LINK ADAPTATION FOR INTERFERENCE ALIGNMENT WITH IMPERFECT CSIT

Mohsen Rezaee†, *Mehrdad Taki*‡, *Maxime Guillaud*†

†Institute of Telecommunications, Vienna University of Technology,
Vienna, Austria – e-mail: {mohsen.rezaee,guillaud}@tuwien.ac.at
‡Electrical Engineering Department, University of Isfahan,
Isfahan, Iran – e-mail: mehrdadtaki@eng.ui.ac.ir

ABSTRACT

In this paper link adaptation for interference alignment (IA) based on imperfect channel state information (CSI) is investigated. We consider a MIMO interference channel where the transmit and receive spaces are determined by IA. We look at maximizing a weighted sum of the average rates provided that a certain set of bit-error-rate and power constraints are satisfied by dynamically adjusting coding, modulation and powers. The problem is quite general and intractable. We resort to some approximations and provide simulations to show the accuracy of the approximations in the regimes with practical interest.

1. INTRODUCTION

Interference alignment (IA) has received a lot of attention due to its promising performance improvement over orthogonal schemes. It has been shown that as long as a set of feasibility conditions are satisfied, IA can achieve the maximum achievable DoF in a K-user constant MIMO IC [1]. A few papers have investigated the effect of imperfect channel state information at the transmitter (CSIT) on the performance of IA [2–6]. However, an improved IA-based scheme that accounts for the imperfect CSIT is still missing. Even though methods have been proposed to reduce the CSIT requirement [7], we need to contain the effect of the residual interference caused by imperfect CSIT and look for methods that can deal with this residual interference properly. Link adaptation methods can be very helpful to deal with residual interference and exploiting the direct channels. Link adaptation improves the performance of a wireless link by adjustment of the rate and/or power at the TX based on the estimated CSI fed back from the RX [8]. Extensive research has been done on link adaptation for point-to-point links. Discrete rate link adaptation for practical systems is introduced in [8] by using adaptive modulation (AM) and is extended to adaptive modulation and coding (AMC) in [9].

In this paper we consider a MIMO IC where the transmit and receive spaces are determined by IA. Considering practical modulation schemes, we use adaptive modulation, coding and power (AMCP) to maximize a weighted sum of the average rates while having a sum-power constraint across the users and ensuring that the bit-error-rate (BER) for decoding every stream in the network remains below a certain threshold. Simulation results are provided to compare the proposed scheme with the orthogonal transmission scheme. [1]

2. SYSTEM MODEL

Consider an interference channel in which K TXs communicate with their respective RXs over a shared medium. For simplicity we assume that every TX is equipped with $\mathrm{N_t}$ antennas while every RX has $\mathrm{N_r}$ antennas. Data symbols are spatially precoded at the TXs. Let d denote the number of data streams sent by each TX. Our results can be extended to more general configurations as long as the feasibility conditions for IA [1] are satisfied. The received signal at RX i reads

$$\mathbf{y}_{\mathrm{i}} = \mathbf{H}_{\mathrm{ii}}\mathbf{V}_{\mathrm{i}}\mathbf{x}_{\mathrm{i}} + \sum_{\substack{1\leq \mathrm{j} \leq \mathrm{K} \\ \mathrm{j}\neq \mathrm{i}}} \mathbf{H}_{\mathrm{ij}}\mathbf{V}_{\mathrm{j}}\mathbf{x}_{\mathrm{j}} + \mathbf{n}_{\mathrm{i}} \tag{1}$$

in which $\mathbf{H}_{\mathrm{ij}} \in \mathbb{C}^{\mathrm{N_r}\times\mathrm{N_t}}$ is the channel matrix between TX j and RX i, $\mathbf{V}_{\mathrm{j}} \in \mathbb{C}^{\mathrm{N_t}\times\mathrm{d}}$ and $\mathbf{x}_{\mathrm{j}} \in \mathbb{C}^{\mathrm{d}}$ are the precoding matrix and the data vector of TX j, respectively. Furthermore, $\mathbf{n}_i \sim \mathcal{CN}(0,\sigma^2\mathbf{I}_{\mathrm{N_r}})$ is the additive noise at RX i. Assuming $\mathrm{E}\{\mathbf{x}_{\mathrm{i}}\mathbf{x}_{\mathrm{i}}^{\mathrm{H}}\} = \mathrm{diag}(\bar{\mathrm{P}}_{\mathrm{i}}(1),...,\bar{\mathrm{P}}_{\mathrm{i}}(\mathrm{d}))$, $\mathrm{i} = 1,\ldots,\mathrm{K}$, the covariance matrix of the signal transmitted by user i is given by $\mathbf{Q}_{\mathrm{i}} = \mathbf{V}_{\mathrm{i}}\mathrm{diag}(\bar{\mathrm{P}}_{\mathrm{i}}(1),...,\bar{\mathrm{P}}_{\mathrm{i}}(\mathrm{d}))\mathbf{V}_{\mathrm{i}}^{\mathrm{H}}$. We assume that all $\mathbf{V}_{\mathrm{i}}$ are truncated unitary matrices, therefore the transmit power for user i is $\mathrm{tr}\left(\mathbf{Q}_{\mathrm{i}}\right) = \sum_{\mathrm{l}=1}^{\mathrm{d}} \bar{\mathrm{P}}_{\mathrm{i}}(\mathrm{l}) = \mathrm{P}_{\mathrm{i}}$.

