ROBUST ADAPTIVE BEAMFORMING BASED ON RESPONSE VECTOR OPTIMIZATION

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ABSTRACT

In this paper, a robust beamforming method is proposed. This method can be viewed as a LCMV beamformer with its response vector further optimized. To generate a better response vector, it is first established as a non-convex quadratically constrained quadratic programming problem, and then is transformed into a semidefinite programming problem which can be efficiently and exactly solved via semidefinite relaxation. This method outperforms the traditional LCMV beamformer with lower sidelobe and well-maintained mainbeam. Moreover, the computation complexity is negligible because the size of the response vector is relatively small. Simulation examples are carried out to demonstrate the effectiveness of the proposed method.

Index Terms—*robust adaptive beamforming, linear constrained minimum variance (LCMV) beamformer; response vector optimization; semidefinite relaxation (SDR).*

1. INTRODUCTION

It is well known that the minimum variance distortionless response (MVDR) beamformer is designed with precious knowledge of the target direction and covariance matrix. Therefore it is sensitive to errors and a robust adaptive beamformer is needed in practical applications. The finite sample support, imprecise knowledge of the desired signal steering vector, and the presence of the desired signal component in training data are the main causes of its performance degradation in adaptive beamforming.

Many existing methods have been developed to overcome the problem aforementioned. One of the most popular methods is the diagonal loading method [1] which can improve the robustness of the MVDR beamformer. However, it is not clear how to choose the diagonal loading factor. In recent years, several uncertainty-set based beamformers are proposed in [2]-[4] with the diagonal loading factor accurately determined. Eigenspace-based beamformer could enjoy robustness against target steering vector errors. However, it is essentially ineffective when the dimension of the signal-plus-interference subspace is high or the SNR is low due to high probability of subspace swaps [5]. Another popular method is so-called linear constrained minimum variance (LCMV) beamformers [6]-[7]. These methods can be used to broaden the mainbeam or the notch. However, the main limitation is its relatively high sidelobes, probably leading to unacceptable output of unknown interference. In [8], the linear constraints are used in robust Capon beamforming to allow arbitrary array steering vector errors. In [9], a phase response constrained LCMV beamformer is proposed with its sidelobe of the beampattern lower than traditional LCMV beamformer. In [10], a robust adaptive beamformer is proposed using two quadratic constraints to force the magnitude of two steering vector exceed unity. This method can be also viewed as an LCMV beamformer with its response vector further optimized.

In this paper, a novel robust approach is proposed based on response vector optimization. Here the number of constrained point can be any integer, while only two is feasible for method proposed in [10]. Firstly, using the adaptive weight of LCMV beamformer we establish the object function with respect to the response vector. Then we consider forcing the mainbeam response of the beamformer to exceed unity, deriving a non-convex quadratic constrained quadratic programming (QCQP) problem. Then the problem is transformed into a semidefinite programming (SDP) using semidefinite relaxation (SDR) with the rank-one constraint ignored. Therefore, this method can enjoy low sidelobe and well-maintained mainbeam of the beampattern.

2. PROBLEM FORMULATION

Consider a monostatic linear array radar with N omni-directional antenna elements and the narrowband signal received by the antenna array can be expressed as

$$\boldsymbol{x}(t)=\boldsymbol{s}(t)+\boldsymbol{i}(t)+\boldsymbol{n}(t) \tag{1}$$

where the desired signal can be expressed as $\boldsymbol{s}(t)=s(t)\boldsymbol{a}_s$, $\boldsymbol{a}_s$ is the corresponding array steering vector. The output of the beamformer can be expressed as

$$y(t)=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(t) \tag{2}$$

The weight vector can be derived as $\boldsymbol{w}=\mu\boldsymbol{R}^{-1}\boldsymbol{a}$ for MVDR beamformer, i.e., $\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w} \quad s.t. \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}=1$, where $\boldsymbol{R}$ is the covariance matrix of the interference and noise. In practice,

This work was supported by the Major State Basic Research Development Program of China (973 Program) (Grant 2010CB731903) and the National Nature Science Foundation of China (NSFC) (Grant 61231017).

the covariance matrix is usually acquired by averaging the outer-product of the snapshot in range.

