AN INFERENCE FRAMEWORK FOR DETECTION OF HOME APPLIANCE ACTIVATION FROM VOLTAGE MEASUREMENTS

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ABSTRACT

We present an inference framework for automatic detection of activations of home appliances based on voltage envelope waveforms. We cast the problem of appliance detection and recognition as an inference problem. When the activation signatures are known, the problem reduces to a simple detection problem. When the activation signatures are unknown, the problem is reformulated as a blind joint delay estimation. Due to the non-convexity of the negative log-likelihood, finding a global optimal solution is a key challenge. Here, we introduce a novel algorithm to estimate the activation templates, which is guaranteed to yield an error within a factor of two of that of the optimal solution. We apply our method to a real-world dataset consisting of voltage waveform measurements of several appliances obtained in multiple homes over a few weeks. Based on ground truth data, we present a quantitative analysis of the proposed algorithm and alternative approaches.

***Index Terms*—** Blind joint delay estimation, detection, home energy management.

1. INTRODUCTION

Electric supply and demand is becoming a source of concern due to the increase in home power consumption [1]. Home energy management can provide further flexibility in energy demand management. A system which can learn, monitor, and control home energy usage patterns is of interest.

Several approaches have been proposed for disaggregated end-use energy [2]. We focus on the problem of appliance recognition based on voltage envelope transient responses [3]. To the best of our knowledge, the use of activation signatures in appliance recognition has only been lightly explored as opposed to complex power analysis [4], spectral signatures [5], or harmonics of current [6].

A key challenge in this paper is how to blindly recognize a signature from multiple waveforms, which contain a noisy version of the appliance activation signature. The time delay estimation problem has been widely explored. Maximum likelihood (ML) estimation is commonly used to estimate signal delays under unchanged scale and shape conditions [7]. In [8], the problem is cast as a blind joint delay estimation and an iterative procedure for ML is provided. A different problem setup considering the change of scale and shape is explored in [9] and [10]. Since the voltage envelope transient response changes slightly in scale and shape, we do not consider the effect of these two parameters in this paper.

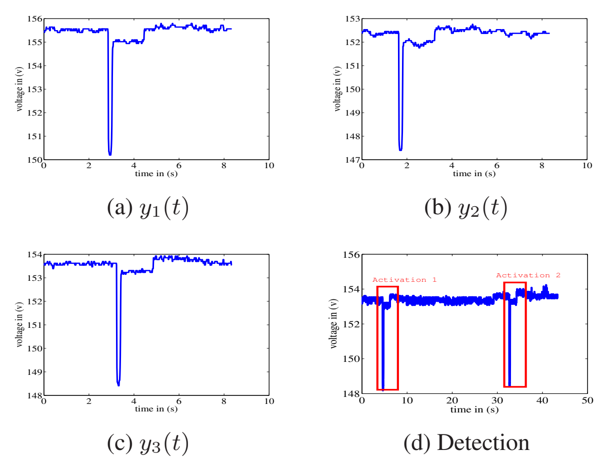
Our contributions in this paper are as follows. We provide (i) a formulation of signature recognition problem as a blind joint delay estimation; (ii) a novel approximation to the solution of the nonconvex ML problem; and (iii) theoretical guarantees on the performance of the proposed approximation.

2. PROBLEM FORMULATION

Home appliance recognition from voltage envelope measurements relies on the unique signatures associated with each appliance. To extract voltage envelope waveforms containing the appliance activation transient response, a power meter measurement of the appliance of interest provides a rough interval in which the activation response is present. Since the precise start time of the activation is unavailable, blind joint delay estimation is key to this problem. In our problem formulation, a set of N signals containing the appliance activation signature are extracted from training data for each appliance,

$$y_i(t), \qquad i = 1, 2, \ldots, N, \quad 1 \le t \le T.$$

Figure 1 (a)-(c) shows three different templates $y_1(t)$ to $y_3(t)$ containing the activation signature from the same appliance.



(a) $y_1(t)$ (b) $y_2(t)$

(c) $y_3(t)$ (d) Detection

Fig. 1: Three air-conditioning activation events (a)-(c) and template detection illustration (d)

Our goal is to detect the presence of an activation signature in a new (test) signal (see Fig. 1 (d)) using the information from the training data $y_i(t)$ for $i = 1, 2, \ldots, n$ and $1 \le t \le T$. We identify two tasks: (i) detect the presence of a signature in a new signal, and (ii) obtain an accurate estimate of the signature present in the multiple training templates.

