OPTIMAL POWER ALLOCATION AND NETWORK BEAMFORMING FOR OFDM-BASED RELAY NETWORKS

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ABSTRACT

We herein consider the problem of optimal power allocation in an OFDM two-way relay network with multiple relays. Assuming two-way relaying is performed using analog network coding, we obtain the optimal power allocation across subcarriers and among a relay and two communicating nodes by minimizing the total power consumption in the network subject to two separate rate constraints for each transceiver. We then present an algorithm to solve the proposed optimization problem. Our simulation result shows that the proposed algorithm significantly outperform an equal power allocation scheme, where all subcarriers at all nodes receive the same levels of power and the total power is equal to that consumed in our proposed solution.

1. INTRODUCTION

Using relay networks in wireless communication has the potential to enhance the capacity and transition range of wireless networks. In two-way relaying, a pair of transceiver nodes aim to send and receive information to/from other with the help of one or more relays witch have two-way relaying capability. Based on the configuration of the two-way relay network, it may enjoy higher spectrum efficiency or less power consumption comparing one-way relaying. Two-way relay networks have been the main focus of some recent studies [1–22]. In majority of the published studies, the channel between each two nodes has been assumed to be flat fading channel. In reality, in a lot of practical situations, the channel is frequency selective which results in inter-symbol interference (ISI) at the two transceiver.

Generally in order to combat ISI in frequency selective channels, there are two different competing approaches. First approach is to use some equalizers in receivers and the second approach is to use multi-carrier transmission. Both of approaches can be implemented in two-way relay networks. In class of filter-and-forward relaying methods single carrier transmission has been combined by distributed and collaborative equalizing (utilizing finite impulse response (FIR) or infinite impulse response (IIR) filters in each network node) [15–18]. To implement the second approach of ISI mitigating in two-way relay networks, the orthogonal frequency division multiplexing (OFDM) technology is employed at all nodes of the communication network. Using cyclic prefix (CP) in OFDM transmission, a frequency selective channel is converted to a number of flat channels and the ISI is completely removed in frequency domains [19–25]. The goal in this method is to optimize a certain performance metric of the network, e.g., the total power consumption or the sum rate, by allocating power to different subcarriers at the two transceivers and at the relays in a optimal manner. Note that in this approach OFDM signaling relays may be used in the relays [20] or simple AF relaying can be done in relays [21, 23].

In this paper, we study the problem of optimal allocating power in an OFDM two-way network system with arbitrary number of relays. Our goal is to obtain the optimal power allocation across subcarriers and among nodes by minimizing the total power consumption of the network under two rate constraints for two transceivers. We show that this minimization can be solved by an iterative method. The proposed iterative method alternates between finding the optimal transceivers' subcarrier powers and optimal relay weights for given subcarrier rates and calculating the optimal subcarrier rates for given relay amplification weights. We show that the first subproblem has a semi-closed-form solution and the second subproblem has a water-filling type of solution. To the best of our knowledge the problem we consider in this paper and the solution we provide to this problem are novel and they have not appeared in the literature.

Throughout this paper, we use the following notations: italic fonts are used for scalar variables, boldface small letters are used for vectors and capital letters are used for matrices. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and the Hermitian operators, respectively. $\mathrm{diag}(\mathbf{a})$ yields a diagonal matrix whose diagonal entries are given by the elements of the vector $\mathbf{a}$.The notation $\mathbf{a} \succeq 0$ requires all the entries of the vector $\mathbf{a}$ to be nonnegative. $\mathrm{E}\{\cdot\}$ represents statistical expectation. We also define $(x)^+ = \min\{0, x\}$ in which x is a real-value number.

2. SYSTEM MODEL

We consider a two-way relay network consisting of a pair of transceivers (TRX1 and TRX2) and n_r relay nodes, as shown in Fig. 1. All nodes use OFDM for data transmission over n_c subcarriers. We assume the same set of subcarriers is used for the relays to receive and re-transmit received from TRX1 and TRX2, and thus, all channels to and from each relay are reciprocal. Let p_{1i} and p_{2i} denote the transmit powers allocated to the ith subcarrier at TRX1 and TRX2, respectively, where $i \in \{1, 2, \ldots n_c\}$. We can write the $n_r \times 1$ complex vector $\mathbf{x}_i$ of the signals received by the relays over the ith subcarrier as