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{x}_l \boldsymbol{x}_l^H \quad (3)$$

where L is the number of sample support. It is known to all that the MVDR-SMI beamformer is sensitive to even small errors. To overcome this problem, LCMV beamformer is proposed to impose multiple unity-gain constraints for a small spread of angles around the nominal look directions [6].

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \qquad s.t. \boldsymbol{w}^{\mathrm{H}} \boldsymbol{C} = \boldsymbol{f}^{\mathrm{T}} \quad (4)$$

where $\boldsymbol{C}$ is the constrain matrix consisted by the M array responses of the corresponding constrained directions. And $\boldsymbol{f}$ is the all-one response vector with each element specifying the desired unity-gain response of each point. The solution of (4) can be expressed as

$$\boldsymbol{w} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\left(\boldsymbol{C}^{\mathrm{H}}\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\right)^{-1}\boldsymbol{f} \quad (5)$$

A significant limitation of the LCMV beamformer is that the sidelobe of the beamformer is relatively high, which induces high sensitivity to the unknown interference and the mainbeam of the beampattern is destroyed.

3. THE PROPOSED ROBUST METHOD BASED ON RESPONSE VECTOR OPTIMIZATION

We will explore the performance improvement by replacing the response vector with an optimal candidate. Here we consider the case that each element of the response vector is complex-valued, thus (4) can be written as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \qquad s.t. \boldsymbol{w}^{\mathrm{H}} \boldsymbol{C} = \boldsymbol{u}^{\mathrm{T}} \quad (6)$$

where $\boldsymbol{u} = \left(u_1, u_2, \cdots, u_M\right)^{\mathrm{T}} = \left(\alpha_1 e^{j\beta_1}, \alpha_2 e^{j\beta_2}, \cdots, \alpha_M e^{j\beta_M}\right)^{\mathrm{T}}$ is the M-dimensional complex-valued response vector and the adaptive weight is

$$\boldsymbol{w} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\left(\boldsymbol{C}^{\mathrm{H}}\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\right)^{-1}\boldsymbol{u}^{*} \quad (7)$$

where the superscript '*' indicates conjugate operator. This weight takes the similar form as in (5). Substitute (7) into the object function in (6), yielding

$$f\left(\boldsymbol{u}\right) = \boldsymbol{u}^{\mathrm{T}}\left(\boldsymbol{C}^{\mathrm{H}}\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\right)^{-1}\boldsymbol{u}^{*} \quad (8)$$

Thus we hope to further decrease the object function under some constraints on the response vector $\boldsymbol{u}$. The motivation comes from the fact that formula (8) contains most of the energy of the interference. Assume that $\boldsymbol{P} = \left(\boldsymbol{C}^{\mathrm{H}}\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\right)^{-1}$, thus $\boldsymbol{P}$ is Hermitian matrix, i.e., $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{H}}$. And also we note that $\boldsymbol{P}$ is a Hermitian semidefinite positive matrix, i.e., $\boldsymbol{P} \succ 0$.

As aforementioned, traditional LCMV beamformer suffers from high sidelobe. This is because the response vector is not matched with the constraint matrix C. Therefore, we treat the response vector as variables and try to search out the suboptimal response vector.

Let consider minimizing $f(\boldsymbol{u})$ and we assume that the magnitude of each element of the response vector exceeds unity. Therefore it can be derived that

$$\min_{\boldsymbol{u}} \boldsymbol{u}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{u}^{*} \qquad s.t. \ \boldsymbol{u}^{\mathrm{H}} \boldsymbol{B}_i \boldsymbol{u} \geq 1, \quad i = 1, 2, \cdots, M \quad (9)$$

where $\boldsymbol{B}_i=\{b_{p,q}\}$ and $b_{p,q}$ is 1 if and only if both p and q both equal to i, otherwise, $b_{p,q}$ is 0. That is

$$\boldsymbol{B}_i = \left\{b_{p,q} \middle| b_{p,q} = 1, \ p = i \text{ and } q = i; b_{p,q} = 0, \quad p \neq i \text{ or } q \neq i\right\} \quad (10)$$

This is a non-convex QCQP problem. Under the constraint that the mainbeam response is no less than 1, we can further decrease the sidelobe output energy.

