TIME-DELAY ESTIMATION FOR TOA-BASED LOCALIZATION OF MULTIPLE SENSORS

Richard Heusdens and Nikolay Gaubitch

Signal and Information Processing lab, Delft University of Technology, the Netherlands
email: {r.heusdens, n.d.gaubitch}@tudelft.nl

ABSTRACT

In many applications using multiple sensors, knowledge of the relative positions of the sensors is required. The locations of the sensors can be obtained from measured time-of-arrivals (TOAs) of events generated by sources. Although several TOA-based localization techniques exist, practical TOA measurements are incomplete because they include an unknown internal delay; the time taken from the signal reaching the sensor to that it is registered as received by the capturing device. In order to localize the sensors properly, these internal delays need to be estimated accurately. In this paper we propose a method for estimating the internal delays by using a data fitting technique based on structured total least squares. Under reasonable assumptions we show that the algorithm is guaranteed to converge to the optimal solution and ultimately achieves a quadratic rate of convergence. Experimental results show that the execution time is less than 1% of the execution time of existing methods while attaining an even higher accuracy.

Index Terms— Auto-localization, time-of-arrival, internal delay estimation, structured total least norm

1. INTRODUCTION

Recent developments in the area of wireless sensors enable the construction of (wireless) sensor networks consisting of a large number of nodes, each having a sensing, data processing, and communication component. Sensor networks facilitate the use of spatial signal processing such as computing temperature densities, concentrations of pollution, or beam forming [1]. An example of beam forming can be found in applications such as speech enhancement where multiple microphones can be used to improve both speech quality and speech intelligibility in noisy environments [2, 3]. In many such applications, knowledge of the relative positions of the sensors is required. This requirement is easily satisfied in conventional sensor arrays where the sensors are positioned in a fixed configuration. In (ad-hoc) sensor networks, however, sensor locations are not known a-priori and, in addition, sensors will be added or removed, usually in an unpredictable way. As a consequence, with (ad-hoc) sensor networks, a method to automatically localize the sensors is necessary.

Many techniques for localizing sensors exist such as methods based on received signal strength [4, 5], time-of-arrival (TOA, sometimes called time-of-flight (TOF)) [6, 7], time-difference-of-arrival (TDOA) [7], angle-of-arrival [8, 9], or diffuse noise field coherence in the case of an acoustic sensor network [10]. In many applications, TOA and TDOA based techniques are most popular since they are less vulnerable to multi path, they can be used using both ultrasound and audible frequencies, and they require only one receiver per sensor. In this paper we will focus on TOA based sensor localization, although the results apply to TDOA based techniques as well.

Given the inter-sensor distances (obtained by multiplying the TOAs by the speed of sound), different techniques exist to localize the sensors. One of the earliest methods for auto-localization is multi-dimensional scaling (MDS) [11, 12, 6], which provides the relative configuration of sensors given the distances between all sensors. MDS implicitly assumes that sensors and sources are co-located, which is of limited applicability for sensor networks. Alternatively, source locations can be found by maximum likelihood estimation of the sensor locations via triangularization [13]. Likelihood maximization, however, is a non-convex problem and has, therefore, possibly multiple solutions. An alternative approach particularly interesting for auto localization in large-scale sensor networks was presented in [14] where it is assumed that sources are in the far-field. It was shown that the sensor locations can be computed analytically, through singular value decomposition of the matrix containing the relative arrival times, up to a $d \times d$ invertible matrix where d denotes the dimension of the space the sensors are located in. Finding the appropriate invertible matrix is a non-linear optimization problem, but is of much lower dimension than the original problem. This method was recently generalized [15, 16] such that it no longer relies on the far-field assumption. In addition, it was shown that if one of the sources is co-located with one of the sensors, a completely closed-form solution to the source localization problem exists.