3. INTERFERENCE ALIGNMENT WITH IMPERFECT CSIT

Assuming that perfect global CSI is available at every TX, the precoders $\mathbf{V}_{\mathrm{i}}$, $\mathrm{i} = 1\ldots\mathrm{K}$ should be designed to align the interference at each RX into a $\mathrm{N_r} - \mathrm{d}$ dimensional space, in order to achieve d interference-free dimensions per user. A solution to the IA problem exists (see [1] and more recently [10, 11] for feasibility criteria – here we will assume that the dimensions and the considered channel realizations are such that the problem is feasible) iff there exist full rank precoding matrices $\mathbf{V}_{\mathrm{j}}$, $\mathrm{j} = 1,...,\mathrm{K}$ and projection matrices $\mathbf{U}_{\mathrm{i}} \in \mathbb{C}^{\mathrm{N_r}\times\mathrm{d}}$, $\mathrm{i} = 1,...,\mathrm{K}$ such that

$$\begin{aligned} &\mathbf{U}_{\mathrm{i}}^{\mathrm{H}}\mathbf{H}_{\mathrm{ij}}\mathbf{V}_{\mathrm{j}} = \mathbf{0} \qquad \forall \mathrm{i},\mathrm{j} \in \{1,...,\mathrm{K}\},\ \mathrm{j}\neq\mathrm{i}, \\ &\mathrm{rank}\left(\mathbf{U}_{\mathrm{i}}^{\mathrm{H}}\mathbf{H}_{\mathrm{ii}}\mathbf{V}_{\mathrm{i}}\right) = \mathrm{d}. \end{aligned} \tag{2}$$

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[1] *Notation:* Nonbold letters represent scalar quantities (italics denote random variables), boldface lowercase and uppercase letters indicate vectors and matrices, respectively. The trace, conjugate, transpose, Hermitian transpose of a matrix or vector are denoted by $\mathrm{tr}(\cdot)$, $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ respectively. The expectation operator is represented by $\mathrm{E}\{\cdot\}$. A diagonal matrix is represented by $\mathrm{diag}(\cdot)$. Absolute value of a scalar is denoted by $|\cdot|$. The mth element of a vector $\mathbf{a}$ and the (m,n)th element of a matrix $\mathbf{A}$ are denoted by $\mathbf{a}(\mathrm{m})$ and $\mathbf{A}(\mathrm{m},\mathrm{n})$ respectively. The complex circularly symmetric Gaussian distribution with mean r and variance σ^2 is denoted by $\mathcal{CN}(\mathrm{r},\sigma^2)$. Finally $\mathrm{var}(.)$ shows the variance of its argument random variable.

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We focus on the first condition in (2) since the second condition is satisfied almost surely when the channel entries are drawn independently from a continuous distribution. It is clear that any truncated unitary matrix that has the same column space as $\mathbf{V}_{\rm i}$ also fulfills (2) and the same argument holds for the RX filters. This means that we can assume without loss of generality that the equivalent direct channels, i.e., $\mathbf{U}_{\rm i}^{\rm H}\mathbf{H}_{\rm ii}\mathbf{V}_{\rm i}$ for $\rm i = 1, ..., K$ are diagonal.

When the CSIT is not perfect at the TXs, the IA filters designed based on imperfect CSIT are denoted by truncated unitary matrices $\hat{\mathbf{V}}_{\rm i}$ and $\hat{\mathbf{U}}_{\rm i}$ where

$$\hat{\mathbf{U}}_{\rm i}^{\rm H}\hat{\mathbf{H}}_{\rm ij}\hat{\mathbf{V}}_{\rm j} = \mathbf{0} \qquad \forall \rm i, j \in \{1, ..., K\},\ j \neq i, \tag{3}$$

where the estimated channels are denoted by $\hat{\mathbf{H}}_{\rm ij}$. Similar to the perfect CSI scenario, we assume that the filters are such that the direct equivalent channels which we denote by $\hat{\mathbf{G}}_{\rm ii} \triangleq \hat{\mathbf{U}}_{\rm i}^{\rm H}\hat{\mathbf{H}}_{\rm ii}\hat{\mathbf{V}}_{\rm i}$ for $\rm i = 1, ..., K$, are diagonal. Using the designed precoders, the received signal after projection by the receive filters can be written as

$$\mathbf{r}_{\rm i} = \hat{\mathbf{U}}_{\rm i}^{\rm H}\mathbf{y}_{\rm i} = \mathbf{G}_{\rm ii}\mathbf{x}_{\rm i} + \sum_{\substack{1\leq \rm j \leq K \\ \rm j\neq i}} \mathbf{G}_{\rm ij}\mathbf{x}_{\rm j} + \hat{\mathbf{U}}_{\rm i}^{\rm H}\mathbf{n}_{\rm i} \tag{4}$$

where $\mathbf{G}_{\rm ii} = \hat{\mathbf{U}}_{\rm i}^{\rm H}\mathbf{H}_{\rm ii}\hat{\mathbf{V}}_{\rm i}$ and $\mathbf{G}_{\rm ij} = \hat{\mathbf{U}}_{\rm i}^{\rm H}\mathbf{H}_{\rm ij}\hat{\mathbf{V}}_{\rm j}$. Clearly the second term is due to the interference leakage from other users which is caused by designing the filters based on imperfect CSI. Also the equivalent channel $\mathbf{G}_{\rm ii}$ for each user is not diagonal anymore which introduces inter-stream interference. To simplify the notation, we denote the qth diagonal elements of $\mathbf{G}_{\rm ii}$ and $\hat{\mathbf{G}}_{\rm ii}$ by $G_{\rm i}^{\rm q}$ and $\hat{G}_{\rm i}^{\rm q}$ respectively, for $\rm q = 1, ..., d$.