The problem can be expressed in higher dimension subspace. The object of (9) can be expressed as

$$\boldsymbol{u}^{\mathrm{H}} \boldsymbol{P} \boldsymbol{u} = \mathrm{tr}\left\{\boldsymbol{u}^{\mathrm{H}} \boldsymbol{P} \boldsymbol{u}\right\} = \mathrm{tr}\left\{\boldsymbol{P}\boldsymbol{U}\right\} \quad (11)$$

where the matrix $\boldsymbol{U}$ is defined as $\boldsymbol{U} = \boldsymbol{u}\boldsymbol{u}^{\mathrm{H}}$, which is Hermitian positive semidefinite and denoted as $\boldsymbol{U} \succeq 0$. It is clear that the rank of $\boldsymbol{U}$ is one, i.e., $\mathrm{rank}\left(\boldsymbol{U}\right) = 1$. Therefore (9) can be equivalently expressed as

$$\min_{\boldsymbol{U}} \mathrm{tr}\left\{\boldsymbol{P}\boldsymbol{U}\right\} \qquad s.t. \ \begin{cases} \mathrm{tr}\left\{\boldsymbol{B}_i \boldsymbol{U}\right\} \geq 1, & i = 1, 2, \cdots, M \\ \boldsymbol{U} \succeq 0, rank\left(\boldsymbol{U}\right) = 1 \end{cases} \quad (12)$$

From (12), we can see that this problem is linear with respect to matrix variable $\boldsymbol{U}$. However, the rank-one constraint is non-convex because that adding two rank-one matrixes will not always derive a rank-one matrix. This problem can be solved by using SDR [11], i.e., dropping the rank-one constraint. Thus yielding

$$\min_{\boldsymbol{U}} \mathrm{tr}\left\{\boldsymbol{P}\boldsymbol{U}\right\} \qquad s.t. \ \begin{cases} \mathrm{tr}\left\{\boldsymbol{B}_i \boldsymbol{U}\right\} \geq 1, & i = 1, 2, \cdots, M \\ \boldsymbol{U} \succeq 0 \end{cases} \quad (13)$$

In summary, we try to explore the optimal response vector for the robust LCMV beamformer. And we converted the original response vector optimization problem in (9) into (13) via SDR which can be efficiently solved. Problem (13) can be easily and exactly solved using standard and highly efficient interior point method software tools, e.g., [12]. Note that after solving the optimization problem (6) and a suboptimal solution $\hat{\boldsymbol{u}}$ can be derived. There are several methods can be utilized to generate the rank-one suboptimal solution [11]. An intuitively reasonable idea is to apply a rank-1 approximation to the SDR solution.

$$\hat{\boldsymbol{U}} = \sum_{i=1}^{r} \lambda_i \boldsymbol{u}_i \boldsymbol{u}_i^{\mathrm{H}} \quad (14)$$

where $\hat{\boldsymbol{U}}$ is the SDR solution, $r = \mathrm{rank}\left(\hat{\boldsymbol{U}}\right)$, $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_r > 0$ are the eigenvalues and $\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_r$ are the respective eigenvectors. Thus the suboptimal response vector can be expressed as

$$\hat{\boldsymbol{u}} = \sqrt{\lambda_1}\boldsymbol{u}_1 \quad (15)$$

It should be clear that the SDR-LCMV beamformer still consume a lot of degree of freedom. However, by properly choosing the constraint matrix, the robust region of SDR-LCMV can be much bigger than other methods.

