# PERFORMANCE ANALYSIS OF DISTRIBUTED COOPERATION UNDER UNCOORDINATED NETWORK INTERFERENCE

*Nikolaos Pappas, Marios Kountouris*

Department of Telecommunications, Supélec
Gif-sur-Yvette, France.
{nikolaos.pappas, marios.kountouris}@supelec.fr

## ABSTRACT

Managing interference is a major technical challenge in large wireless networks. Distributed cooperation techniques, such as Interference Alignment (IA), exploit the available spatial degrees of freedom of the interference channel holding promise of enhanced spectral efficiency. Most prior results, however, consider isolated network settings, neglecting the interference from nodes that are not participating in the cooperation scheme. This paper analyzes the performance of IA in the presence of uncoordinated interference from a heterogeneous network. Specifically, we analyze perfect downlink IA in a fixed-size cell, where the interfering nodes are distributed according to a spatial point process, and compare it with a non-cooperative MIMO scheme. Furthermore, the performance gains by using a guard zone between the IA cluster and the interference field are evaluated and design guidelines for the necessary isolation distance from out-of-cluster interferers are provided.

***Index Terms***— Interference Alignment, distributed cooperation, stochastic geometry.

## 1. INTRODUCTION

Consumer demand for cellular data has been growing tremendously, fueled by the ubiquity of wireless devices, such as smartphones, tablets, and laptops. This has placed a greater strain on conventional cellular network infrastructure, and as a result, current cellular networks are becoming dense and heterogeneous [1]. One of the major technical challenges in deploying heterogeneous cellular networks is managing the intra- and the inter-cluster interference. Cooperation techniques among base stations (BSs) sharing the same resources in a cell/cluster is a promising method to reduce or cancel the intra-cell interference. Distributed cooperation schemes, such as interference alignment (IA) [2], hold the promise of increased spectral efficiency, thus attracting recently vivid academic and practical interest. IA is an elegant information-theoretic concept, which ensures that the interference at each receiver aligns along a certain subspace leaving the remaining dimensions interference free.

In [3] the fundamental performance of IA in the context of a large cellular network is investigated and compared with that of non cooperative multi-antenna (MIMO) systems. Interestingly, [4] has established the fundamental limitations of cooperative schemes. It is shown that in systems with pilot-assisted channel estimation, the spectral efficiency upper bound does not depend on the transmit powers, thus increasing the power does not result in higher ergodic rates. Cooperation is then possible only within clusters of limited size, which are subject to out-of-cluster interference (OCI) whose power scales with that of the in-cluster signals. The paper concludes that cooperation cannot in general change an interference-limited network to a noise-limited one. Furthermore, IA in random access networks has been analyzed using tools from stochastic geometry in [5] and the performance of IA in cooperative cellular networks has been assessed in [6].

In this paper we analyze the performance of IA within a fixed-size cluster in the context of a large heterogeneous network (HetNet) and compare it with that of non-cooperative MIMO. The effect of HetNet interference in the cluster is quantified, where the interfering BSs are distributed according to a homogeneous Poisson point process (PPP) accounting for the spatial randomness of interferers. Additionally, the performance gains by introducing an exclusion region between the cluster and the interference field are evaluated and design guidelines for the necessary isolation from out-of-cluster interferers are provided. IA is shown to be sensitive to OCI and practical IA systems should operate far from strong co-channel interferers, e.g. a scenario in which IA is performed inside an airport terminal, which is quite protected from strong macrocell interference due to airport regulations.

## 2. NETWORK MODEL

We consider a wireless network within a cluster (cell) of interest with radius $R$, where $K$ BSs serve one receiver each

This work was supported by the Future and Emerging Technologies (FET) project HIATUS within the Seventh Framework Programme for Research of the European Commission under FET-Open grant number: 265578.

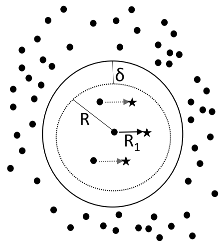


**Fig. 1**. Network model: transmission within a cluster of interest for in presence of OCI.

(downlink). Each BS is equipped with $N_t$ transmit antennas and each receiver has $N_r$ receive antennas, and each user receives $d \leq \min(N_t, N_r)$ signal streams. In Fig. 1 the case of $K = 3$ is depicted. Without loss of generality (WLOG), we assume that the first BS is located at the center of the cluster. A guard region of radius $\delta$ from the cell/cluster edge is imposed around the fixed cell in which no transmitters can occupy. The role of the guard region is the isolation from the interference coming from the uncoordinated OCI [7].