$$\mathbf{x}_i = \sqrt{p_{1i}}\mathbf{f}_{1i}s_{1i} + \sqrt{p_{2i}}\mathbf{f}_{2i}s_{2i} + \boldsymbol{\nu}_i \tag{1}$$

where $\boldsymbol{\nu}_i$ is an $n_r \times 1$ complex vector representing the relay noises on the ith subcarrier, s_{1i} (s_{2i}) is the information symbol transmitted by TRX1 (TRX2) over the ith subcarrier, and $\mathbf{f}_{1i}$ ($\mathbf{f}_{2i}$) is the $n_r \times 1$ complex vector of the channel coefficients between TRX1 (TRX2) and the relays corresponding to the ith subcarrier.

We assume that $\mathrm{E}\{|s_{1i}|^2\} = \mathrm{E}\{|s_{2i}|^2\} = 1$. Each relay multiplies the signal it receives over the ith subcarrier, by a complex weight to adjust the amplitude and the phase of the signal received

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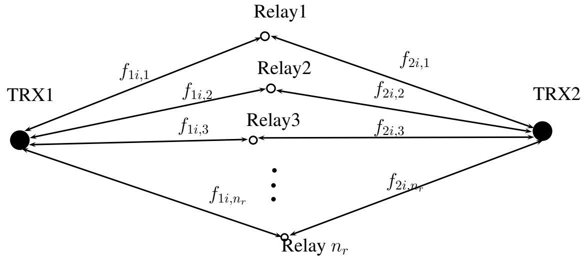

Fig. 1. A two-way relay network.

over that subcarrier. The $n_r \times 1$ vector $\mathbf{t}_i$ of the relay signals transmitted over the ith subcarrier is given by

$$\mathbf{t}_i = \mathbf{w}_i \text{diag}(\mathbf{x}_i) \tag{2}$$

where $\mathbf{w}_i$ is the $n_r \times 1$ vector of the relay complex weights used in the ith subcarrier. The signals received at TRX1 and TRX2 are, respectively, given by

$$\begin{aligned} y_{1i} &= \sqrt{p_{1i}}\mathbf{w}_i^H \mathbf{F}_{1i}\mathbf{f}_{1i}s_{1i} + \sqrt{p_{2i}}\mathbf{w}_i^H \mathbf{F}_{1i}\mathbf{f}_{2i}s_{2i} \\ &\quad + \mathbf{w}_i^H \mathbf{F}_{1i}\boldsymbol{\nu}_i + n_{1i} \qquad (3) \\ y_{2i} &= \sqrt{p_{1i}}\mathbf{w}_i^H \mathbf{F}_{2i}\mathbf{f}_{1i}s_{1i} + \sqrt{p_{2i}}\mathbf{w}_i^H \mathbf{F}_{2i}\mathbf{f}_{2i}s_{2i} \\ &\quad + \mathbf{w}_i^H \mathbf{F}_{2i}\boldsymbol{\nu}_i + n_{2i} \qquad (4) \end{aligned}$$

where n_{1i} and n_{2i} are the measurement noises over the ith subcarrier at TRX1 and TRX2, respectively, and $\mathbf{F}_{ki} \triangleq \text{diag}\{\mathbf{f}_{ki}\}$ for $k \in \{1, 2\}$. At TRX1 and TRX2, self-interference cancelation is performed and the residual signals are given by [26]

$$\tilde{y}_{1i} \triangleq \sqrt{p_{2i}}\mathbf{w}_i^H \mathbf{F}_{1i}\mathbf{f}_{2i}s_{2i} + \mathbf{w}_i^H \mathbf{F}_{1i}\boldsymbol{\nu}_i + n_{1i} \tag{5}$$
$$\tilde{y}_{2i} \triangleq \sqrt{p_{1i}}\mathbf{w}_i^H \mathbf{F}_{2i}\mathbf{f}_{1i}s_{1i} + \mathbf{w}_i^H \mathbf{F}_{2i}\boldsymbol{\nu}_i + n_{2i}. \tag{6}$$