Although several TOA-based localization techniques exist, practical TOA measurements are incomplete because they include an unknown source onset time (the time the source event was generated) and an unknown internal delay (the time taken from the signal reaching the sensor to that it is registered as received by the capturing device). In [17], a solution to this problem is presented which is based on alternating minimization [18], where the source onset times and internal delays are found by an iterative procedure involving low-rank approximation and (non-linear) least-squares optimization. Although alternating minimization is a simple algorithm, it can be slow in terms of convergence rate. In addition, since the problem is non-convex, alternating minimization, without taking special precautions, can easily end up in local minima. An alternative approach has been proposed in [19] where the problem is tackled using truncated nuclear norm regularization. Here, low-rank approximation is formulated as a constrained nuclear norm minimization problem which is solved using the alternating direction method of multipliers (ADMM). Similar to alternating minimization, ADMM can converge slowly to high accuracy [20]. The slow convergence of both alternating minimization and ADMM distinguishes it from algorithms such as Newtons method (or, for constrained problems, interior-point methods), where high accuracy can be attained in a reasonable amount of time (quadratic convergence rate). In this pa-

This work was supported by Google Inc.

per we solve the above mentioned problem using a data fitting technique based on *structured total least squares* [21, 22, 23]. Under reasonable assumptions we will show that the algorithm is guaranteed to converge to the optimal solution and ultimately achieves a quadratic rate of convergence.

This paper is organized as follows. In Section 2 we will formalize the problem at hand and introduce notations. In Section 3 we describe the actual algorithm whereas in Section 4 we present experimental results obtained by computer simulations. Finally, in Section 5, we draw conclusions.

2. PROBLEM FORMULATION

Consider the situation where we have to localize some sources (e.g. loudspeakers) $s_1, \ldots, s_N$ and receivers (e.g. microphones) $r_1, \ldots, r_M$, possibly randomly distributed in a d-dimensional space. Without loss of generality, we will assume $N \geq M$. Moreover, let τ_j and δ_i denote the onset time of source $s_j \in \mathbb{R}^d$ and internal delay of receiver $r_i \in \mathbb{R}^d$, respectively. With this, the measured TOA of the event generated by source s_j at receiver r_i is given by

$$t_{ij} = \frac{\|r_i - s_j\|}{c} + \tau_j + \delta_i, \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm, c is the sound velocity and we assume the measurements are noise free. We will discuss the effect of measurement errors later in Section 4. In this work we will assume that the source onset times are known a-priori (at least with respect to a reference source), so that we can include them in the TOAs t_{ij}. This assumption is met when we generate the source signals at known time instances, for example by using periodically generated wavelets [24]. Hence, without loss of generality, we will assume $\tau_j = 0$ for all j. With this, the inter sensor distances satisfy (assuming $c = 1$)

$$\|r_i - s_j\|^2 = (t_{ij} - \delta_i)^2, \quad \text{for all } i, j.$$

Subtracting the corresponding equations for $i = 1$ and $j = 1$ successively, we arrive at

$$\begin{aligned}(r_i - r_1)^T (s_j - s_1) = \delta_i (t_{ij} - t_{i1}) - \delta_1 (t_{1j} - t_{11}) \\ - (t_{ij}^2 - t_{i1}^2 - t_{1j}^2 + t_{11}^2)/2,\end{aligned} \tag{2}$$

which is bilinear in the sensor and source locations and allows for a factorization similar to what has been proposed in [14, 15, 17]. To do so, let $R = (r_2 - r_1, \ldots, r_M - r_1) \in \mathbb{R}^{d \times (M-1)}$ and $S = (s_2 - s_1, \ldots, s_N - s_1) \in \mathbb{R}^{d \times (N-1)}$. Moreover, let $T \in \mathbb{R}^{(M-1)\times(N-1)}$ be defined as $T_{i-1,j-1,} = -(t_{ij}^2 - t_{i1}^2 - t_{1j}^2 + t_{11}^2)/2$ for $i = 2, \ldots, M, j = 2, \ldots, N$, $W \in \mathbb{R}^{M \times (N-1)}$ as $W_{i,j-1} = t_{ij} - t_{i1}$ for $i = 1, \ldots, M, j = 2, \ldots, N$, and

$$\Delta(\delta) = \begin{pmatrix} -\delta_1 & \delta_2 & 0 & \cdots & 0 \\ -\delta_1 & 0 & \delta_3 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ -\delta_1 & 0 & \cdots & 0 & \delta_M \end{pmatrix} \in \mathbb{R}^{(M-1)\times M}, \tag{3}$$

where $\delta = (\delta_1, \ldots, \delta_M)^T \in \mathbb{R}^{M \times 1}$. With this we can express (2) as