2.1. The Detection Problem

We are interested in detecting the presence of a known fixed length template $s(t)$ in an observed signal. We further assume that $s(t)$ is nonzero for $t = 1, 2, \dots T_0$ ($T_0 \leq T$), i.e., for a portion of the analyzed signal. If the template is present, we assume that the observed signal $y_{test}(t)$ is given by

$$\begin{aligned} H_1 : y_{test}(t) &= s(t-\tau) + B + n(t) \\ n(t) &\overset{\text{i.i.d}}{\sim} N(0, \sigma^2), \qquad t = 1, 2, \dots, T, \end{aligned}$$

where $\tau \in \{0, 1, \dots, T - T_0\}$ is an unknown delay, B is a voltage offset, and $n(t)$ is a zero mean, σ^2 variance, additive white Gaussian noise. If the appliance activation template is not present in the observed signal, the observed signal is given by

$$H_0 : y_{test}(t) = B + n(t),$$

where B and $n(t)$ are as defined for H_1. Our goal in the detection problem is to determine which hypothesis was used to produce the observed signal $y_{test}(t)$.

2.2. Activation Signature Estimation Problem

Since the template is not available, we have to estimate it from the training data as described in the beginning of this section.

We assume a collection of voltage envelope waveforms $y_i(t)$ for $1 \leq t \leq T$ and $i = 1, 2, \dots, N$, each containing a single activation corrupted by noise following the model

$$y_i(t) = s(t - \tau_i) + B_i + n_i(t), \quad i = 1, 2, \dots, N \quad 1 \leq t \leq T,$$

where $s(t)$ is the previously defined unknown activation template, B_i is the voltage offset associated with the ith observed signal, and $n_i(t)$ are iid following $\mathcal{N}(0, \sigma^2)$. In this setup, the problem is reformulated as a blind joint delay estimation, in which the delayed template is unknown. Consequently, our goal is to jointly estimate $\tau_1, \dots, \tau_n$, $B_1, \dots, B_n$, and $s(t)$ for $1 \leq t \leq T_0$. Note that all other values of $s(t)$ are assumed to be zero.

3. INFERENCE SOLUTION FRAMEWORK

In the following section, we present a generalized likelihood ratio test (GLRT) framework for solving the detection problem and ML estimation approach for solving the template estimation problem.

3.1. Generalized Likelihood Ratio Test

The GLRT framework is a common and powerful statistical test method to determine between multiple hypothesis models which involve unknown parameters. The GLRT [11] for observation vector x is given by:

$$\frac{\max_{\theta_1}\ p(x|H_1, \theta_1)}{\max_{\theta_0}\ p(x|H_0, \theta_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \rho, \tag{1}$$

where θ_0 and θ_1 are the unknown parameters associated with the statistical model under hypothesis H_0 and H_1, respectively, and ρ is the non-negative test threshold. The test can be rephrased in terms of the negative log-likelihood as $\min_{\theta_1}(-\log p(x|H_1, \theta_1)) - \min_{\theta_0}(-\log p(x|H_0, \theta_0)) \underset{H_1}{\overset{H_0}{\gtrless}} \rho'$, where $\rho' = -\log \rho$ is a real-valued threshold [11].

Based on the Gaussian model for H_0 and H_1, we can directly obtain the detector as a correlation test [11]:

$$\max_{\tau} \sum_{t=1}^{T} (y_{test}(t) - \bar{y}_{test}(t))(s(t-\tau) - \overline{s(t-\tau)}) \underset{H_0}{\overset{H_1}{\gtrless}} \rho'' \tag{2}$$

where $\rho'' = \rho' \sigma^2 - \frac{1}{2} \sum_t (s(t-\tau) - \overline{s(t-\tau)})^2)$. The resulting detector compares the maximum sample cross-covariance function to a threshold to determine the presence or absence of the template s. It is closely related to the well-known matched filter [11, p. 95] in which a test signal is correlated with a given template.

