T} = \tilde{h}_{kk}^{nT} \tilde{h}_{kl}^{n_p T} = -\tilde{h}_{kl}^{nT}, \ (\forall k, l \in \{1, 2, ..., K\}, \ k \neq l),$$
 (5)

then S_k at fading block n_p retransmits the signal which was transmitted at fading block n, i.e. $X_k^{n_pT+i} = X_k^{nT+i}$, where $i = KT_{\tau} + 1, ..., KT_{\tau} + T_d$. To avoid measure zero events, the channel pairing in (5) can be performed based on a quantized version of the estimated channel gain using sufficiently fine quantization [3]. The destination D_k receives the following signals

$$Y_{d,k}^{nT+i} = h_{kk}^{nT} X_k^{nT+i} + \sum_{l=1, l \neq k}^{K} h_{kl}^{nT} X_l^{nT+i} + Z_k^{nT+i}$$
(6)

$$Y_{d,k}^{n_{p}T+i} = h_{kk}^{n_{p}T} X_{k}^{nT+i} + \sum_{l=1, l \neq k}^{K} h_{kl}^{n_{p}T} X_{l}^{nT+i} + Z_{k}^{n_{p}T+i}.$$
 (7)

Then, it forms the following signal

$$\begin{split} \overline{Y}_{d,k}^{nT+i} &= Y_k^{nT+i} + Y_k^{n_pT+i} \\ &= \left(2 \tilde{h}_{kk}^{nT} + \left(\varepsilon_{kk}^{nT+i} + \varepsilon_{kk}^{n_pT+i} \right) \right) X_k^{nT+i} \\ &+ \sum_{l=1,l \neq k}^K \! \left(\varepsilon_{kl}^{nT+i} \! \! + \! \varepsilon_{kl}^{n_pT+i} \right) X_l^{nT+i} \! + \! \left(Z_k^{nT+i} \! \! + \! Z_k^{n_pT+i} \right). \end{split}$$

It decodes its message after receiving all N^\prime segments of the transmitted codeword.

Each source transmits at power P. The transmission power of pilot symbols (P_{τ}) and the one for data symbols (P_d) can be different in general. Let $P_d = \beta P$, where $0 \le \beta \le 1/(1 - \alpha)$ is a *power allocation factor*. Because of energy conservation, we have

$$\alpha T P_{\tau} / K + (1 - \alpha) T P_d = T P.$$
(8)

Therefore, $P_{\tau} = K \left((1 - (1 - \alpha) \beta) / \alpha \right) P$.

3. ACHIEVABLE RATE REGION

In this section, we present an achievable rate region.

Theorem 1 In the considered K-user time-varying IC, an achievable rate region $(R_1, R_2, ..., R_K)$ is

$$R_{k} = \frac{1-\alpha}{2} \mathbb{E}_{\tilde{h}_{kk}} \left[\log \left(1 + \frac{2\beta |\tilde{h}_{kk}|^{2} P}{N_{0} + \frac{\beta K P}{1 + (1-\beta(1-\alpha))TP/N_{0}}} \right) \right]$$
(9)
where $\tilde{h}_{kk} \sim \mathcal{CN} \left(0, \frac{TP(1-\beta(1-\alpha))/N_{0}}{1 + TP(1-\beta(1-\alpha))/N_{0}} \right).$

Proof The proof follows from Proposition 2 in [11] by using the fact that the estimation error of MMSE estimator is uncorrelated with the estimated channel. The variance of the estimation error given in (3) can be simplified by substituting $T_{\tau} = \alpha T$ and P_{τ} given in (8).

This achievable rate region can be further simplified by calculating the expectation in (9) and using equation (34) in [12] as presented in the next proposition.