The BS locations outside of the $R+\delta$ radius area are distributed according to a homogeneous PPP $\Phi$ of intensity $\lambda$, which may model homogeneous interference from one network tier as well as interference from HetNets. In the latter case, the OCI is the superposition of multi-tier interference, where each tier's BSs are located according to a PPP with different density and transmit power. All the base stations (inside and outside the guard zone) in our system transmit with power $P$. A singular, distance-dependent pathloss model $\ell(x) = \|x\|^{-\alpha}$ with pathloss exponent $\alpha > 2$ is considered, and the distance between the BS and the $i$-th user is denoted as $R_i$. Channel fading between any BS and user is assumed to be Rayleigh with unit mean and independent across nodes.

## 3. INTERFERENCE ALIGNMENT VS. NON-COOPERATIVE MIMO

In this section, we analyze the performance of distributed cooperation technique, namely IA, in terms of success probability and average rate. We compare IA with a non cooperative (NC) scheme, in which each BS in the cluster transmits $d = \min(N_t, N_r)$ independent spatial streams (spatial multiplexing) using uniform power allocation and zero-forcing (ZF) receivers are employed by the users. Although suboptimal, we consider ZF receivers in this paper because they are more tractable than MMSE receivers. Moreover, the rates achieved by ZF and MMSE receivers converge at asymptotically high SNR (interference-limited regime). For exposition convenience and WLOG, we consider a cluster with $K = 3$ BS-user links and the performance is evaluated at a user served by a BS located at the cell center.

### 3.1. Non-Cooperative MIMO

In the non-cooperative case (spatial multiplexing with ZF receivers), the total interference $I$ comes from two independent sources, the intra-cluster interference $I_{\mathrm{in}}$ and the out-of-cluster interference (outside the guard region) $I_{\mathrm{oci}}$, where BS locations are modeled as PPP.

The per-stream signal-to-interference ratio (SIR) at the receiver of interest assuming independence over streams is given by

$$\mathtt{SIR}_1 = \frac{\frac{P}{d} h_1 R_1^{-a}}{I_{\mathrm{oci}} + I_{\mathrm{in}}}, \tag{1}$$

where $R_1$ is the distance between BS and first user and $h_1 \sim \exp(1)$ denotes the small-scale fading.

The OCI from the PPP is given by

$$I_{\mathrm{oci}} = \sum_{r \in \Phi \setminus r < R^*} P h_r r^{-\alpha}, \tag{2}$$

where, $h_i \sim \exp(1)$, for the small ball approximation (upper bound) $R^* = R + \delta - R_1$ for having a small ball approximation (upper bound on interference), and $R^* = R + \delta + R_2$ for the large ball approximation (lower bound on interference). The $I_{\mathrm{oci}}$ is characterized by its Laplace transform along the lines of [8]

$$\begin{aligned} \mathcal{L}_{I_{\mathrm{oci}}}(s) &= \mathbb{E}_I \left\{ e^{-sI_{\mathrm{oci}}} \right\}, \\ &= \exp\left\{ -2\pi\lambda \left[ \int_{R^*}^{\infty} \left(1 - \mathbb{E}_h(-sPhr^{-\alpha})\right) r \mathrm{d}r \right] \right\} \end{aligned} \tag{3}$$

After some algebraic manipulations $\mathcal{L}_{I_{\mathrm{oci}}}$ is given by

$$\mathcal{L}_{I_{\mathrm{oci}}}(s) = \exp\left\{ \pi\lambda (R^*)^2 - 2\pi\lambda \times \times \mathbb{E}_h \left[ \frac{(sh)^{\frac{2}{\alpha}}}{\alpha} \left( \Gamma\left(-\frac{2}{\alpha}, sh(R^*)^{-\alpha}\right) - \Gamma\left(-\frac{2}{\alpha}\right) \right) \right] \right\}. \tag{4}$$

The intra-cluster interference is given by

$$I_{\mathrm{in}} = \frac{P}{d} R_2^{-\alpha} \sum_{i=1}^{d} h_{2i} + \frac{P}{d} R_3^{-\alpha} \sum_{i=1}^{d} h_{3i}, \tag{5}$$

where $h_{ij} \sim \exp(1)$. For $R = R_2 = R_3$ and $H = \sum_{i=1}^{d} h_{2i} + \sum_{i=1}^{d} h_{3i}$, we have

$$I_{\mathrm{in}} = \frac{P}{d} R^{-\alpha} H, \tag{6}$$

where $H \sim \Gamma(2d, 1)$. Thus, the Laplace transform of $I_{\mathrm{in}}$ is given by

$$\mathcal{L}_{I_{\mathrm{in}}}(s) = \left(1 + s\frac{P}{d} R^{-\alpha}\right)^{-2d}. \tag{7}$$