$$R^T S = T + \Delta(\delta) W. \tag{4}$$

A key observation in the localization problem is that, in the absence of measuring errors, the matrix $R^T S$ has rank r, where $r \leq d$ denotes the dimension of the linear manifold the sensors are located in. As a consequence, $R^T S$ has a singular value decomposition given by

$$R^T S = U \Sigma V^T,$$

with $U \in \mathbb{R}^{(M-1)\times r}, V \in \mathbb{R}^{(N-1)\times r}$ and $\Sigma \in \mathbb{R}^{r \times r}$, which determines R up to a $r \times r$ invertible matrix. That is, the receiver locations are given by $R = (UC)^T$ where the matrix C can be obtained by non-linear optimization [14, 15] or, in the case one source is co-located with one of the receivers, by least-squares approximation [15]. In the case where the sources are located in the same linear manifold the receivers are located in, the source locations are given by $S = C^{-1} \Sigma V^T$. Note that the locations thus obtained are unique up to a unitary transform (rotation, reflection) and translation. In order to determine the sensor positions, we first need to determine the internal delays δ after which we can compute R and S using the method described above.

3. ESTIMATING INTERNAL DELAYS

As mentioned before, the matrix $R^T S$ has rank r. As a consequence, the unknown time delays can be found by searching for $\delta \in \mathbb{R}^M$ such that $T + \Delta(\delta) W$ is of rank r. In [17], the delays are found by an alternating minimization procedure involving rank-r approximation by truncated singular value decomposition followed by least-squares optimization of δ to enforce the structure (4). In [19], the rank reduction is obtained by solving a regularized nuclear norm minimization problem, where the structure (4) is enforced by introducing proper constraints to the set of feasible solutions. Solving the unknown internal delay problem using alternating minimization or truncated nuclear norm regularization can be very slow and can easily end up in local minima. In this paper we solve the above mentioned problem by finding a rank-r approximation of $R^T S$ that preserves its specific structure by using a data fitting technique based on *structured total least squares* [21, 22, 23].

Rank approximation problems have an analytic solution in terms of the singular value decomposition (SVD). The optimal solution is given by the Eckart-Young-Mirsky theorem [25] which states that the best rank-r approximation of a matrix X having SVD $X = U \Sigma V^T$ is given by $X_r = U \Sigma_r V^T$, where $\Sigma_r = \operatorname{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots, 0)$ is obtained from Σ by setting the singular values $\sigma_n = 0$ for $n > r$. A problem that is closely related is the *total least squares* (TLS) problem [26]. Given $A \in \mathbb{R}^{m \times r}, B \in \mathbb{R}^{m \times n}$, $m > r$, we would like to find $X \in \mathbb{R}^{r \times n}$ that minimizes error matrices E and R for A and B, respectively. That is, we want to find

$$\arg\min_{X,E} \|[E\ R]\|_F, \tag{5}$$

where $R = (A + E)X - B$. The matrix $[A + E\ B + R]$ is rank deficient since $B + R \in \operatorname{range}(A + E)$ and the minimum perturbation $[E\ R]$ (in the Frobenius-norm sense) is given by [26]

$$[E\ R] = -\sum_{i=r+1}^{p} \sigma_i u_i v_i^T,$$

where $\sigma_1, \ldots, \sigma_p, p = \min(m, n + r)$, are the singular values of $[A\ B]$ and u_i, v_i are the corresponding left and right singular vectors, respectively. As a consequence, the rank deficient matrix $[A + E\ B + R]$ can be expressed as

$$[A + E\ B + R] = \sum_{i=1}^{r} \sigma_i u_i v_i^T,$$

which is the best rank-r approximation of $[A\ B]$ by the Eckart-Young-Mirsky theorem. The *structured total least norm* (STLN) is an extension to TLS in the sense that it permits a known structure in $[A\ B]$ to be preserved in $[A+E\ B+R]$. Requirements of this type are important in, for example, system identification problems, where the matrix $[A\ B]$ has Toeplitz or Hankel structure. In this section, we will use STLN to find a low-rank approximation of $T+\Delta(\delta)W$ that preserves its specific structure.