Proposition 1 The achievable rate R_k given in (9) is

$$R_k = \frac{1-\alpha}{2}\log_2(e)\exp\left(1/\mathsf{SNR}_{\mathsf{eq}}\right)\mathsf{E}_1\left(1/\mathsf{SNR}_{\mathsf{eq}}\right) \tag{10}$$

where $E_1(x) = \int_1^\infty \frac{1}{t} e^{-xt} dt$, x > 0, and

$$SNR_{eq} = \frac{2(1 - (1 - \alpha)\beta)T\beta P^2}{N_0^2 + (1 - (1 - \alpha)\beta)TPN_0 + K\beta PN_0}.$$
 (11)

4. THE OPTIMUM POWER ALLOCATION

If transmitters are capable to transmit at different powers during the channel training and the data transmission phases, then power can be allocated such that the achievable rate region in **Theorem 1** be enlarged.

Proposition 2 In the considered system, the optimum power allocation is

$$P_{d,\text{opt}} = \beta_{\text{opt}} P, \ P_{\tau,\text{opt}} = K\left(\left(1 - (1 - \alpha)\beta_{\text{opt}}\right)/\alpha\right) P, \qquad (12)$$

where

$$\beta_{\text{opt}} = \frac{1}{1 - \alpha} \left(1 + \sqrt{\frac{1 + KP/(1 - \alpha)}{1 + PT/N_0}} \right)^{-1}.$$
 (13)

Proof Since R_k in (10) is a monotonic increasing function of SNR_{eq} , it is sufficient to maximize SNR_{eq} . We can prove that SNR_{eq} is a strictly concave function of β . Therefore, a unique β that maximizes R_k can be found by solving KKT conditions [13].

5. ACHIEVABLE DEGREES OF FREEDOM REGION

In this section, we study the achievable degrees of freedom region.

Theorem 2 In the K-user IC with coherence time T, if no CSI is a priori available at terminals, then the degrees of freedom region $(d_1, d_2, ..., d_K)$ is achievable where

$$d_k = \begin{cases} \frac{1}{2} \left(1 - \frac{K}{T} \right), & K < T \\ 0 & K \ge T \end{cases} .$$
(14)

Proof Using (9), we have

$$d_{k} = \lim_{P \to \infty} R_{k} / \log P$$

$$\stackrel{(a)}{=} \frac{1 - \alpha}{2} \mathbb{E}_{\tilde{h}_{kk}} \left[\lim_{P \to \infty} \log \left(1 + \frac{2\beta |\tilde{h}_{kk}|^{2} P}{1 + \frac{2\beta |\tilde{h}_{kk}|^{2} P}{1 + \frac{\beta K P}{N_{0}} (1 - \beta (1 - \alpha))}} \right) / \log P \right]$$

$$= \frac{1 - \alpha}{2}, \qquad (15)$$

where (a) follows the monotone convergence theorem [14]. To maximize the achievable degrees of freedom in (15) we set α equal to its minimum possible value which is K/T.

According to **Theorem 2**, if $T \gg K$, then the achievable degrees of freedom by the interference alignment scheme with perfect CSI can be preserved. However, the achievable degrees of freedom decays by increasing the number of users. The achievable total degrees of freedom depends on both the number of the users and the achievable degrees of freedom per user. A specific number of users maximizes the achievable total degrees of freedom. Thus, we select only a subset of users called *active users* to transmit.

Theorem 3 In the K-user ergodic block fading Gaussian IC with coherence time T, the achievable total degrees of freedom is

$$d_{\Sigma} = \frac{1}{2} K_{\text{opt}} \left(1 - \frac{K_{\text{opt}}}{T} \right), \tag{16}$$

where K_{opt} is the number of the active users

$$K_{\text{opt}} = \min\left\{\frac{T}{2}, K\right\}.$$
(17)

Proof Let K' < T denote the number of the active users, then according to **Theorem 2** we have

$$d_{\Sigma} = \sum_{i=1}^{K} d_{k} = \frac{1}{2} K' \left(1 - \frac{K'}{T} \right) = -\frac{1}{2T} \left(K' - \frac{T}{2} \right)^{2} + \frac{T}{8}.$$

We can observe that d_{Σ} is maximized when $K' = \min \{K, \frac{T}{2}\}$.