The success probability, after some manipulations, is given by

$$P_s(\gamma) = \mathbb{P}\left(\mathtt{SIR} > \gamma\right) = \mathcal{L}_{I_{\text{in}}}\left(\frac{\gamma R_1^{\alpha} d}{P}\right)\mathcal{L}_{I_{\text{oci}}}\left(\frac{\gamma R_1^{\alpha} d}{P}\right). \quad (8)$$

The average achievable rate (treating interference as noise and using Gaussian codebooks) is given by

$$\mathbb{E}\log(1+\mathtt{SIR}) = \int_0^{\infty} \mathbb{P}\left(\log(1+\mathtt{SIR}) > \gamma\right) \mathrm{d}\gamma$$
$$= \int_0^{\infty} \mathbb{P}\left(\mathtt{SIR} > e^{\gamma}-1\right) \mathrm{d}\gamma = \int_0^{\infty} P_s\left(e^{\gamma}-1\right) \mathrm{d}\gamma$$
$$= \int_0^{\infty} \mathcal{L}_{I_{\text{in}}}\left(\frac{(e^{\gamma}-1) R_1^{\alpha} d}{P}\right)\mathcal{L}_{I_{\text{oci}}}\left(\frac{(e^{\gamma}-1) R_1^{\alpha} d}{P}\right) \mathrm{d}\gamma$$

### 3.2. Interference Alignment

When IA is applied inside the cluster, the intra-cluster interference is suppressed. WLOG, we consider $N_t = N_r = 2$ antennas, for which IA is feasible if and only if the number of streams $d = 1$ [2]. The SIR for the first receiver is given by

$$\mathtt{SIR}_1 = \frac{P h_1 R_1^{-\alpha}}{I_{\text{oci}}}. \quad (9)$$

The success probability in that case can be expressed as

$$P_s(\gamma) = \mathbb{P}\left(\mathtt{SIR} > \gamma\right) = \mathcal{L}_{I_{\text{oci}}}\left(\frac{\gamma R_1^{\alpha} d}{P}\right), \quad (10)$$

and the average rate is given by

$$\mathbb{E}\left[\log(1+\mathtt{SIR})\right] = \mathcal{L}_{I_{\text{oci}}}\left(\frac{(e^{\gamma}-1) R_1^{\alpha} d}{P}\right) \mathrm{d}\gamma. \quad (11)$$

## 4. NUMERICAL RESULTS

The performance of IA and non cooperative MIMO (referred to as NC) is assessed through numerical results. For the IA scheme, we plot both small and large ball approximations (denoted IA-SB and IA-LB respectively in the plots), whereas for the case of spatial multiplexing (NC), we only plot the small ball approximation (NC-SB). In the numerical results, we assume transmit power $P = 7$W, pathloss exponent $\alpha = 4$, and cluster cell radius $R = 300$m.

In Fig 2, the success probability is plotted versus $\delta$ (isolation radius) for $R_1 = 100$m and different density values, i.e. $\lambda = 10, 50, 90 \times \frac{1}{16\times 300^2}$. When the intra-cluster interferers are close to the cell edge ($R_{\text{in}} = 250$m), for density of interferers $\lambda$ increasing, the success probability evidently decreases. As $\delta$ increases, the success probability increases due to the decreasing amount of out-of-cluster interference. Furthermore, we observe that above a certain value of $\delta$, the achieved gain has a ceiling behavior and saturates

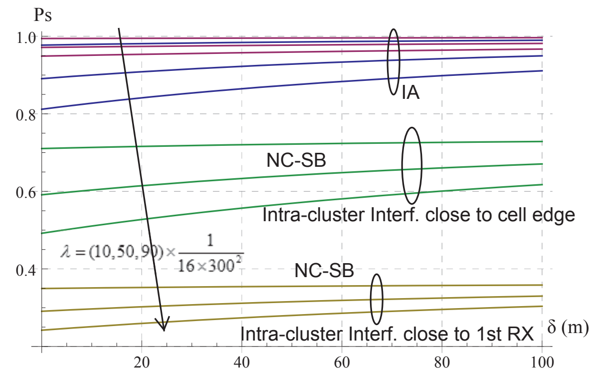


**Fig. 2**. Success probability vs. $\delta$, for $\lambda = 10, 50, 90 \times \frac{1}{16\times 300^2}$ and $R_1 = 100$m.

in a constant value. For the IA case, as $\delta$ increases, the success probabilities tend to be same for different $\lambda$ due to the fact that beyond a value of $\delta$, the OCI becomes negligible. When the intra-cluster interferers are closer to the receiver ($R_{\text{in}} = 180$m), thus causing more interference to the receiver of interest, the success probability for NC MIMO is decreased, however the performance gap between IA and NC is increased.