In order to find a rank-r matrix that is of the form (4), we write $T=[A\ B]$, where $A\in\mathbb{R}^{(M-1)\times r}, B\in\mathbb{R}^{(M-1)\times(N-1-r)}$ and $W=[F\ G]$, where $F\in\mathbb{R}^{M\times r}, G\in\mathbb{R}^{M\times(N-1-r)}$. Since both T and W are given (they contain the measured TOAs), the perturbation matrix for T is given by EW where $E(\delta)$ is of the form (3). We will assume that T and W have full rank and, in addition, that $\mathrm{rank}(A)=\mathrm{rank}(F)=r$. With this, our structured rank-r approximation problem can be expressed as a constrained minimization problem given by

$$\begin{aligned}&\min_{X,\delta}\|EW\|_F,\\&\text{subject to } (A+EF)X=B+EG.\end{aligned}\tag{6}$$

In order to solve (6), we need the precise relation between E and δ. Let $P_i\in\mathbb{R}^{M\times M}$, $i=1,\ldots,M-1$, be defined as

$$P_i(j,k)=\begin{cases}1, & \text{if } E(i,k)=\delta_j,\\ 0, & \text{otherwise,}\end{cases}$$

so that $E=(P_1^T\delta,\ldots,P_{M-1}^T\delta)^T$. As a consequence, we have

$$\begin{aligned}\|EW\|_F^2&=\|\delta^T(P_1W,\ldots,P_{M-1}W)\|_2^2\\&=\delta^TZ\delta,\end{aligned}$$

where $Z=\sum_{i=1}^{M-1}P_iWW^TP_i^T$. Since W has full rank M, we conclude that $\|EW\|_F^2\geq 0$ with equality if and only if $E=O$, and thus $\delta^TZ\delta\geq 0$ with equality if and only if $\delta=o$, where O and o denote the zero element in $\mathbb{R}^{M\times(M-1)}$ and $\mathbb{R}^M$, respectively. Hence, $Z\succ 0$ (positive definite) and symmetric having an eigenvalue decomposition $Q\Lambda Q^T$ with Q unitary and $\Lambda\succ 0$. As a consequence, Z has a (unique) decomposition $Z=D^TD$ with $D=Q\Lambda^{\frac{1}{2}}Q^T\succ 0$ symmetric. With this, (6) can be expressed as

$$\begin{aligned}&\min_{X,\delta}\|D\delta\|_2,\\&\text{subject to } (A+EF)X=B+EG.\end{aligned}\tag{7}$$

By introducing a sufficiently large penalty value ω, we can solve (7) by minimizing the (non-convex) unconstrained problem

$$\min_{X,\delta}\left\|\begin{array}{c}D\delta\\ \omega\mathrm{vec}(\rho(X,\delta))\end{array}\right\|_2,\tag{8}$$

where $\rho(X,\delta)=B+EG-(A+EF)X$ and $\mathrm{vec}(\cdot)$ is the vectorization operator; it stacks all columns of its input argument to form a single column vector.

Solving (8) can be stated as minimizing the differentiable function ξ given by

$$\xi=\frac{1}{2}\|D\delta\|_2^2+\frac{\omega^2}{2}\|\mathrm{vec}(\rho(X,\delta))\|_2^2.$$

The first-order optimality condition for a (local) minimum of (8) is that the gradient of ξ with respect to the vector variables δ and $\mathrm{vec}(X)$ vanishes. In order to compute the gradient, we need to express EFX and EG in terms of δ explicitly. Let $Z=(z_1,\ldots,z_n)\in\mathbb{R}^{M\times n}$ for some $n>0$. We then have

$$\begin{aligned}Ez_i&=(P_1^T\delta,\ldots,P_{M-1}^T\delta)^Tz_i\\&=(P_1z_i,\ldots,P_{M-1}z_i)^T\delta\\&=\Theta_{z_i}\delta,\end{aligned}$$

so that $\mathrm{vec}(EZ)=\Theta_Z\delta$ with $\Theta_Z=(\Theta_{z_1}^T,\ldots,\Theta_{z_n}^T)^T$. Moreover, let $q=N-1-r$ and $\mathrm{diag}_q(Z)$ denote a block-diagonal matrix with q block-diagonal elements Z. With this, $\mathrm{vec}(\rho(X,\delta))$ can be expressed as

$$\begin{aligned}\mathrm{vec}(\rho(X,\delta))&=\mathrm{vec}(B)+\Theta_G\delta-\mathrm{diag}_q(A+EF)\mathrm{vec}(X),\\&=\mathrm{vec}(B)+\Theta_{G-FX}\delta-\mathrm{diag}_q(A)\mathrm{vec}(X).\end{aligned}$$