4. SIMULATION RESULTS

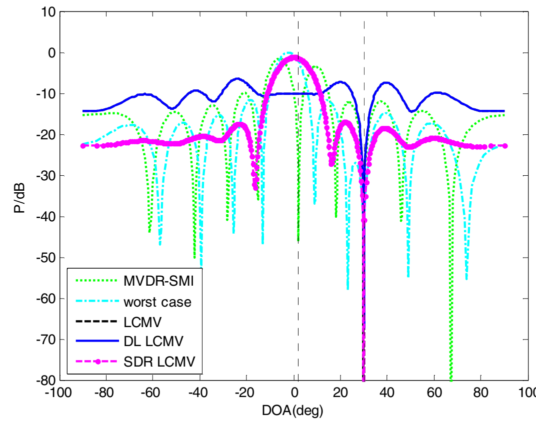
In this section, simulation experiments are carried out to evaluate the proposed method. The simulation parameters are listed in Table.1.

Table.1 Parameters for 1D adaptive beamforming

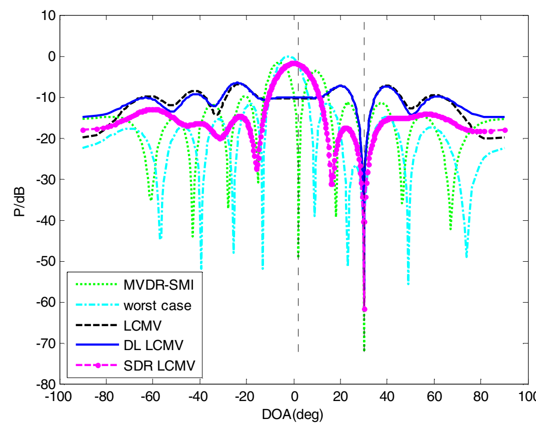
Parameters	Values	parameters	values
wavelength	0.03 m	Number of channels	10
Inter-element space	0.015 m	Number of snapshots	200
Target DOA	2°	Interference DOA	30°
SNR	10dB	Interference-to-noise ratio (INR)	30dB

A. comparison of adaptive beamforming

In this simulation, we compare the beamforming characteristic of MVDR-SMI beamformer, the worst case optimization based beamformer, LCMV beamformer, diagonal loading LCMV beamformer and the proposed SDR based LCMV beamformer. As we know in practice, the assumption of an accurate knowledge of target parameters is not completely realistic. In this part, we assume a mismatched target DOA is utilized, i.e, we assume the nominal direction of the beampattern is 0° while the real target direction is 2° as shown in Table.1. The constraint points are -3°, -1°, 0°, 1°, and 3°, respectively. The theoretical covariance is utilized in Fig.1(a) while the estimated covariance matrix is adopted in Fig.1(b). As shown in Fig.1, because of the mismatch between the presumed DOA and the actual DOA of the target, the MVDR-SMI beamformer misinterprets the target signal as interference and makes effort to suppress it, thus causing target self-nulling and severe performance degradation. On the other hand, the worst case optimization based beamformer with accurate diagonal loading factor could avoid the self-nulling effect. And both traditional LCMV beamformers and the proposed SDR based LCMV beamformers could also overcome this performance degradation. Unfortunately, there are two disadvantages of the traditional LCMV beamformer: one is the loss in mainbeam and the second is the high sidelobe of the beampattern. On the contrary, the proposed beamformer as show in Fig.1 (b), could maintain the mainbeam of the beampattern and simultaneously the sidelobe of the beampattern is lower. This is the benefit of further optimization of the response vector. It should be noted that the beamformer of the proposed method outperform other methods especially in case of the theoretical covariance matrix. This means that the well estimated covariance matrix could induce a further performance improvement of the proposed method.