The case of dense OCI (e.g. $\lambda = 90/(16 * 300^2)$) and where intra-cluster interferers are near to the cell-edge ($R_{\text{in}} = 250$m) and close to the first receiver ($R_{\text{in}} = 180$m) is plotted in Fig. 3. For $R_{\text{in}} = 250$m, NC transmission is shown to be better than the IA when the intensity of the OCI is relatively high and the isolation area is not very large. However, for approximately $\delta > 240$m, the performance of IA is higher than that of NC scheme. This is justified as for perfect IA in the cluster, the only source of interference is the OCI from the external Poisson field, hence when for high $\lambda$, a relatively large guard zone is required to provide sufficient isolation from external interferers. On the other hand, for the NC case, the main source of the interference is the intra-cluster interference, thus as $\delta$ increases the average rate is not increasing sufficiently fast since the intra-cluster interference is unaffected by $\delta$. When $R_{\text{in}} = 180$m, i.e. intra-cluster interferers are closer to the first receiver, using IA is always better than the NC MIMO, the reason being that $I_{\text{in}} >> I_{\text{oci}}$, thus it is of primal importance to suppress the intra-cluster interference.

Fig. 4 shows the case where the OCI intensity is relatively low ($\lambda = 10/(16 * 300^2)$) and $R_{\text{in}} = 180, 250$m. In this case, the IA average rate is again superior to NC rate, as $I_{\text{in}} >> I_{\text{oci}}$ for both cases of intra-cluster interferers. Thus, the average rate of NC MIMO remains almost the same for $\delta$ increasing, because the cell radius itself is sufficient to provide isolation from the PPP interference compared to the intra-cluster one.

In Fig. 5, the parameters are the same with Fig. 4 ex-

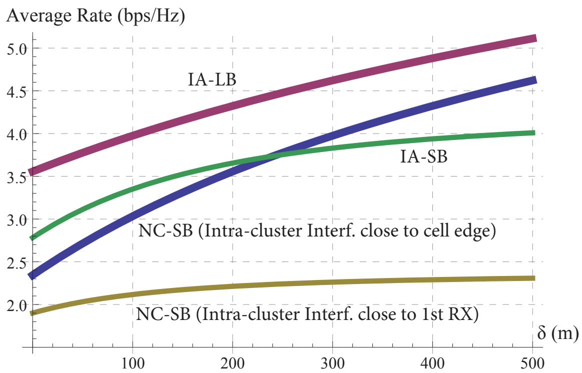


**Fig. 3**. Average rate vs. $\delta$ for $\lambda = 90/(16 * 300^2)$ and $R_1 = 100$m.

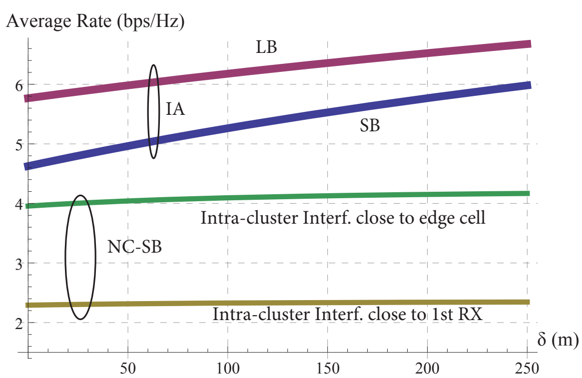


**Fig. 4**. Average rate vs. $\delta$ for $\lambda = 10/(16 * 300^2)$ and $R_1 = 100$m.

cept the distance between the first receiver and its serving BS, which is $R_1 = 75$m instead of $R_1 = 100$m. When the intra-cell interferers are close to the cell edge, for relatively small $\delta < 20$m (approximately), the NC scheme provides better performance than perfect IA. When $R_{\text{in}} = 180$m, IA average rate is higher than that of the NC scheme for any value of $\delta$.

Fig. 6 illustrates the average rate versus $\delta$ for $\lambda = 90/(16 * 300^2)$ and $R_1 = 75$m. When $R_{\text{in}} = 250$m, then the non cooperative transmission results in higher average rate for $\delta \leq 570$m approximately, meaning that intra-cluster interference cancelation using IA results in lower rates. The average rate of NC scheme using small ball approximation is higher even compared with the IA average rate using large ball approximation for $\delta < 380$m. However, for $R_{\text{in}} = 180$m, then IA is better than non cooperative scheme for all $\delta$ values.