3.2. Signature Maximum Likelihood Estimation

In order to identify the activation pattern from voltage envelope measurements, we need to estimate the offset parameter b_i and the delay τ_i for each observed noisy template y_i. Following the Gaussian iid assumption with $n_i(t) \sim \mathcal{N}(0, \sigma^2)$, the negative log-likelihood [12] of the observation can be written as $\frac{1}{2\sigma^2} \sum_{i,t} \|y_i(t) - s(t-\tau_i) - b_i\|^2 + const$. Hence, the optimization associated with ML is equivalent to the following minimization problem:

$$\min_{\theta} \sum_{i=1}^{N} \sum_{t=1}^{T} \|y_i(t) - (s(t-\tau_i) + b_i)\|^2, \tag{3}$$

where $\theta = [\tau_1, \dots, \tau_n, b_1, \dots, b_n, s(1), \dots, s(T_0)]^T$ is the vector of unknown parameters. To perform the minimization, we propose to eliminate the b_i's, then the $s(t)$ and finally the τ_i's. By [12], the resulting ML estimate of the b_i's is given by $\hat{b}_i^{ML} = \bar{y}_i - \overline{s(t-\tau_i)}$. Substituting $\hat{b}_i^{ML}$ for $i = 1, 2 \dots, n$ into (3) yields

$$\min_{\tau, \tilde{s}} \sum_{i=1}^{N} \sum_{t=1}^{T} (\tilde{y}_i(t) - \tilde{s}(t-\tau_i))^2, \tag{4}$$

where $\tau = [\tau_1, \dots, \tau_n]^T$, $\tilde{y}_i(t) = y_i(t) - \bar{y}_i$, $\tilde{s}(t) = s(t) - \overline{s(t)}$ and $\tilde{s} = [\tilde{s}(1), \dots, \tilde{s}(T)]^T$. Note that $\sum_t \tilde{s}(t) = 0$. Next, we exploit the fact that $s(t) = 0$ for $t \notin \{1, \dots, T_0\}$. Consequently $\tilde{s}(t) = -\bar{s}$ for $t \notin \{1, \dots, T_0\}$ and (4) can be rewritten as

$$\min_{\tau, \tilde{s}} \sum_{i=1}^{N} \sum_{t=1}^{T_0} (\tilde{y}_i(t+\tau_i) - \tilde{s}(t))^2 + \sum_{i=1}^{N} \sum_{t \in T(\tau_i)} (\tilde{y}_i(t) + \bar{s})^2, \tag{5}$$

where $T(\tau_i) = [1, \tau_i] \cup [\tau_i + T_0 + 1, T]$. Next, if we expand $\sum_{t \in T(\tau_i)} (\tilde{y}_i(t) + \bar{s})^2$ as $\sum_{t \in T(\tau_i)} (\tilde{y}_i(t) - \bar{y}_{iT(\tau_i)})^2 + \sqrt{T - T_0}(\bar{y}_{iT(\tau_i)} + \bar{s})^2$ then we can rewrite (5) as

$$\min_{\tau, \tilde{s}} \Big(\sum_{i=1}^{N} \sum_{t=1}^{T_0} (\tilde{y}_i(t+\tau_i) - \tilde{s}(t))^2 + \sqrt{T - T_0}(\bar{y}_{iT(\tau_i)} + \bar{s})^2\Big) + \sum_{i=1}^{N} \sum_{t \in T(\tau_i)} (\tilde{y}_i(t) - \bar{y}_{iT(\tau_i)})^2. \tag{6}$$

We construct the $(T_0 + 1) \times (T - T_0 + 1)$ matrix Y_i such that its kth column given by $[y_i(k), \dots, y_i(k + T_0 - 1), \sqrt{T - T_0}\bar{y}_{iT(k)}]^T$ and vector $\tilde{\mathbf{s}} = [\tilde{s}(1), \dots, \tilde{s}(T_0), -\sqrt{T - T_0}\bar{s}]^T$ and rewrite (6) as

$$\min_{\tilde{\mathbf{s}}, \tau} \sum_{i=1}^{N} \|Y_i e_{\tau_i} - \tilde{\mathbf{s}}\|^2 + \sum_{i=1}^{N} \phi_i(\tau_i), \tag{7}$$