To maximize the total degrees of freedom (and the network throughput at high SNR), in large networks (K > T/2), **Theorem 3** suggests to first apply a user selection scheme, and then perform channel training and interference alignment only within the subset of active users. Since the network is symmetric, a random user selection is sufficient. In addition, this theorem crystallizes the dependency of the optimum number of active users to be selected on the coherence time of channel.

6. NUMERICAL EVALUATION

In this section, we numerically evaluate the analytical results presented in the previous sections. **Fig. 2** shows the optimum power allocation factor β_{opt} given in (13) as a function of the number of users for different values of *T*. We set $P/N_0 = 20$ dB, and $\alpha = 0.1$. It can be observed that β_{opt} decays as the number of users increases. The intuition behind this observation is that in large networks the impact of residual interference due to imperfect interference alignment become more important, thus, it is recommended to allocate more power to pilot symbols to acquire CSI more accurately. Also, we can observe that β_{opt} increases by increasing *T*. This implies that as the channel coherence time increases, a larger power should be allocated for data transmission. It is clear from (3) that to preserve



Fig. 2: The optimum value of β versus K.



Fig. 3: The achievable rate of the interference alignment scheme and the TDMA scheme.

a given variance of the channel estimation error, a lower P_{τ} is required as T and consequently T_{τ} increases. Thus, a larger power can be devoted for data transmission.

Fig. 3 shows the achievable rate per user of the pilot-assisted interference alignment scheme in both cases that power allocation is optimized ($\beta = \beta_{opt}$) and when there is no power optimization ($\beta = 1$). The network parameters are K = 40, T = 1000, and we set $\alpha = 0.04$. We plot the achievable rate of the TDMA with pilot-based channel training scheme and that of the interference alignment scheme with perfect CSI for comparison. This figure shows that, for the given parameters, the pilot-assisted interference alignment scheme can achieve almost the same (slightly less) degrees of freedom as the interference alignment scheme with perfect CSI. This confirms the result in **Proposition 2** when K is sufficiently smaller than T. A large gap between the achievable rate of the interference alignment scheme and that of the TDMA scheme can be seen. Furthermore, 2 dB gain can be seen using the optimum power allocation compared to the case with uniform power allocation.



Fig. 4: The achievable sum-rate versus K.

Fig. 4 shows the achievable sum-rate of the pilot-assisted interference alignment scheme (IA) and that of the user selection and pilot-assisted interference alignment scheme (US-IA) for different number of users. In this example, T = 100, $P/N_0 = 10$ dB, and $\beta = \beta_{opt}$. For IA, we set $\alpha = K/T$ and for US-IA we set $\alpha = K_{opt}/T$, where K_{opt} is given in (17). It can be seen that the achievable sum-rate of IA is maximized for a specific number of users. This observation coincides with **Theorem 3**. The intuition behind this result is that, by increasing the number of the active users, in one hand the number of the independent transmitted symbols increases, and on the other hand the achievable rate per users decreases due to the less available resources for the channel training and consequently more interference. It can be seen that the US-IA scheme outperforms the IA scheme in large networks (K > T/2).

7. CONCLUSION

We have derived the achievable rate region of the Gaussian timevarying IC when no CSI is a priori available at terminals. Our study reveals that the sum degrees of freedom $K_{opt}(1 - K_{opt}/T)/2$ is achievable when the number of the active users is selected to be min $\{T/2, K_{opt}\}$. Thus, it can be recommended that, in large networks (K > T/2), to perform a user selection, and apply interference alignment only within the set of the active users. In addition, we have derived the optimum problem allocation to the channel training and the data transmission. Our results shows that to increase the achievable rate in large networks more power should be allocated to the channel training instead of the data transmission.

The provided results reveals the inherent performance limits of wireless interference channel due to the intrinsic requirements for the radio resources dedicated for channel training. These results provide intuitions for the design of the coordinated transmission schemes over time varying interference channels when no CSI is a priori available at terminals.

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