Hence, the gradient of ξ is given by

$$\begin{aligned}\nabla_\delta\xi&=D^TD\delta+\omega^2\Theta_{G-FX}^T\mathrm{vec}(\rho(X,\delta)),\\\nabla_{\mathrm{vec(X)}}\xi&=-\omega^2\mathrm{diag}_q(A+EF)^T\mathrm{vec}(\rho(X,\delta)),\end{aligned}$$

or, equivalently, by

$$\nabla\xi=\begin{pmatrix}\nabla_\delta\xi\\ \nabla_{\mathrm{vec}(X)}\xi\end{pmatrix}=J_\xi(X,\delta)^T\begin{pmatrix}D\delta\\ \omega\mathrm{vec}(\rho(X,\delta))\end{pmatrix},\tag{9}$$

where $J_\xi(X,\delta)$ is the Jacobian matrix of ξ given by

$$J_\xi(X,\delta)=\begin{pmatrix}D & O\\ \omega\Theta_{G-FX} & -\omega\mathrm{diag}_q(A+EF)\end{pmatrix}.$$

We want to find X and δ such that (9) is zero, which can be done iteratively using the Gauss-Newton method. In that case, the steps $\Delta\delta$ and ΔX towards the minimum are given by

$$J_\xi(X,\delta)^TJ_\xi(X,\delta)\begin{pmatrix}\Delta\delta\\ \Delta X\end{pmatrix}=-J_\xi(X,\delta)^T\begin{pmatrix}D\delta\\ \omega\mathrm{vec}(\rho(X,\delta))\end{pmatrix},\tag{10}$$

where we can start the iterations with initial values $E=0$ and $X=A^+B$, where A^+ denotes the pseudo-inverse of A. Despite the fact that only first-order derivatives are used, the Gauss-Newton method will ultimately achieve a quadratic rate of convergence [27]. It can be shown [24] that under reasonable assumptions (W having full rank and $\mathrm{rank}(A+EF)=r$) $J_\xi^TJ_\xi\succ 0$, which implies that (10) has a unique solution which will be zero if and only if the left-hand side of (10) is zero. Hence, convergence of the algorithm ($\Delta\delta,\Delta X\to 0$) is equivalent to reaching a (local) minimum of ξ. The resulting algorithm for solving (8) is summarized in Algorithm 1.

Some remarks are in place here. First of all, the fact that $J_\xi^TJ_\xi\succ 0$, regardless of the values of X and δ, implies that if $\|\rho\|_F$ is sufficiently small with respect to ω, we have that the Hessian matrix of ξ $\nabla^2\xi\succ 0$ for all X,δ [27], and the algorithm converges to the global minimum at a quadratic rate of convergence. In practical situations this requirement is always met since $\rho^{(k)}$, the residual at iteration k, satisfies $\|\rho^{(k)}\|_F\leq\|\rho^{(0)}\|_F=\|B+EG-(A+EF)A^+B\|_F$ which will be relatively small for practical values of $\omega\gg 1$. Secondly, the assumptions for which $J_\xi^TJ_\xi\succ 0$ hold, namely W having full rank and $\mathrm{rank}(A+EF)=r$, are generally satisfied in practical situations since both W, A and F contain measured TOAs and can be made rank deficient only by special construction.

Algorithm 1 Gauss-Newton method for solving (8).