The residual signals $\tilde{y}_{1i}$ and $\tilde{y}_{2i}$ will be processed at TRX1 and TRX2, respectively, to extract s_{2i} and s_{1i}. Using (5) and (6), the ith subcarrier SNRs at TRX1 and TRX2 are respectively given by

$$\text{SNR}_{1i}(p_{2i}, \mathbf{w}_i) = \frac{p_{2i}\mathbf{w}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_i}{1 + \mathbf{w}_i^H \mathbf{D}_1 \mathbf{w}_i}, \tag{7}$$

$$\text{SNR}_{2i}(p_{1i}, \mathbf{w}_i) = \frac{p_{1i}\mathbf{w}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_i}{1 + \mathbf{w}_i^H \mathbf{D}_2 \mathbf{w}_i} \tag{8}$$

where $\mathbf{h}_i \triangleq \mathbf{F}_{1i}\mathbf{f}_{2i} = \mathbf{F}_{2i}\mathbf{f}_{1i}$, $\mathbf{D}_1 = \mathbf{F}_{1i}\mathbf{F}_{1i}^H$, $\mathbf{D}_2 = \mathbf{F}_{2i}\mathbf{F}_{2i}^H$, and we have assumed $\text{E}\{|\nu_i|^2\} = \text{E}\{|n_{1i}|^2\} = \text{E}\{|n_{2i}|^2\} = 1$. The total relay power consumed over the ith subcarrier, can be written as [26]

$$p_{\text{r},i} = \mathbf{w}_i^H (p_{1i}\mathbf{D}_1 + p_{2i}\mathbf{D}_2 + \mathbf{I})\mathbf{w}_i. \tag{9}$$

We define $\mathbf{p}_1 \triangleq [p_{11}\ p_{12} \cdots p_{1n_c}]^T$, $\mathbf{p}_2 \triangleq [p_{21}\ p_{22} \cdots p_{2n_c}]^T$ and $\mathcal{W} \triangleq \{\mathbf{w}_i\}_{i=1}^{n_c}$. The total consumed power in the network is then given by

$$P_T(\mathbf{p}_1, \mathbf{p}_2, \mathcal{W}) = \sum_{i=1}^{n_c} p_{1i} + p_{2i} + p_{\text{r},i}. \tag{10}$$

Our goal is to minimize $P_T(\mathbf{p}_1, \mathbf{p}_2, \mathcal{W})$ subject to some constraints on the data-exchange rate in both direction. The corresponding optimization problem is shown in the next section.

3. TOTAL POWER MINIMIZATION

In this section, we aim to minimize the total transmit power subject to two separate constraints on the rates of the end users. We propose an optimization problem as bellow

$$\begin{aligned} \min_{\mathbf{p}_1, \mathbf{p}_2, \mathcal{W}} \quad & P_T(\mathbf{p}_1, \mathbf{p}_2, \mathcal{W}) \\ \text{subject to} \quad & \frac{1}{2}\sum_{i=1}^{n_c} \log(1 + \text{SNR}_{1i}(p_{2i}, w_i)) \geq r_1^{\max} \\ & \frac{1}{2}\sum_{i=1}^{n_c} \log(1 + \text{SNR}_{2i}(p_{1i}, w_i)) \geq r_2^{\max} \end{aligned} \tag{11}$$

where $r_1^{\max}$ and $r_2^{\max}$ are the minimum required sum-rates for TRX1 and TRX2, respectively. It can be easily shown that, at the optimum, the two rate constraints in (11) must be satisfied with equality. To solve (11), we can rewrite it as

$$\begin{aligned} \min_{\mathbf{r}_1, \mathbf{r}_2} \quad & \sum_{i=1}^{n_c} \min_{p_{1i}, p_{2i}, \mathbf{w}_i} p_{1i} + p_{2i} + p_{\text{r},i}(p_{1i}, p_{2i}, \mathbf{w}_i) \\ \text{subject to} \quad & \frac{1}{2}\log(1 + \text{SNR}_{1i}(p_{2i}, \mathbf{w}_i)) \geq r_{1i} \\ & \frac{1}{2}\log(1 + \text{SNR}_{2i}(p_{1i}, \mathbf{w}_i)) \geq r_{2i} \\ & \mathbf{1}^T \mathbf{r}_1 = r_1^{\max},\ \mathbf{r}_1 \succcurlyeq \mathbf{0} \\ & \mathbf{1}^T \mathbf{r}_2 = r_2^{\max},\ \mathbf{r}_2 \succcurlyeq \mathbf{0} \end{aligned} \tag{12}$$