4. STATISTICAL MODEL OF THE CSIT

We assume that the entries of the true channels $\mathbf{H}_{\rm ij}$ and the channel estimates available at the TX side $\hat{\mathbf{H}}_{\rm ij}$ are modeled as complex Gaussian random variables with zero mean and variance $\Delta_{\rm ij}$. We assume the following CSI imperfection model,

$$\mathbf{H}_{\rm ij} = \rho_0\hat{\mathbf{H}}_{\rm ij} + \sqrt{1-\rho_0^2}\mathbf{E}_{\rm ij} \tag{5}$$

where we have assumed that the entries of the perturbation matrix $\mathbf{E}_{\rm ij}$ have the same distribution as the true channel and they are independent from the entries of $\hat{\mathbf{H}}_{\rm ij}$. With this model the parameter ρ_0 represents the correlation between the true channel elements and the estimated ones. Denoting the real and imaginary parts of $\mathbf{H}_{\rm ij}$ by $\mathbf{X}_{\rm ij}$ and $\mathbf{Y}_{\rm ij}$ respectively and that of $\hat{\mathbf{H}}_{\rm ij}$ by $\hat{\mathbf{X}}_{\rm ij}$ and $\hat{\mathbf{Y}}_{\rm ij}$ respectively, due to the circular symmetry, the entries of the real matrices are independent from the imaginary matrices. Further, we have

$$\rm E\{\mathbf{H}_{ij}(m,n)|\hat{\mathbf{H}}_{ij}(m,n)\} = \rho_0\hat{\mathbf{H}}_{ij}(m,n)$$
$$\rm E\{|\mathbf{H}_{ij}(m,n)|^2|\hat{\mathbf{H}}_{ij}(m,n)\} = (1-\rho_0)^2\Delta_{ij} + \rho_0^2|\hat{\mathbf{H}}_{ij}(m,n)|^2.$$

5. IA WITH AMCP USING IMPERFECT CSIT

In the context of link adaptation, transmission rate and power of the TX is adjusted depending on the instantaneous channel condition. There are two different approaches for link adaptation in the literature namely, continuous and discrete link adaptation. In the first case, capacity achieving codes with vanishing error probability are employed and it is assumed that the rate can be adjusted continuously according to the channel condition. In this case if we denote the equivalent channel gain, the average power of noise (and interference) and the transmission power by $h_{\rm eq}$, $\delta_{\rm eq}$ and p respectively, then the spectral efficiency of the link is equal to

$$\log_2(1 + ps_{\rm eq}) \quad \rm (bit/Sec/Hz) \tag{6}$$

where $s_{\rm eq} = \frac{|h_{\rm eq}|^2}{\delta_{\rm eq}}$ is the equivalent SINR of the link.

In the context of discrete link adaptation, adaptive coding is used along with adaptive modulation which improves the performance significantly. In this method, $\rm M + 1$ transmission modes are considered where every mode $\rm m$ corresponds to a pair of modulation and coding configuration and finally provides a transmission rate denoted by $\rm R_m$. The rates for different modes are sorted as $\rm 0 = R_0 < R_1 < ... < R_M$. Mode 0 represents the case where no information is transmitted. In mode $\rm m$ the BER is approximated by the following expression [12]:

$$\rm p_e(\mathit{ps_{eq}}, R_m) = A_m exp(-A'_m \mathit{ps}_{eq}),\ \ \forall \mathit{m} = 0, ..., M,\ \ s_{eq} \geq 0 \tag{7}$$

where $\rm A_m$ and $\rm A'_m$ are constants that are determined by the transmission mode. This model is based on approximating the BER curve which is realized using Monte Carlo simulation over a large range of SNR. The process is repeated for every mode $\rm m$ which yields the parameters $\rm A_m$ and $\rm A'_m$ via curve fitting. In transmission mode $\rm m$, the minimum required SINR to ensure a maximum BER of $\rm B_0$ is denoted by $\rm g_{B_0}(m)$, i.e.,

$$ps_{\rm eq} \geq \rm g_{B_0}(m) \Rightarrow p_e(\mathit{ps}_{eq}, m) \leq B_0 \tag{8}$$

where $\rm g_{B_0}(m) \triangleq \frac{-1}{A'_m}\ln(\frac{B_0}{A_m})$ for $\rm B_0 \leq \min(1/2, A'_m)$. If the CSI is not perfect, direct use of imperfect CSI instead of the true CSI in IA and AMCP will be highly sub-optimal. Based on our assumptions, since the available CSI at each TX is a degraded version of the true CSI, therefore the IA and link adaptation methods should be revisited taking the effect of the residual interference into account. Here we present a new scheme where AMCP is performed using imperfect CSIT to improve IA performance.