where $\phi_i(\tau_i) = \sum_{t\in T(\tau_i)}(\tilde{y}_i(t) - \bar{y}_{iT(\tau_i)})^2$ and e_k is the canonical vector with 1 at the k^{th} place and 0 otherwise. Next, we obtain the ML estimate of $\tilde{\mathbf{s}}$ by differentiating (7) with respect to $\tilde{\mathbf{s}}$ and setting to zero. The resulting ML estimate for $\tilde{\mathbf{s}}$ is given by $\tilde{\mathbf{s}} = \frac{1}{N}\sum_{i=1}^{N} Y_i e_{\tau_i}$. After substituting the ML estimate of $\tilde{\mathbf{s}}$ in (7), we obtain a minimization only with respect to τ

$$\min_{\tau} \sum_{i=1}^{N} \|Y_i e_{\tau_i} - \frac{1}{N}\sum_{j=1}^{N} Y_j e_{\tau_j}\|^2 + \sum_{i=1}^{N}\phi_i(\tau_i). \qquad (8)$$

While the resulting minimization involves only τ, it is still non-trivial. The τ_i's are integers and hence the domain of the problem is non-convex leading to a non-convex optimization problem. Note that (8) can also be written as

$$\min_{\tau} \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N} \|Y_i e_{\tau_i} - Y_j e_{\tau_j}\|^2 + \sum_{i=1}^{N}\phi_i(\tau_i). \qquad (9)$$

In the reformulation of (9), each term in the summation involves only two delay terms τ_i and τ_j. The equivalence between (8) and (9) is due to the following result. For vectors $u_1, u_2, \ldots, u_n$ we have $\frac{1}{2N}\sum_{ij}\|u_i - u_j\|^2 = \sum_i \|u_i - \bar{u}\|^2$ where $\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i$. This can be proven by expanding both LHS and RHS into the term $\sum_i \|u_i\|^2 - \|\bar{u}\|^2$.

Denote the number of delays for each τ_i by $M = T - T_0$. The computational complexity of minimizing (9) with respect to the delays τ in a brute-force manner is $\mathcal{O}(M^N)$ [13]. For example, if $N = 30$ and the number of delays is $M = 100$, then $M^N = 10^{60}$. This prompts us to propose an approximate solution with significantly lower computational complexity. The proposed solution guarantees no more than twice of the global minimum achieved by the objective in (9). This approach is the core contribution of this paper.

3.3. Graph-based ML approximation

The objective in (9) can be viewed as a sum of edge weight in a graph given by $D_{ij} = \|Y_i e_{\tau_i} - Y_j e_{\tau_j}\|^2$ and a sum of node penalties $\phi_i(\tau_i)$. Since the sum runs over all pairs of (i, j), the graph is a complete graph. We propose to replace the single complete graph by N bipartite graphs [14] (see Fig. 2). The ith bipartite graph contains only $N - 1$ edges placed between the ith node and all other nodes.

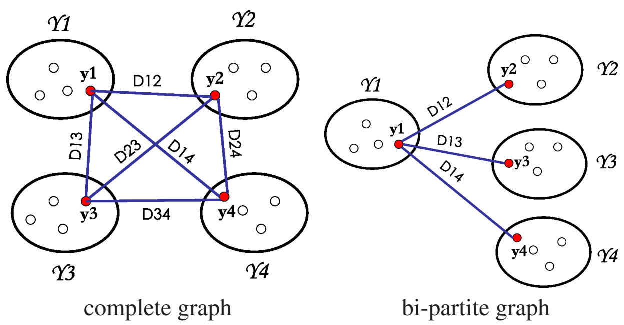


Fig. 2: Graphical representation of two approaches: (9) and (11)

To obtain the approximate ML solution $\hat{\tau}_{AML}$, we begin by solving N minimizations. The ith minimization is given by

$$\tau^i = \arg\min_{\tau} f_i(\tau), \text{ where} \qquad (10)$$

$$f_i(\tau) = \sum_{j=1\neq i}^{N} \left(\|Y_i e_{\tau_i} - Y_j e_{\tau_j}\|^2 + \phi_j(\tau_j)\right). \qquad (11)$$