Set $E = 0, \delta = o$.
Compute $X = A^+B$ and $\rho(X, \delta) = B + EG - (A + EF)X$.
while $(\|\Delta\delta\| + \|\Delta X\|) >$ threshold **do**

$$\begin{pmatrix} \Delta\delta \\ \Delta X \end{pmatrix} = J_\xi^+(X, \delta) \begin{pmatrix} D\delta \\ \omega \mathrm{vec}(\rho(X, \delta)) \end{pmatrix};$$

$\delta := \delta + \Delta\delta$;
$X := X + \Delta X$;
Construct E from δ and Θ_{G-FX} from X;
Compute $\rho(X, \delta)$ and update $J_\xi(X, \delta)$;
end while

STLS		AM [17]		TNNR [19]	
τ(s)	$\|\delta - \hat{\delta}\|_2^2$	τ(s)	$\|\delta - \hat{\delta}\|_2^2$	τ(s)	$\|\delta - \hat{\delta}\|_2^2$
0.03	$6.8\ 10^{-17}$	5.46	$1.0\ 10^{-12}$	x	x
0.03	$4.2\ 10^{-14}$	5.12	$1.0\ 10^{-12}$	32.57	$8.2\ 10^{-13}$
0.03	$1.6\ 10^{-13}$	1.05	$1.0\ 10^{-12}$	11.00	$3.3\ 10^{-13}$
0.03	$9.7\ 10^{-13}$	x	x	4.79	$7.7\ 10^{-13}$
0.03	$7.1\ 10^{-18}$	0.60	$1.0\ 10^{-12}$	5.23	$7.1\ 10^{-13}$
0.04	$8.6\ 10^{-16}$	0.97	$1.0\ 10^{-12}$	5.47	$5.8\ 10^{-13}$
0.02	$5.7\ 10^{-20}$	1.63	$1.0\ 10^{-12}$	5.74	$8.8\ 10^{-13}$
0.08	$8.7\ 10^{-15}$	1.16	$1.0\ 10^{-12}$	14.97	$8.5\ 10^{-13}$
0.03	$3.2\ 10^{-14}$	2.70	$1.0\ 10^{-12}$	x	x
0.02	$2.1\ 10^{-16}$	0.52	$1.0\ 10^{-12}$	6.06	$3.9\ 10^{-13}$

Table 1. Convergence results for the STLN, alternating minimization (AM) [17] and truncated nuclear norm regularization (TNNR) [19] algorithm.

4. EXPERIMENTAL RESULTS

In this section we present experimental results obtained by computer simulations. The first experiment compares the convergence behavior of the proposed method to the performance of the alternating minimization (AM) approach presented in [17], and the truncated nuclear norm regularization (TNNR) algorithm of [19]. To do so, we randomly distributed 7 receivers and 25 sources uniformly in a room of dimensions $4 \times 4 \times 2.5$ m. For each receiver, internal delays are generated according to a uniform distribution over the time interval $[0, 100]$ ms. The sound velocity was set to $c = 343$ m/s and the penalty value ω was chosen to be 10^9. Table 1 shows the results of 10 such experiments for the proposed structured total least squares (STLS) algorithm, the AM algorithm, and the TNNR algorithm. For each algorithm the total execution time τ(s) (left column) and the resulting error $\|\delta - \hat{\delta}\|_2^2$ (right column) are presented. In order to make a fair comparison between the methods, we stopped the iterations whenever the error in δ is less that 10^{-12}. The reason that the STLS results have a much higher accuracy than 10^{-12} is because they only took 5-10 iterations, each iteration improving the result at a quadratic rate. The AM and TNNR algorithm, on the other hand, typically take a few thousands of iterations. In some cases, the AM and TNNR algorithm did not converge within the maximum number of allowed iterations (which was set to 100.000), which is indicated by the symbol x in the table. Clearly, the STLN algorithm has superior convergence properties as compared to both the AM and TNNR algorithm where we have a gain in execution time of a factor larger than 100.

The second experiment shows the sensitivity of the proposed method to measurement errors. To do so, we assume that measurement errors are due to a finite sampling rate of the sensor hardware. That is, we can measure the TOAs up to an accuracy of $\pm T_s/2$, where T_s denotes the sampling period. We randomly distributed 8 receivers and 15 sources uniformly in a room of dimensions $4 \times 4 \times 2.5$ m, where we co-located the first source with one of the receivers. This is done since in that case there exists a closed-form expression for the source and receiver locations so that we exclude possible errors introduced by the non-linear optimization needed in case we did not co-locate one of the sources. For each receiver, internal delays are generated according to a uniform distribution over the time interval $[0, 10]$ ms and the sound velocity was set to $c = 343$ m/s. The measured TOAs are then computed as