5.1. Problem Formulation

According to Section 2, every TX has $\rm d$ data streams. For stream $\rm q$ of user $\rm i$, we denote the average power of interference and noise which affects this stream by $\delta_{\rm i}^{\rm q}$. Furthermore, the instantaneous true SINR and the estimate of the instantaneous SINR (which is available at the transmitters) are denoted respectively by $\gamma_{\rm i}^{\rm q} = \frac{|G_{\rm i}^{\rm q}|^2}{\delta_{\rm i}^{\rm q}}$ and $\hat{\gamma}_{\rm i}^{\rm q} = \frac{|\hat{G}_{\rm i}^{\rm q}|^2}{\delta_{\rm i}^{\rm q}}$. The instantaneous rate and power for stream $\rm q$ of user $\rm i$ are denoted by $k_{\rm i}^{\rm q} = \Phi(\hat{\gamma}_{\rm i}^{\rm q})$ and $p_{\rm i}^{\rm q} = \Psi(\hat{\gamma}_{\rm i}^{\rm q})$ respectively, for $\rm 1 \leq i \leq K$ and $\rm 1 \leq q \leq d$. Based on the available CSIT at the TXs ($\hat{\mathbf{H}}_{\rm ij},\ \forall \rm i, j$), the TXs choose the precoders $\hat{\mathbf{V}}_{\rm j}$ according to (3), while the rate and power are determined by the mappings $\Phi(\cdot)$ and $\Psi(\cdot)$ which should be designed such that the average sum-power is at most $\rm P_0$ and the BER at each RX is less or equal to $\rm B_0$, while a weighted sum of the average rates is maximized. The weights for different users are denoted by $\omega_{\rm i}$, $\rm i = 1, ..., K$. With these definitions, the overall optimization problem can be formulated as follows

$$\begin{aligned}
&\max_{\Phi(\cdot),\Psi(\cdot)} \sum_{i=1}^{\mathrm{K}} \omega_{\mathrm{i}} \sum_{q=1}^{\mathrm{d}} \mathrm{E}_{\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}}\{\Phi(\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}})\} \\
&\text{s.t.} \quad \sum_{i=1}^{\mathrm{K}} \sum_{q=1}^{\mathrm{d}} \mathrm{E}_{\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}}\{\Psi(\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}})\} \le \mathrm{P}_0 \\
&\qquad \mathrm{E}_{\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}},\gamma_{\mathrm{i}}^{\mathrm{q}}}\{\mathrm{p_e}(\Psi(\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}})\gamma_{\mathrm{i}}^{\mathrm{q}}, \Phi(\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}))\} \le \mathrm{B}_0, \ \forall \mathrm{i}, \mathrm{q}.
\end{aligned} \tag{9}$$

Solving the exact optimization problem in (9) is intractable especially since the optimization is performed using imperfect information about the channels. We divide the optimization problem in (9) into the following three steps.

5.2. Effect of Imperfect CSI on IA

Here we investigate the properties of the residual interference affecting the decoding of every stream when the precoders are designed using IA. For simplicity we define $\hat{\mathbf{v}}_{\mathrm{i,q}}$ and $\hat{\mathbf{u}}_{\mathrm{i,q}}$ as the qth column of $\hat{\mathbf{V}}_{\mathrm{i}}$ and $\hat{\mathbf{U}}_{\mathrm{i}}$ respectively. Therefore from (4) we have $\mathbf{r}_{\mathrm{i}}(\mathrm{q}) = G_{\mathrm{i}}^{\mathrm{q}}\mathbf{x}_{\mathrm{i}}(\mathrm{q}) + I_{\mathrm{i,q}} + I'_{\mathrm{i,q}} + \mathbf{n'}_{\mathrm{i}}^{\mathrm{q}}$ where $G_{\mathrm{i}}^{\mathrm{q}} = \hat{\mathbf{u}}_{\mathrm{i,q}}^{\mathrm{H}}\mathbf{H}_{\mathrm{ii}}\hat{\mathbf{v}}_{\mathrm{i,q}}$ is the qth diagonal element of $\mathbf{G}_{\mathrm{ii}}$, $I_{\mathrm{i,q}} = \sum_{\substack{1\le \mathrm{l}\le \mathrm{d}\\ \mathrm{l}\neq \mathrm{q}}} \hat{\mathbf{u}}_{\mathrm{i,q}}^{\mathrm{H}}\mathbf{H}_{\mathrm{ii}}\hat{\mathbf{v}}_{\mathrm{i,l}}\mathbf{x}_{\mathrm{i}}(\mathrm{l})$ is the interference from other streams of the same user, $I'_{\mathrm{i,q}} = \sum_{\substack{1\le \mathrm{j}\le \mathrm{K}\\ \mathrm{j}\neq \mathrm{i}}} \hat{\mathbf{u}}_{\mathrm{i,q}}^{\mathrm{H}}\mathbf{H}_{\mathrm{ij}}\hat{\mathbf{V}}_{\mathrm{j}}\mathbf{x}_{\mathrm{j}}$ is the interference from other users and $\mathbf{n'}_{\mathrm{i}}^{\mathrm{q}} = \hat{\mathbf{u}}_{\mathrm{i,q}}^{\mathrm{H}}\mathbf{n}_{\mathrm{i}}$ is the equivalent noise for this stream. If the channels are perfectly known ($\hat{\mathbf{H}}_{\mathrm{ij}} = \mathbf{H}_{\mathrm{ij}}, \ \forall \mathrm{i}, \mathrm{j}$), then the terms $I_{\mathrm{i,q}}$ and $I'_{\mathrm{i,q}}$ will be zero due to the alignment equations. In the following, we derive the statistical properties of $G_{\mathrm{i}}^{\mathrm{q}}$ (given the information about $\hat{G}_{\mathrm{i}}^{\mathrm{q}}$) which can be used to derive the statistics of the true SINR. Then we derive the power of each interference term depending on the level of imperfectness of the channels controlled by ρ_0.