where the auxiliary variables r_{1i} and r_{2i} are rates of TRX1 and TRX2, respectively, over the ith subcarrier, and we define $\mathbf{r}_j \triangleq [r_{j1}\ r_{j2} \cdots r_{jn_c}]^T$, for $j = 1, 2$. Using the results of [26] and [27], for any given r_{1i} and r_{2i}, the inner minimization in (12) is equivalent to the following minimization problem:

$$\min_{\mathbf{w}_i} \quad (2^{2r_{1i}} + 2^{2r_{2i}} - 2)\xi_i(\mathbf{w}_i) + \mathbf{w}_i^H \mathbf{w}_i. \tag{13}$$

where $\xi_i(\mathbf{w}_i) = \dfrac{(1 + \mathbf{w}_i^H \mathbf{D}_{1i}\mathbf{w}_i)(1 + \mathbf{w}_i^H \mathbf{D}_{2i}\mathbf{w}_i)}{\mathbf{w}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_i}$. We now use (13) to write the optimization problem (12) as

$$\begin{aligned} \min_{\mathbf{r}_1, \mathbf{r}_2, \mathcal{W}} \quad & \sum_{i=1}^{n_c} (2^{2r_{1i}} + 2^{2r_{2i}} - 2)\xi_i(\mathbf{w}_i) + \mathbf{w}_i^H \mathbf{w}_i \\ \text{subject to} \quad & \mathbf{1}^T \mathbf{r}_1 = r_1^{\max},\ \mathbf{r}_1 \succcurlyeq \mathbf{0} \\ & \mathbf{1}^T \mathbf{r}_2 = r_2^{\max},\ \mathbf{r}_2 \succcurlyeq \mathbf{0}. \end{aligned} \tag{14}$$

The optimization problem in (14) is not convex and may not have a computationally efficient solution. We propose a two-step iterative method to tackle this problem. In the first step, we obtain $\mathbf{w}_i$ by solving the minimization problem (13) for a given feasible pair of $\mathbf{r}_1$ and $\mathbf{r}_2$ such that $\mathbf{1}^T \mathbf{r}_1 = r_1^{\max}$ and $\mathbf{1}^T \mathbf{r}_2 = r_2^{\max}$. This problem is guaranteed to have a unique solution given as bellow [28]

$$\mathbf{w}_i^{\circ}(r_{1i}, r_{2i}) = \rho_i(\mu_i \mathbf{D}_{1i} + \mathbf{I})^{-1/2}\mathbf{u}_i \tag{15}$$

where $\rho_i \triangleq \sqrt{\frac{g_{1i} + g_{2i}}{\lambda_i}}$, $g_{ji} \triangleq 2^{2r_{ji}} - 1$, for $j = 1, 2$, and λ_i and $\mathbf{u}_i$ are, respectively, the principal eigenvalue and the normalized principal eigenvector of the matrix

$$\begin{aligned} \mathbf{P}_i \triangleq (\mu_i \mathbf{D}_{1i} + \mathbf{I})^{-1/2}&(\mu_i \mathbf{h}_i \mathbf{h}_i^H - (g_{1i} + g_{2i})\mathbf{D}_{2i}) \\ &(\mu_i \mathbf{D}_{1i} + \mathbf{I})^{-1/2}, \end{aligned} \tag{16}$$

where $\mu_i \in [\frac{(g_{1i}+g_{2i})}{\|\mathbf{f}_{1i}\|^2}, +\infty]$ is the unique solution to the following equation

$$\frac{\mu_i^{-2} - \lambda_i \mathbf{h}_i^H (g_{1i}+g_{2i})\mathbf{D}_{2i} + \lambda_i(\mu_i \mathbf{D}_{1i} + \mathbf{I}))^{-2}\mathbf{D}_{1i}\mathbf{h}_i}{\lambda_i^2 \mathbf{h}_i^H((g_{1i}+g_{2i})\mathbf{D}_{2i} + \lambda_i(\mu_i\mathbf{D}_{1i}+\mathbf{I}))^{-2}(\mu_i\mathbf{D}_{1i}+\mathbf{I})\mathbf{h}_i} = \frac{1}{(g_{1i}+g_{2i})}. \tag{17}$$

Once the optimal value $\mathbf{w}_i$ is obtained as in (15) for a feasible pair of $\mathbf{r}_1$ and $\mathbf{r}_2$, in the second step, we use the so-obtained $\mathbf{w}_i$, and solve the minimization to obtain a new pair of $\mathbf{r}_1$ and $\mathbf{r}_2$. Then, these two steps are repeated until the value of the objective function in (14) converges. It is worth mentioning that for a fixed $\mathbf{w}_i$, the minimization over $\mathbf{r}_1$ and $\mathbf{r}_2$ is a convex problem, as the constraints in (14) linear in $\mathbf{r}_1$ and $\mathbf{r}_2$ and we can show that the Hessian matrix of the objective is diagonal with positive diagonal entries. Thus, we can solve the optimization problem w.r.t. $\mathbf{r}_1$ and $\mathbf{r}_2$, using a water-filling type of solution, as shown below.