$$t_{ij} = \frac{\|r_i - s_j\|}{c} + \delta_i + \nu_{ij},$$

where the measurement errors ν_{ij} are drawn randomly from a uniform distribution over the interval $[-T_s/2, T_s/2]$. The locations of sources and receivers were computed using the method described in [15]. Figure 1 shows the results (averaged over 100 realizations) in terms of the accuracy of the location (top plot) and the accuracy of the internal delays (bottom plot) and as a function of sampling frequency where the error bars indicate the standard deviation. At a sampling frequency of $f_s = 48$ KHz, the average error in sensor location is less than 0.6 mm (standard deviation less than 0.4 mm).

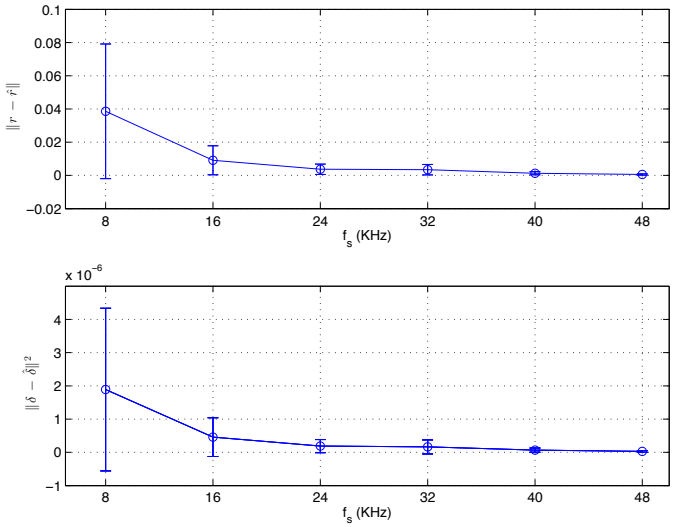

Fig. 1. Accuracy of the internal delays (top plot) and location (bottom plot) as a function of sampling frequency.

5. CONCLUSIONS

In this paper we considered the estimation of unknown internal delays in time-of-arrival measurements for localization of multiple sensors. In order to localize the sensors properly, the internal delays need to be estimated accurately. Existing methods for solving this problem are based on alternating minimization or the alternating direction method of multipliers, which both are known to converge slowly to accurate solutions. As an alternative, we proposed a method based on structured total least squares. We showed that under reasonable assumptions the algorithm is guaranteed to converge to the optimal solution and ultimately achieves a quadratic rate of convergence. Experimental results based on computer simulations showed the superior convergence behavior of the algorithm; the execution time is less than 1% of the execution time of existing methods while attaining an even higher accuracy.

6. REFERENCES

[1] S. Haykin and K.J. Ray Liu, Eds., *Handbook on array processing and sensor networks*, Wiley, 2009.

[2] M.S. Brandstein and D.B. Ward, Eds., *Microphone Arrays: Signal Processing Techniques and Applications*, Springer-Verlag, Berlin, Germany, 2001.

[3] K. Eneman et al., "Evaluation of signal enhancement algorithms for hearing instruments," in *Proc. European Signal Processing Conference 2008*, Lausanne, Switzerland, August 25-29 2008.

[4] M. Chen, Z. Liu, L.W. He, P. Chou, and Z. Zhang, "Energy-based position estimation of microphones and speakers for ad hoc microphone arrays," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz, NY, USA, 2007.

[5] Z. Liu, Z. Zhang, L.W. He, and P. Chou, "Energy-based sound source localization and gain normalization for ad hoc microphone arrays," in *Proc. ICASSP 2007*, Honolulu, Hawaii, USA, April 15-20 2007, pp. 761–764, Vol. II.

[6] J.N. Ash and R.L. Moses, *Self-localization of sensor networks*, chapter 13, pp. 409 – 437, In Haykin and Ray Liu [1], 2009.

[7] R. Kaune, "Accuracy studies for tdoa and toa localization," in *15th International Conference on Information Fusion (FUSION) 2012*, 2012, pp. 408–415.

[8] R. Peng and M.L. Sichitiu, "Angle of arrival localization for wireless sensor networks," in *3rd Annual IEEE Communications Society on SECON 2006*, 2006, pp. 374–382, Vol. I.