5.2.1. *Statistics of the direct channel power gain*

It can be shown that $|G_{\mathrm{i}}^{\mathrm{q}}|^2$ and $|\hat{G}_{\mathrm{i}}^{\mathrm{q}}|^2$ are approximately distributed as exponential random variables (this holds exactly in the single-stream case) [13]. Henceforth we work under the assumption that $|G_{\mathrm{i}}^{\mathrm{q}}|^2$ and $|\hat{G}_{\mathrm{i}}^{\mathrm{q}}|^2$ are exponentially distributed with parameter $\frac{1}{\Delta_{\mathrm{ii}}}$. Note that the value of $\hat{G}_{\mathrm{i}}^{\mathrm{q}}$ is the estimation of the direct channel gain at the transmitter and this value is used to have an instantaneous estimate of the SINR.

5.2.2. *Calculating the power of interference plus noise*

In this part we calculate the power of interference plus noise for every stream at each RX. Clearly, assuming $\hat{\mathbf{U}}_{\mathrm{i}}, \hat{\mathbf{V}}_{\mathrm{i}}, \hat{\mathbf{H}}_{\mathrm{ij}} \ \forall \mathrm{i}, \mathrm{j}$ are given, $I_{\mathrm{i,q}}$ is a sum of $\mathrm{d}-1$ random variables and $I'_{\mathrm{i,q}}$ is a sum of $\mathrm{K}-1$ random variables. It can be shown that every two distinct elements of $I_{\mathrm{i,q}}$, $I'_{\mathrm{i,q}}$ and $\mathbf{n'}_{\mathrm{i}}^{\mathrm{q}}$ are independent. This gives

$$\delta_{\mathrm{i}}^{\mathrm{q}} = \mathrm{E}\{|I_{\mathrm{i,q}}|^2\} + \mathrm{E}\{|I'_{\mathrm{i,q}}|^2\} + \mathrm{E}\{|\mathbf{n'}_{\mathrm{i}}^{\mathrm{q}}|^2\}. \tag{10}$$

It can be shown that the power of the interference terms is

$$\mathrm{E}\{|I_{\mathrm{i,q}}|^2\} = \Delta_{\mathrm{ii}}(1-\rho_0^2) \sum_{\substack{1\le \mathrm{l}\le \mathrm{d}\\ \mathrm{l}\neq \mathrm{q}}} \bar{P}_{\mathrm{i}}(\mathrm{l}), \tag{11}$$

$$\mathrm{E}\{|I'_{\mathrm{i,q}}|^2\} = (1-\rho_0^2) \sum_{\substack{1\le \mathrm{j}\le \mathrm{K}\\ \mathrm{j}\neq \mathrm{i}}} \Delta_{\mathrm{ij}} \sum_{1\le \mathrm{l}\le \mathrm{d}} \bar{P}_{\mathrm{i}}(\mathrm{l}). \tag{12}$$

Also it is clear that the noise variance will not change after projection by the (truncated unitary) receive filters.

5.2.3. *Statistics of $\gamma_{\mathrm{i}}^{\mathrm{q}}$ conditioned on $\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}$*

In the considered link adaptation method, transmission rate and power are designed based on the SINR of the link. Since only $\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}$ is available at the TX, in order to do link adaptation, we need to find the relationship between $\gamma_{\mathrm{i}}^{\mathrm{q}}$ and $\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}$ or, more precisely, the statistics of $\gamma_{\mathrm{i}}^{\mathrm{q}}$ given $\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}$ should be determined. We know that $|G_{\mathrm{i}}^{\mathrm{q}}|^2$ and $|\hat{G}_{\mathrm{i}}^{\mathrm{q}}|^2$ and hence $\gamma_{\mathrm{i}}^{\mathrm{q}}$ and $\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}$ have exponential distribution approximately. The pdf of two correlated exponential random variables reads

$$\begin{aligned}
f_{\gamma_{\mathrm{i}}^{\mathrm{q}}|\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}}(\mathrm{x}_1|\mathrm{x}_2) \quad = \quad & \frac{1}{\Gamma_{\mathrm{i}}^{\mathrm{q}}(1-\rho)} \exp\left\{\frac{-\rho}{1-\rho}\frac{\mathrm{x}_2}{\hat{\Gamma}_{\mathrm{i}}^{\mathrm{q}}}\right\} \\
\cdot \quad & \mathrm{I}_0\left\{\frac{2\sqrt{\rho}}{1-\rho}\sqrt{\frac{\mathrm{x}_2}{\Gamma_{\mathrm{i}}^{\mathrm{q}}\hat{\Gamma}_{\mathrm{i}}^{\mathrm{q}}}}\sqrt{\mathrm{x}_1}\right\} \exp\left\{\frac{-\mathrm{x}_1}{\Gamma_{\mathrm{i}}^{\mathrm{q}}(1-\rho)}\right\}
\end{aligned} \tag{13}$$