The conclusion as far as the average rate performance is concerned is that the non cooperative transmission can provide higher average rate than perfect IA when the OCI is dense or the distance between the first receiver and its intended BS is low and the isolation area radius is small or the

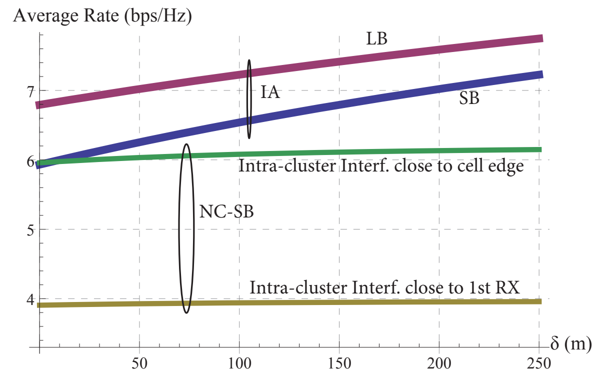


**Fig. 5**. Average rate vs. $\delta$ for $\lambda = 10/(16 * 300^2)$ and $R_1 = 75$m.

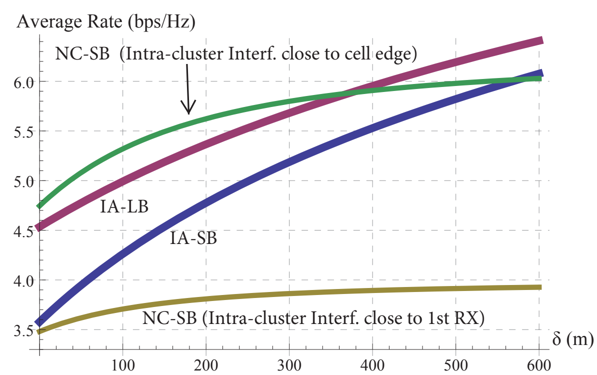


**Fig. 6**. Average rate vs. $\delta$ for $\lambda = 90/(16 * 300^2)$ and $R_1 = 75$m.

intra-cluster interferers are close to the first receiver. Otherwise, perfect IA provides better performance than non cooperative transmission.

## 5. CONCLUSIONS

In this paper, we investigated the downlink performance of IA in a fixed-size cell with guard zone in the presence of out-of-cluster, heterogeneous network interference. Using tools from stochastic geometry, we evaluated the success probability and average rate performance of IA, and we compared it with non cooperative MIMO transmission. Furthermore, we showed the effect of a guard region around the cluster proving sufficient isolation from OCI. The main takeaway of this work is that non cooperative transmission can provide higher average rate than perfect IA when the OCI is dense or the distance between the first receiver and its intended BS is small and the isolation area radius is small or the intra-cluster interferers are close to the first receiver. Otherwise, perfect IA provides better performance than non cooperation.

## 6. REFERENCES


[1] T. Quek, G. de la Roche, I. Guvenc, and M. Kountouris (Eds.), "Small cell networks: Deployment, PHY techniques, and resource management," in *Cambridge University Press*, May 2013.

[2] S. A. Jafar, "Interference alignment: A new look at signal dimensions in a communication network," in *Foundations and Trends in Commun. and Inform. Theory*, vol. 7, no. 1, June 2011.

[3] R. Mungara, G. George, and A. Lozano, "System-level performance of distributed cooperation," in *Proc. 46th Asilomar Conf. on Signals, Systems and Comp. (ASILOMAR)*, Nov. 2012, pp. 1561–1565.

[4] A. Lozano, R. Heath, and J. Andrews, "Fundamental limits of cooperation," *IEEE Trans. on Information Theory*, vol. 59, no. 9, pp. 5213–5226, Sep. 2013.

[5] B. Nosrat-Makouei, J. Andrews, R. Heath, and R. Ganti, "MIMO interference alignment in random access networks," in *Proc. 45th Asilomar Conf. on Signals, Systems and Comp. (ASILOMAR)*, Nov. 2011, pp. 641–645.

[6] D. Aziz, F. Boccardi, and A. Weber, "System-level performance study of interference alignment in cellular systems with base-station coordination," in *2012 IEEE 23rd International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2012, pp. 1155–1160.

[7] R. Heath, M. Kountouris, and T. Bai, "Modeling heterogeneous network interference using Poisson point processes," *IEEE Trans. on Signal Processing*, vol. 61, no. 16, pp. 4114–4126, Aug. 2013.

[8] J. Andrews, F. Baccelli, and R. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. on Communications*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.