[9] P. Brida, J. Machaj, J. Benikovsky, and J. Duha, "A new complex angle of arrival location method for ad hoc networks," in *Proc. 7th Workshop on Positioning, Navigation and Communication (WPNC) 2010*, 2010, pp. 284–290.

[10] I. McCowan, M. Lincoln, and I. Himawan, "Microphone array shape calibration in diffuse noise fields," *IEEE Trans. on Audio, Speech and Language Processing*, vol. 16, no. 3, pp. 666–670, March 2008.

[11] W.S. Torgerson, "Multidimensional scaling: I. theory and method," *Psychometrika*, vol. 17, no. 4, pp. 401–419, December 1952.

[12] S.T. Birchfield and A. Subramanaya, "Microphone array position calibration by basis-point classical multidimensional scaling," *IEEE Trans. on Speech and Audio Processing*, vol. 13, no. 5, pp. 1025–1034, September 2005.

[13] Y.T. Chan, H.Y.C. Hang, and P.C. Ching, "Exact and approximate maximum likelihood localization algorithms," *IEEE Trans. on Vehicular Technology*, vol. 55, no. 1, pp. 10–16, January 2006.

[14] S. Thrun, "Affine structure from sound," in *Advances in Neural Information Processing Systems 18*, Y. Weiss, B. Schölkopf, and J. Platt, Eds., pp. 1353–1360. MIT Press, Cambridge, MA, 2006.

[15] M. Crocco, A. Del Bue, and V. Murino, "A bilinear approach to the position self-calibration of multiple sensors," *IEEE Trans. on Signal Processing*, vol. 60, no. 2, pp. 660 – 673, February 2012.

[16] M. Crocco, A. Del Bue, M. Bustreo, and V. Murino, "A closed form solution to the microphone position self-calibration problem," in *Proc. ICASSP 2012*, Kyoto, Japan, March 25-30 2012, pp. 2597 – 2600.

[17] N.D. Gaubitch, W.B. Kleijn, and R. Heusdens, "Auto-localization in ad-hoc microphone arrays," in *Proc. ICASSP 2013*, Vancouver, Canada, May 26-31 2013, pp. 106–110.

[18] I. Csiszár and G. Tusnády, "Information geometry and alternating minimization procedures," *Statistics and Decisions*, pp. 205–237, 1984, Supplemental Issue Number 1.

[19] F. Jiang, Y. Kuang, and K. Åström, "Time delay estimation for TDOA self-calibration using truncated nuclear norm regularization," in *Proc. ICASSP 2013*, Vancouver, Canada, May 26-31 2013, pp. 3885–3889.

[20] S. Boyd, N. Parikh, E. Chu, B. Paleato, and J. Eckstein, *Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers*, vol. 3, pp. 1–122, 2010, Foundations and Trends in Machine Learning.

[21] J.B. Rosen, H. Park, and J. Glick, "Total least norm formulation and solution for structured problems," *SIAM Journal on Matrix Analysis and Applications*, vol. 17, no. 1, pp. 110 – 128, 1996.

[22] S. van Huffel, H. Park, and J.B. Rosen, "Formulation and solution of structured total least norm problems for parameter estimation," *IEEE Trans. on Signal Processing*, vol. 44, no. 10, pp. 2464–2474, October 1996.

[23] H. Park, L. Zhang, and J.B. Rosen, "Low rank approximation of a Hankel matrix by structured total least squares," *BIT*, vol. 39, no. 4, pp. 757 – 779, 1999.

[24] R. Heusdens and N.D. Gaubitch, "Time-delay estimation for TOA-based localization of multiple sensors," Technical Report TR-2013-03, Delft University of Technology, Delft, the Netherlands, 2013.

[25] C. Eckart and G. Young, "The approximation of one matrix by another of lower rank," *Psychometrika*, vol. 1, pp. 211–218, 1936.

[26] G.H. Golub and C.F. van Loan, "An analysis of the total least squares problem," *SIAM J. Num. Anal.*, vol. 17, pp. 883 – 893, 1980.

[27] P.E. Gill, W. Murray, and M.H. Wright, *Practical Optimization*, Academic Press, London, first edition, 1981.