where $\Gamma_{\mathrm{i}}^{\mathrm{q}} \triangleq \mathrm{E}\{\gamma_{\mathrm{i}}^{\mathrm{q}}\}$ and $\hat{\Gamma}_{\mathrm{i}}^{\mathrm{q}} \triangleq \mathrm{E}\{\hat{\gamma}_{\mathrm{i}}^{\mathrm{q}}\}$, therefore $\Gamma_{\mathrm{i}}^{\mathrm{q}} = \hat{\Gamma}_{\mathrm{i}}^{\mathrm{q}} = \frac{\Delta_{\mathrm{ii}}}{\sigma^2+\mathrm{E}\{|I_{\mathrm{i,q}}|^2\}+\mathrm{E}\{|I'_{\mathrm{i,q}}|^2\}}$. Also it can be shown that the correlation coefficient is equal to $\rho = \rho_0^2$ [13].

5.3. Point-to-Point Link with Imperfect CSIT

5.3.1. *Point-to-Point Link with average power and BER constraints*

Denoting the true SINR and the estimated SINR by γ and $\hat{\gamma}$ respectively and Γ and $\hat{\Gamma}$ their respective expectations, the conditional distribution $f_{\gamma|\hat{\gamma}}$ is according to (13). The goal is to design the transmission scheme using adaptive power and rate $p = \Psi(\hat{\gamma})$ and $k = \Phi(\hat{\gamma})$ which are functions of the estimated SINR. The design parameters are the functions $\Psi(\cdot)$ and $\Phi(\cdot)$ such that the average rate is maximized while the average power and average BER constraints are satisfied as formulated in the following optimization problem

$$\begin{aligned}
&\max_{\Phi(\cdot),\Psi(\cdot)} \mathrm{E}\{\Phi(\hat{\gamma})\} \\
&\quad \text{s.t.} \quad \mathrm{E}\{\Psi(\hat{\gamma})\} \le \mathrm{P}^{\max} \qquad (\mathrm{C'1}) \\
&\qquad\quad \mathrm{E}_{\gamma,\hat{\gamma}}\{\mathrm{p_e}(\Psi(\hat{\gamma})\gamma, \Phi(\hat{\gamma}))\} \le \mathrm{B}_0. \quad (\mathrm{C'2})
\end{aligned} \tag{14}$$

Condition $(\mathrm{C'2})$ is complicated and instead we enforce the same condition for any instance of $\hat{\gamma}$, i.e., $\mathrm{E}_{\gamma}\{\mathrm{p_e}(p\gamma, k)|\hat{\gamma}\} \le \mathrm{B}_0$. By satisfying this condition, $(\mathrm{C'2})$ is always satisfied.

Lemma 1 *Defining* $\mathrm{A}_0 = \max(\mathrm{A}_1, \cdots, \mathrm{A}_{\mathrm{M}})$, *to satisfy the BER condition* $(\mathrm{C'2})$ *it is sufficient to have* $\frac{\mathrm{g}_{\mathrm{B}_0}(\mathrm{m})}{\mathrm{Q}_1\hat{\gamma}+\mathrm{Q}_2} \le \Psi(\hat{\gamma})$, *where* $\mathrm{Q}_1 = \rho_0^2\frac{\Gamma}{\hat{\Gamma}}$ *and* $\mathrm{Q}_2 = (1-\rho_0^2)\Gamma\ln(\frac{\mathrm{B}_0}{\mathrm{A}_0})$.

Proof: See [13].

Considering the limitation on the transmit power (condition $\mathrm{C'1}$), Lemma 1 gives $\Psi(\hat{\gamma}) = \frac{\mathrm{g}_{\mathrm{B}_0}(\mathrm{m})}{\mathrm{Q}_1\hat{\gamma}+\mathrm{Q}_2}$. Clearly when $\rho_0 \to 1$ (more accurate channels), we get (8) which was derived for perfect channels. Similar to [9], where link adaptation is performed using perfect SINR estimates, we divide the range of $\hat{\gamma}$ using thresholds $\mathrm{t_m}$, $0 \le \mathrm{m} \le \mathrm{M}$ such that when $\hat{\gamma} \in [\mathrm{t_m}, \mathrm{t_{m+1}})$ then the transmission rate is chosen to be $\mathrm{R_m}$. Therefore the optimization problem (14) can be reformulated as

$$\begin{aligned}
&\max_{\{\mathrm{t_m}\}_{\mathrm{m}=0}^{\mathrm{M}}} \sum_{\mathrm{m}=0}^{\mathrm{M}} \mathrm{R_m} \int_{\mathrm{t_m}}^{\mathrm{t_{m+1}}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \\
&\text{s.t.} \quad \sum_{\mathrm{i}=0}^{\mathrm{M}} \mathrm{g}_{\mathrm{B}_0}(\mathrm{i}) \int_{\mathrm{t_i}}^{\mathrm{t_{i+1}}} \frac{f_{\hat{\gamma}}(\hat{\gamma})}{\mathrm{Q}_1\hat{\gamma}+\mathrm{Q}_2} d\hat{\gamma} \le \mathrm{P}^{\max}.
\end{aligned} \tag{15}$$

Note that we have implicitly included the BER constraint. Using the Lagrangian method we can show that the optimal thresholds are given by

$$\mathrm{t_m} = \frac{\lambda \frac{(\mathrm{g_{B_0}(m) - g_{B_0}(m-1)})}{\mathrm{R_m - R_{m-1}}} - \mathrm{Q_2}}{\mathrm{Q_1}}, \quad 0 \le \mathrm{m} \le \mathrm{M} \tag{16}$$

The Lagrangian multiplier λ is computed such that the power constraint is satisfied with equality.