Then, $\tau^{AML} = \tau^{i^*}$, where

$$i^* = \arg\min_i f_i(\tau^i). \qquad (12)$$

Although the objectives $f_i(\tau)$ differ from our original objective in (9) they are tightly connected. For both estimators, we establish a lower and upper bounds:

$$\frac{1}{2N}\sum_i f_i(\tau^i) \le f(\tau^*) \le \min_i f_i(\tau^i).$$

The bound holds for both $\tau^* = \tau^{ML}$ and $\tau^* = \tau^{AML}$. Due to space limitations, we omit the proof. Since $N \min_i f_i(\tau^i) \le \sum_i f_i(\tau^i)$, we can further bound the lower bound by $\frac{1}{2}\min_i f_i(\tau^i)$. Therefore,

$$\frac{1}{2}\min_i f_i(\tau^i) \le f(\tau^{ML}) \le f(\tau^{AML}) \le \min_i f_i(\tau^i).$$

This sandwich inequality guarantees $f(\tau^{ML}) \le f(\tau^{AML}) \le 2f(\tau^{ML})$. This bound suggests that the proposed approach yields a solution objective within a factor of 2 from the optimal solution objective.

The main advantage of the proposed algorithms is the relatively low computational complexity. The minimization in (11) can be implemented as follows. For each of the M values of τ_i, $N - 1$ separate minimizations over M values of τ_j can be performed yielding a computational complexity of the order $\mathcal{O}(M^2 N)$. Since this minimization is applied for every i, the overall computational complexity is $\mathcal{O}((MN)^2)$. This is the computational complexity obtained by comparing every one of M delay windows in every one of N observed sequences with every one of the M delay windows in all other $N - 1$ observed sequences.

4. NUMERICAL EVALUATION

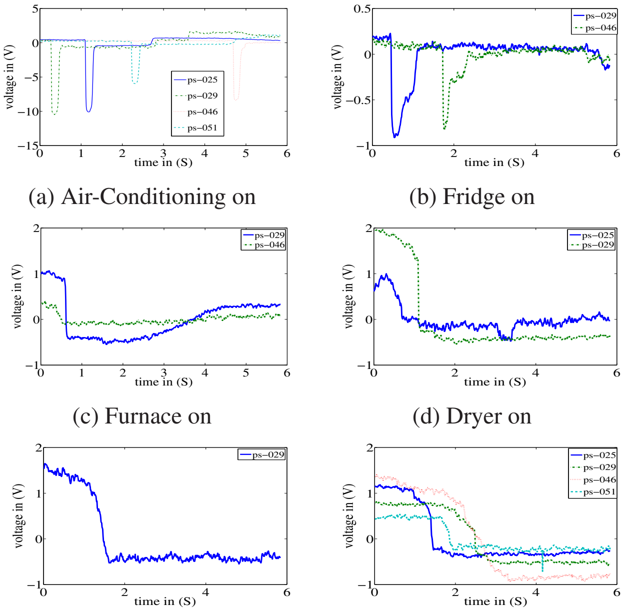
In this section, we evaluate our proposed method and compare it with Woody's method [8]. We first deploy our signature estimation procedure and compute sum of squared errors for both methods. Then we use the estimated signature to detect activation events of multiple devices from voltage measurements taken from multiple homes.

4.1. Data Description and Preprocessing

In our experiments, we use the Pecan Street dataset (*Source: Pecan Street Research Institute*). The dataset contains four homes of disaggregated, time-sampled electricity usage data with 120 sampling frequency. The data set includes voltage and apparent power readings for both the whole home and disaggregated household appliances in a period of 25 days. Since the voltage peak to peak (V_{pp}) waveform is corrupted by spike noise, we apply a five-tap median filter to despike the voltage waveforms.

4.2. Experiment Setup

Our goal is to learn the activation signature for each appliance using the training data and to test the detection performance obtained using a detector which uses the estimated signature. In our experiment, we split four home data into training data (in the period $11/17/2012$-$11/25/2012$ with around 50 activations per appliance) and test data (in the period $11/26/2012$-$12/11/2012$ with around 80 activations per appliance). The ground truth (based on the independent measurement from a commercial power meter) regarding the activation



(a) Air-Conditioning on (b) Fridge on

(c) Furnace on (d) Dryer on

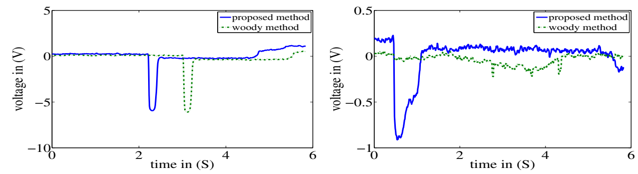
(e) Microwave on (f) Oven on

Fig. 3: Activation patterns of six household appliances from four homes.

events is obtained by identifying a power increase from 0 to 80 watt or more.