For a fixed $\mathbf{w}_i$, we can split the optimization problem (14), into two minimization problems, for $j = 1, 2$, as

$$\begin{aligned} \min_{\mathbf{r}_j} \quad & \sum_{i=1}^{n_c} 2\xi_i 2^{2r_{ji}} \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{r}_j = r_j^{\max},\ \mathbf{r}_j \succcurlyeq \mathbf{0}. \end{aligned} \tag{18}$$

Since $\mathbf{w}_i$ is fixed in the second step of our iterative method, in (18) and hereafter, we use the notion ξ_i instead of $\xi_i(\mathbf{w}_i)$. The Lagrangian function of the optimization problem (18) is given as

$$\mathcal{L}(\mathbf{r}_j, \kappa_j, \boldsymbol{\mu}_j) = \sum_{i=1}^{n_c} 2\xi_i 2^{2r_{ji}} - \kappa_j(\mathbf{1}^T\mathbf{r}_j - r_j^{\max}) - \boldsymbol{\mu}_j^T\mathbf{r}_j \tag{19}$$

where κ_j is the Lagrangian multiplier corresponding to the equality constraint (18) and $\boldsymbol{\mu}_j \triangleq [\mu_{j1} \cdots \mu_{jn_c}]^T$ is the vector of Lagrange multipliers corresponding to the non-negative constraints on the elements of $\mathbf{r}_j$. Differentiating $\mathcal{L}(\mathbf{r}_j, \kappa_j, \boldsymbol{\mu}_j)$ with respect to the ith entry of $\mathbf{r}_j$ yields

$$\frac{\mathcal{L}(\mathbf{r}_j, \kappa_j, \boldsymbol{\mu}_j)}{\partial r_{ji}} = 4\xi_i 2^{2r_{ji}} - \kappa_j - \mu_{ji}. \tag{20}$$

Equating this derivative to zero we obtain that

$$4\xi_i 2^{2r_{ji}} - \kappa_j - \mu_{ji} = 0. \tag{21}$$

It follows from the complementary slackness condition $\mu_{ji} r_{ji} = 0$ that either $\mu_{ji} = 0$ and $r_{ji} \geq 0$, or $\mu_{ji} > 0$ and $r_{ji} = 0$. If $\mu_{ji} = 0$ and $r_{ji} \geq 0$, then we obtain from (21) that

$$r_{ji} = \frac{1}{2}\log_2 \frac{\kappa_j}{4\xi_i} \tag{22}$$

In this case, the condition $r_{ji} \geq 0$ implies that $\lambda_j \geq 4\xi_i$. If $\mu_{ji} > 0$ and $r_{ji} = 0$, then we have

$$0 < \mu_{ji} = -\kappa_j + 4\xi_i. \tag{23}$$

Hence, in this case $\kappa_j < 4\xi_i$ must hold true. Hence, combining these two cases, we can write

$$r_{ji} = \frac{1}{2}\left(\log_2 \frac{\kappa_j}{4\xi_i}\right)^+. \tag{24}$$

To obtain κ_j, we can use the sum-rate constraint for Transceiver j to write

$$\sum_{i=1}^{n_c} r_{ji} = \sum_{i=1}^{n_c} \frac{1}{2}\left(\log_2 \kappa_j - \log_2 4\xi_i\right)^+ = r_j^{\max}. \tag{25}$$

From (25), we can interpret $\log_2 \kappa_j$ as the water level in a water-filling algorithm and it can obtained using a bisection method.