5.3.2. Approximation of the average rate in a point-to-point link

Inspired by the Gaussian channel, we approximate the average rate achievable in the link by $\mathrm{E}\{k\} = \mathrm{L_1} \log(1 + \mathrm{L_2}\Gamma \mathrm{P^{max}})$ where $\mathrm{L_1}$ and $\mathrm{L_2}$ are functions of the transmission modes m, maximum error probability $\mathrm{B_0}$ and the CSI uncertainty parameter ρ_0. We have used a curve-fitting method to find the values for $\mathrm{L_1}$ and $\mathrm{L_2}$. We verified the accuracy of this approximation compared to the actual average rate via extensive simulations (see [13]).

5.4. Optimization of the approximate average rates

Using the approximated closed-form expression for the average rate we equivalently look at the following problem

$$\begin{aligned} &\max_{\bar{\mathrm{P}}_\mathrm{i}(\mathrm{q}),\, 1\le \mathrm{i} \le \mathrm{K},\ 1 \le \mathrm{q} \le \mathrm{d}} \sum_{\mathrm{i}=1}^{\mathrm{K}} \omega_\mathrm{i} \sum_{\mathrm{q}=1}^{\mathrm{d}} \bar{K}_\mathrm{i}(\mathrm{q}) \\ &\text{s.t.} \qquad \sum_{\mathrm{i}=1}^{\mathrm{K}} \sum_{\mathrm{q}=1}^{d} \bar{\mathrm{P}}_\mathrm{i}(\mathrm{q}) \le \mathrm{P_0} \end{aligned} \tag{17}$$

where $\bar{K}_\mathrm{i}(\mathrm{q}) = \mathrm{L_1} \log(1 + \mathrm{L_2}\Gamma_\mathrm{i}^\mathrm{q} \bar{\mathrm{P}}_\mathrm{i}(\mathrm{q})) \approx \mathrm{E}\{k_\mathrm{i}^\mathrm{q}\}$. Note that $\Gamma_\mathrm{i}^\mathrm{q}$ is also a function of the average powers. The above problem is a non-convex constrained optimization problem. We resort to numerical optimization methods to find a (locally) optimum solution. The optimization problem is solved using the active-set method [14]. We used the standard MATLAB implementation. Therefore, we find the average powers to be assigned to different streams of different users. After finding the average powers, the instantaneous rates and powers can be chosen independently for each stream similar to the point-to-point scenario discussed in the previous subsection.

6. ORTHOGONAL TRANSMISSION IN THE K-USER MIMO IC USING AMCP

Due to the imperfectness of the CSI and sub-optimality of IA, it is natural to ask whether we are better off performing IA rather than simply using an orthogonal resource-sharing transmission scheme. In orthogonal transmission we assume that the channel resources (time/bandwidth) are divided equally among different users in the network. In order to have a fair comparison with the proposed scheme, we consider a similar problem where we look for maximizing a weighted sum of the average rates of the users while having a total transmission power constraint $\mathrm{P_0}$ and a maximum average BER of $\mathrm{B_0}$. Here we have K point-to-point MIMO links where for each link $\mathrm{d'} = \min(\mathrm{N_t}, \mathrm{N_r})$ parallel streams are transmitted. Therefore we will have a similar optimization problem except that the filters are not determined by IA equations but they are chosen such that the direct channels are diagonalized. It should be noted that in this case every RX only has to consider inter-stream interference when computing the SINR distribution. Similar to the case of interfering transmission we can find the statistics of the SINR and proceed with solving the problem.

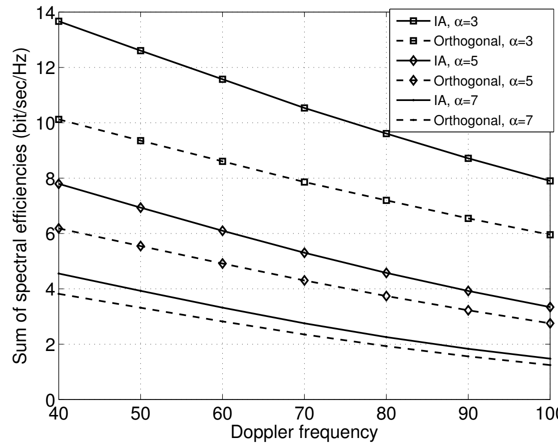

Fig. 1. Rate comparison for different values of Doppler and distance, $\mathrm{K} = 5, \mathrm{N_t} = \mathrm{N_r} = 3$