For the training phase, we obtain activation events from the training data by extracting a segment $y_i(t)$ of $T = 1000$ samples around the reported activation time for each event i in the training dataset. We consider an activation signature window size of $T_0 = 700$. We use (10)-(12) to find the delay of the activation signature within each segment. For each segment, we extract the portion associated with the activation signature and average following (7). Similarly, we apply the Woody's method [8] to obtain a signature for each device. The mean square error (MSE) $\frac{1}{N}\sum_{i=1}^{N}\|Y_i e_{\tau_i} - \tilde{\mathbf{s}}\|^2$ is presented in the Table 1.

After the training process, we generate distinct activation patterns of each appliance in each home. In Fig. 3, we present activation patterns of six appliances in four homes (PS-025, PS-029, PS-046, and PS-051). Based on the activation patterns estimated during the training phase, we apply the detector in (2) to the test data. We apply the detection scheme to each hourly file in a period of more than ten days and acquire the receiver operating characteristic (ROC) curve for each appliance in all homes. We present the area under the ROC curve (AUC) for each of the appliances available in each of the homes in Table 1. We observe that for most of the appliances the AUC is over 80%. Additionally, we observe that for devices which have a distinct single consistent activation pattern such as air-conditioning, both the proposed method and the Woody's method achieve AUC of over 0.9 (e.g., see air-conditioning signature in Fig. 4(a)). However, we notice that for some of the other appliances, Woody's method fails to find the activation pattern yielding a low AUC of 0.5 (e.g., see fridge signature in Fig. 4(b)). Moreover, when a given appliance has more than one activation pattern, the detection performance degrades for all algorithms tested. The template obtained by averaging over the multiple activation patterns may not resemble either of the patterns. Additionally, when one of the activation signatures is prominent, the average follows it closely. However, during the test phase, the less prominent activation signatures of a given appliance may not be detected.



(a) Air-conditioning (b) Fridge

Fig. 4: Template comparison for the proposed method and the Woody's method [8].

House ID	App. Name	MSE. Our Method	MSE. Woody's Method	AUC Our Method	AUC Woody's Method
PS-025	Air-Cond.	2517.93	3201.17	0.95066	0.90309
PS-025	Oven	1812.61	3243.28	0.52177	0.38571
PS-029	Air-Cond.	5356.00	3723.36	0.91496	0.88241
PS-029	Fridge	1573.47	4605.86	0.71906	0.30876
PS-029	Furnace	1582.34	2201.17	0.86338	0.39473
PS-029	Dryer	3812.68	7316.96	0.99142	0.55087
PS-029	Microwave	2168.59	5440.45	0.87869	0.47560
PS-029	Oven	1953.54	2323.37	0.91030	0.53450
PS-046	Air-Cond.	1548.87	2366.34	0.84892	0.85404
PS-046	Fridge	1303.00	2142.41	0.49252	0.49213
PS-046	Furnace	623.93	690.28	0.53887	0.55045
PS-046	Oven	4193.05	5024.09	0.91824	0.49346
PS-051	Air-Cond.	2730.66	2569.54	0.91311	0.92936
PS-051	Oven	2115.58	2599.95	0.78501	0.47497

Table 1: MSE for the estimated template and AUC for the proposed method and for Woody's method [8]

5. CONCLUSION AND FUTURE WORK

In this paper, we provided a formulation of the problem of automatic detection of electric appliance activation from voltage measurements as a blind joint delay estimation. We presented a GLRT detection scheme base on activation signatures estimated in the maximum likelihood framework. We provided a feasible approximation for the maximum likelihood estimate and proved that the approximation produces an objective value (based on the negative log likelihood) which is no more than twice the value obtained by the exact maximum likelihood solution. In our experimental study, we achieved a better detection performance than Woody's method. For most appliances, the AUC achieved was over 80%.

One challenge in practice involves multiple activation patterns which are produced by the same appliance. We are currently focussing our efforts on models that can account for multiple activations of a given appliance. Additionally, we are interested in solving the problem when outliers are present (e.g., spike noise or missing activations).

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