It is worth mentioning that if we have $r_1^{\max} = r_2^{\max}$, then solving (25) leads us to $\kappa_1 = \kappa_2$, and consequently, to $r_{1i} = r_{2i}$, for all i. In other words, the rates of both transceivers at each subcarrier are balanced, so are the corresponding SNRs. This SNR balancing property implies that at that subcarrier, the two transceivers consume half of the total power consumed in the whole network at that subcarrier, and the relays collectively consume the remaining half of the total power at that subcarrier [10]. Hence, we can conclude that the total power used by the two transceivers over all subcarriers is half of the total consumed power and the relays consume the remaining half of the total consumed power.

Special case: In a single-relay network, i.e., when $n_r = 1$, the weight vector $\mathbf{w}_i$ reduces to a scalar w_i, and hence, $\mathcal{W} = \{w_i\}_{i=1}^{n_c}$ is a set of such scalar weights. In addition, the vector $\mathbf{h}_i$ and the matrices $\mathbf{D}_{1i}$ and $\mathbf{D}_{2i}$ become scalars, and they are denoted as h_i, d_{1i}, and d_{2i}, respectively. In this case, the solution to (13) has a closed form. To show this, note that the matrix $\mathbf{P}_i$ in (16) reduces to a scalar, which is equal to its eigenvalue. That is

$$\lambda_i = \frac{(\mu_i d_{1i} - (g_{1i}+g_{2i}))d_{2i}}{\mu_i d_{1i} + 1} \tag{26}$$

and $\mathbf{u}_i$ in (16) reduces to a scalar which is equal to 1. Using (26), we then simplify (17) as

$$\frac{d_{1i}}{d_{2i}} \frac{(g_{1i}+g_{2i})(g_{1i}+g_{2i}+1)}{(\mu_i d_{1i} - (g_{1i}+g_{2i}))^2} = 1. \tag{27}$$

From (27), we can obtain the unique solution for $\mu_i > \frac{g_{1i}+g_{2i}}{d_{1i}}$ as

$$\mu_i = \frac{g_{1i}+g_{2i}}{d_{1i}} + \sqrt{\frac{(g_{1i}+g_{2i})(g_{1i}+g_{2i}+1)}{d_{1i}d_{2i}}} > \frac{g_{1i}+g_{2i}}{d_{1i}}. \tag{28}$$

Using (26) and (28), the parameters w_i, p_{1i} and p_{2i} can be calculated. Therefore, in the case of single relay network, we have a closed form solution for the first stage of the proposed algorithm.

It is worth mentioning that the proposed iterative algorithm is guaranteed to converge, however, convergence to global optimality cannot be claimed at this time.

4. SIMULATION RESULTS

We consider an OFDM system with $N_c = 128$ subcarriers and $N_r = 10$ relays. We assume that the channel coefficients at each subcarrier are i.i.d complex Gaussian random variables with zero means and unit variances. The average channel power gain is assumed to be 1, and therefore $E\{|f_{1i}|^2\} = 1$ and $E\{|f_{2i}|^2\} = 1$. Fig. 2 compares the performance of the proposed power-minimization method for different values of rate constraints (assuming the same rate constraint for both transceivers i.e., $r_1^{\max} = r_2^{\max}$) with an equal power allocation scheme where the total power is equally distributed over all subcarriers and among all nodes (i.e., $p_{1i} = p_{2i} = p_{r,i}$, for $i = 1, 2, ..., N_c$). As can be seen from Fig. 2, our power-minimization method outperforms the equal power allocation method significantly.

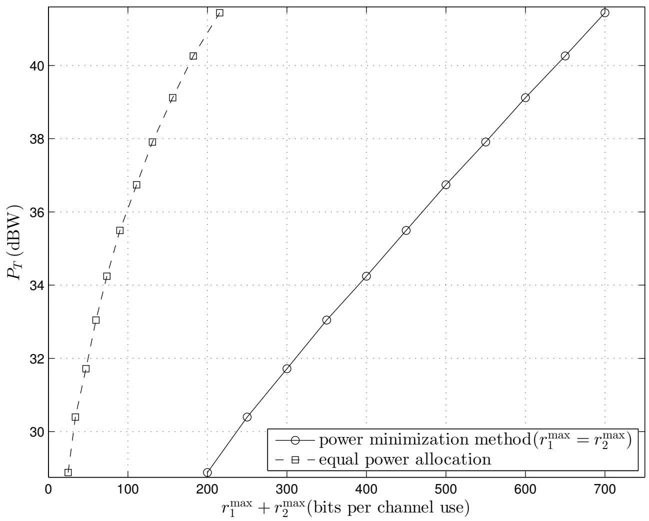

Fig. 2. The average minimum transmit power of the proposed method and that of the equal power allocation scheme versus $r_1^{\max} + r_2^{\max}$.