7. SIMULATION RESULTS

To evaluate the performance of the proposed scheme, we use the AMC modes defined in the IEEE 802.11-a standard. In this setting there are 8 different modes each of which is associated to a particular modulation and coding pair. These modes provide the set of rates $\{0, .5, .75, 1, 1.5, 2, 3, 4\}$. The parameters of the BER estimation according to (7) are extracted from [12]. We assume that the TXs and the RXs are placed such that the distance between TX j and RX i equals $\sqrt{\alpha^2 + (\mathrm{i}-\mathrm{j})^2\beta^2}$ and the variance of the channel entries equals $\mathrm{E}\{|\mathbf{H}_\mathrm{ij}|^2\} = \Delta_\mathrm{ij} = \frac{\Psi_0}{(\alpha^2 + (\mathrm{i}-\mathrm{j})^2\beta^2)^{1.5}}$ where Ψ_0, α and β are constants. α is the distance between a pair of TX and RX. We assume that the TXs and the RXs are equipped with the same number of antennas and that the weights for different users are equal ($\omega_\mathrm{j} = 1, \forall \mathrm{j}$). Also we set $\beta = 1$ and a total power constraint of $\mathrm{P_0} = 1$ is considered and the noise variance is 0.1. Even though the uncertainty model introduced previously is quite general, we assume in this section that the imperfection is introduced by feedback latency and Doppler effects. The maximum Doppler frequency is denoted by $\mathrm{f_d}$ and the delay between the current transmission and channel estimation at the RX is denoted by $\mathrm{T_d}$. According to the Jakes model [15], the coefficient ρ_0 is derived from $\rho_0 = \mathrm{J_0}(2\pi \mathrm{f_d T_d})$ where $\mathrm{J_0}(\cdot)$ is the Bessel function of the first kind of order 0. Further, we assume that α is related to $\mathrm{T_d}$ as $\mathrm{T_d} = 10^{-4} \times \alpha$ in our setting. In Figure 1, assuming $\mathrm{K} = 3$, $\mathrm{N_t} = \mathrm{N_r} = 4$ and $\mathrm{d} = 2$, the sum rate is plotted for different values of Doppler $\mathrm{f_d}$ and for different values of distance α. Clearly, by increasing the Doppler and distance, the CSI becomes more outdated and it degrades the overall performance. It is evident that the proposed scheme outperforms the orthogonal scheme.

8. CONCLUSION

Interference alignment based on imperfect CSI was investigated. We considered a MIMO interference channel where the transmit and receive spaces are determine by IA. We looked at maximizing a weighted sum of the average rates provided that a certain set of bit-error-rate and power constraints are satisfied by choosing appropriate modulation coding and powers.

9. REFERENCES

[1] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, "On feasibility of interference alignment in MIMO interference networks," *IEEE Trans. Signal Processing*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.

[2] R. Tresch and M. Guillaud, "Cellular interference alignment with imperfect channel knowledge," in *Proc. IEEE International Conference on Communications (ICC)*, Dresden, Germany, Jun. 2009.

[3] O. El Ayach and R. W. Heath, "Interference alignment with analog channel state feedback," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 626–636, Feb. 2012.

[4] S. H. Park, H. Park, Y. D. Kim, and I. Lee, "Regularized interference alignment based on weighted sum-MSE criterion for MIMO interference channels," in *Proc. IEEE Int. Conf. Commun.*, Cape Town, South Africa, May 2010.

[5] B. Nosrat-Makouei, J. Andrews, and R. Heath, "A simple SINR characterization for linear interference alignment over uncertain MIMO channels," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Austin, TX, June. 2010.

[6] H. M. B. Xie, Y. Li and A. Nosratinia, "Adaptive interference alignment with CSI uncertainty," *IEEE Trans. Comm.*, vol. 61, no. 2, pp. 792–801, Feb. 2013.

[7] M. Rezaee and M. Guillaud, "Limited feedback for interference alignment in the K-user MIMO interference channel," in *Proc. IEEE Inf. Theory. Workshop (ITW)*, Lausanne, Switzerland, 2012.

[8] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Trans. Commun.*, vol. 49, no. 9, pp. 1561–1571, Sep. 2001.

[9] K. J. Hole, H. Holm, and G. E. Oien, "Adaptive multidimensional coded modulation over flat fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, 2000.

[10] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, "On the degrees of freedom achievable through interference alignment in a MIMO interference channel," *IEEE Trans. Signal Processing*, vol. 60, no. 2, 2012.

[11] G. Bresler, D. Cartwright, and D. Tse, "Settling the feasibility of interference alignment for the MIMO interference channel: the symmetric square case," Apr. 2011, preprint, http://arxiv.org/abs/1104.0888.

[12] J. S. Harsini and F. Lahouti, "QoS constrained throughput optimization for joint adaptive transmission with ARQ over block-fading channels," *IET Communications*, vol. 3, no. 6, pp. 1030–1040, Jun. 2009.

[13] M. Taki, M. Rezaee, and M. Guillaud, "Adaptive modulation and coding for interference alignment with imperfect CSIT," *Submitted to Trans. Wireless Comm*, 2013.

[14] J. Nocedal and S. J. Wright, *Numerical optimization*. Springer Verlag, Berlin, New York, 2nd edition, 2006.

[15] W. C. Jakes, *Microwave Mobile Communication*. NY: Wiley, 1974.