(a) theoretical covariance matrix

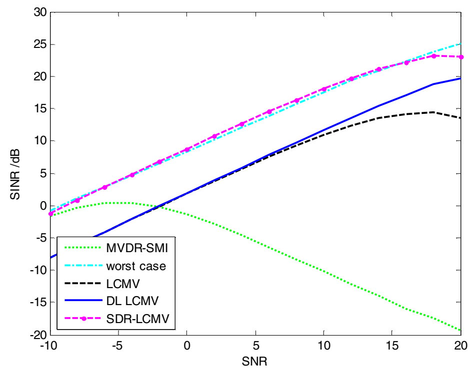


(b) estimated covariance matrix

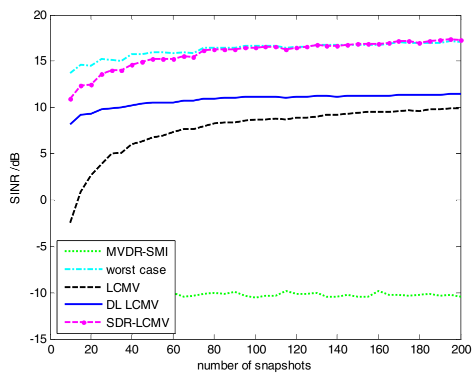
Fig.1 comparison of adaptive beamforming

B. performance comparison in case of DOA errors

In this part, we test the performance in case of DOA errors. As shown in Fig.2 (a), the output SINR performance is compared with respect to the input SNRs. The snapshot number in this simulation is 200. The MVDR-SMI beamformer is sensitive to the input SNRs and the performance degrades substantially at high SNRs. We can see that the worst case optimization based beamformer, the traditional LCMV, and the proposed SDR LCMV beamformers are all robust against the input SNRs. Also we can see that the proposed SDR-LCMV beamformer is not only robust against the input SNRs but also enjoy substantial performance improvement. Note that a diagonal loading factor is utilized in the proposed method. In Fig.2 (b), we evaluate the SINR performance with respect to the number of snapshots. The performance of the traditional LCMV beamformer is not as good as desired. The performance of the MVDR-SMI beamformer is even worse due to the target self-nulling. On the other hand, the proposed SDR-LCMV beamformer outperforms the traditional LCMV beamformer.



(a) SINR versus input SNRs



(b) SINR versus number of snapshots

Fig.2 performance comparison of output SINR

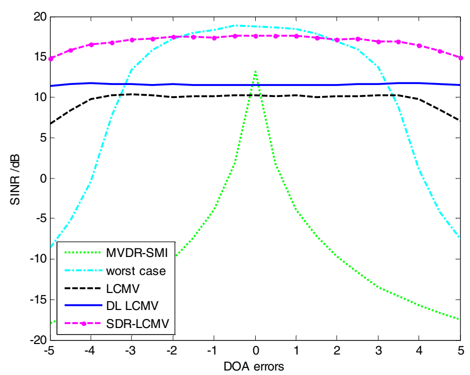


Fig.3 performance of DOA error tolerance

Fig.3 compares the target DOA error tolerance of these beamformers. As is shown in this figure, even small mismatch could induce severe performance degradation for MVDR-SMI beamformer. The worst case optimization based beamformer is robust for a limited region. When the DOA error is much larger, the performance degrades dramatically. Nevertheless, the traditional LCMV and the proposed SDR-LCMV beamformers are all robust against target DOA mismatch. The performance of the proposed SDR-LCMV is nearly optimal in the whole tested error range, and the maximum value of the performance loss is less than 4dB when the difference of the presumed and actual DOA is less than 5°. Thus the proposed method has a large robust region which can be adjusted.

5. CONCLUSIONS

One of the main limitations of LCMV beamformer is the high sidelobe of the beampattern which would induce high false alarm probability and the mainbeam loss which causes performance degradation. In this paper, we propose a robust implementation of LCMV beamformer based on response vector optimization. The performance improvement against traditional LCMV method is substantial with much lower sidelobe and better maintained mainbeam. In addition, the proposed method can be utilized for arbitrary geometry array.

6. REFERENCES

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