For example, in Fig. 2, to achieve a sum-rate of around 200 bits per channel use, the equal power allocation method needs at least 12 dB more power compared to our method. Fig. 3, shows the average power of each of two transceivers and that of relays system in our power-minimization method, assuming the same rate constraint for both transceivers i.e., $r_1^{\max} = r_2^{\max}$. As can seen from Fig. 3, the relays consume half of the total power and each transceiver in average (taken over different channel realizations) consumes $\frac{1}{4}$ of the total power. It can be seen that that the power consumption of the relays is always half of the total power for any channel realization but transceiver powers are equal only in average. Fig. 4 shows the average of the minimum transmit power versus iteration number, for $r_1^{\max} = r_2^{\max} = 20$ bits/cu. This figures shows that the convergence of our iterative method is very fast as after 3 to 5 iterations, the average of the minimum transmit power does not change significantly.

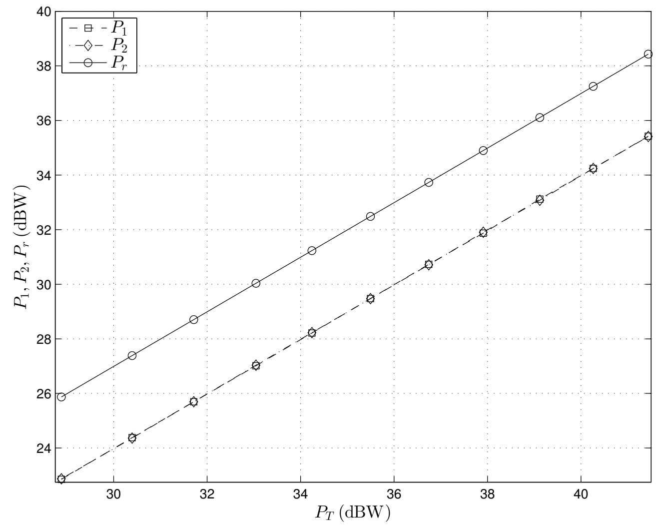

Fig. 3. The average transmit powers of the two transceivers and that of the relays versus the average minimum transmit power.

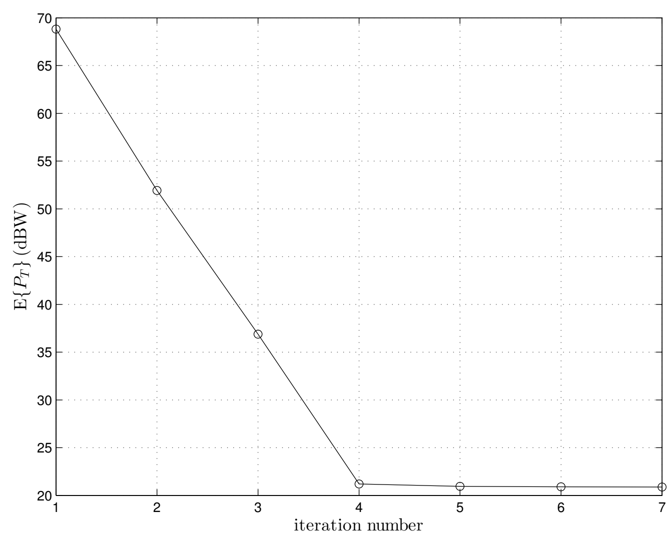

Fig. 4. Average of the minimum transmit power versus iteration number, for $r_1^{\max} = r_2^{\max} = 20$ bits/cu

5. CONCLUSION

we considered a two-way relay-assisted communication system, where OFDM is employed at all nodes to combat inter-symbol-interference caused by the frequency selectivity of the channels. We formulated the problem of optimal power allocation as a power consumption minimization subject to separate rate constraint for each transceiver. Solving the proposed optimization problem, optimal power allocation across subcarriers and among the relays and the two communicating end nodes can be obtained. We solved this minimization by splitting it into two sub-problems and iterating between these these subproblems. We then showed that each of subproblem can be solved using a low complexity method. Our numerical results show the efficiency of the proposed method compared to an equal power allocation scheme, where all subcarriers at all nodes receive the same levels of power and the total power is equal to that consumed in our proposed solution.

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