MISALIGNMENT ANALYSIS AND INSIGHTS INTO THE PERFORMANCE OF CLIPPED-INPUT LMS WITH CORRELATED GAUSSIAN DATA

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ABSTRACT

The three-level clipped input least-mean-square (CLMS) adaptive algorithm is known to have low complexity that is suitable for the identification of long finite impulse response of unknown systems. In this paper we analyze the performance of CLMS which allows one to gain insights into its convergence property and the amount of steady-state misalignment error for both time-invariant and time-varying systems perturbed by correlated Gaussian input. Arising from our analysis, we derive the optimal step-size for CLMS to achieve the minimum possible steady-state misalignment and compare its results with the performance of LMS adaptive algorithm. The accuracy of our derivations is evaluated with simulation results.

Index Terms— Adaptive filter, Clipped input LMS, Misalignment, Tracking.

1. INTRODUCTION

Adaptive system identification is a well-known topic in signal processing [1]. In applications such as acoustic echo cancellation (AEC) and network echo cancellation [2], adaptive algorithms have been employed to estimate a long (unknown) impulse response. For such applications, the high computational requirement results in the demand for low-complexity adaptive algorithms. Several efficient variants of the least-mean-square (LMS) adaptive filtering algorithm including signed-regressor LMS (SR-LMS) [3][4], sign-error LMS [5][6], sign-sign LMS [7][8], sequential-LMS [9][10], periodic-LMS [11], MMax-LMS [12][13] as well as the three-level clipped LMS (CLMS) algorithm [14] have been proposed for system identification.

Among these low-complexity adaptive algorithms, CLMS is one of the most efficient [14][15]. The CLMS algorithm achieves complexity reduction by employing a three-level quantized input signal for updating its filter coefficients. As shown in [14], compared to the SR-LMS algorithm, CLMS can achieve higher convergence rate with lower computational load. As the convergence behavior of CLMS depends on the step-size and clipping threshold, analysis of its convergence performance is of practical importance. Such analysis not only allows one to gain insights into its behavior, it also allows one to select suitable CLMS parameters in order to achieve the desired performance under various operating conditions.

The convergence and steady-state misalignment of the CLMS algorithm has recently been analyzed for i.i.d. input signals [16][17]. As opposed to that presented in [16], we derive, in this work the steady-state misalignment for correlated Gaussian input data. Analysis of such input is important since many practical signals such as speech, music and biological signals, which can be modeled as Gaussian signals, are correlated [18][19][20]. In addition, we perform the analysis for both time-invariant (TI) and time-varying (TV) systems and show that, the *tracking* capability of SR-LMS is higher than CLMS (with a clipping threshold of greater than zero). On the contrary, for a TI system, the *convergence* capability of CLMS is superior than SR-LMS. In addition, our analysis reveals that the effect of the clipping threshold varies under different signal-to-noise ratio (SNR) conditions. Unlike the work of [16], we further derive an approximate optimal step-size for CLMS to achieve the minimum possible steady-state misalignment for both TI and TV systems. With this step-size, we compare the steady-state performance of CLMS as well as its complexity with those of the LMS algorithm.

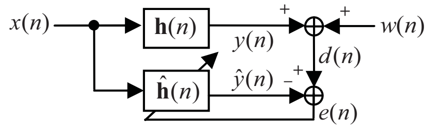

Fig. 1. Adaptive system identification.

2. PROBLEM DEFINITION AND REVIEW OF CLMS

Figure 1 shows a schematic of adaptive filtering in the context of system identification application. The input signal $x(n)$ is first filtered by the unknown time-varying impulse response $\mathbf{h}(n) = [h_0(n), \ldots, h_{L-1}(n)]^T$ of length L and the output signal is given by $y(n) = \mathbf{h}^T(n)\mathbf{x}(n)$, where $\mathbf{x}(n) = [x(n), \ldots, x(n-L+1)]^T$. The time variation of $\mathbf{h}(n)$ can be modeled by [21]

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mathbf{c}(n) \tag{1}$$

where $\mathbf{c}(n)$ determines the amount of time variation of $\mathbf{h}(n)$ such that for a TI system, $\mathbf{c}(n) = \mathbf{0}_{L\times 1}$. For mathematical tractability, we assume that $\mathbf{c}(n)$ is statistically stationary with $E\{\mathbf{c}(n)\} = 0$.

The output signal of the unknown system is often corrupted by ambient noise and the desired signal can be expressed as

$$d(n) = y(n) + w(n) \tag{2}$$

where $w(n)$ is the uncorrelated ambient noise which may include measurement and/or environmental noise. The output signal of the adaptive filter can be expressed as

$$\widehat{y}(n) = \mathbf{x}^T(n)\widehat{\mathbf{h}}(n) \tag{3}$$

where $\widehat{\mathbf{h}}(n) = [\widehat{h}_0(n), \ldots, \widehat{h}_{L-1}(n)]^T$ is the adaptive filter coefficient vector of length L. The aim of CLMS is to estimate $\mathbf{h}(n)$ using

$$\widehat{\mathbf{h}}(n+1) = \widehat{\mathbf{h}}(n) + \mu e(n)\widetilde{\mathbf{x}}(n) \tag{4}$$

where μ is the step-size of CLMS which controls its convergence rate and the error signal $e(n)$ is defined as

$$e(n) = d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n). \tag{5}$$

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In addition, $\widetilde{\mathbf{x}}(n) = [\widetilde{x}(n), \ldots, \widetilde{x}(n-L+1)]^T$ in (4), is the clipped tap-input vector with elements being obtained by the three-level clipping of $\mathbf{x}(n)$ according to the following relation

$$\widetilde{x}(n-k) = \begin{cases} 1 & x(n-k) > \delta \\ 0 & -\delta < x(n-k) \leq \delta \\ -1 & x(n-k) \leq -\delta \end{cases}, \; 0 \leq k \leq L-1$$

where δ is the clipping threshold.

We therefore note that when $\delta = 0$, the three-level clipping mechanism is equivalent to two-level clipping and hence, CLMS is equivalent to SR-LMS. On the other hand, when δ increases, more elements in $\widetilde{\mathbf{x}}(n)$ becomes zero resulting in less frequent updating of $\widehat{\mathbf{h}}(n)$ in (6). When δ approaches the upper bound of $|\mathbf{x}(n)|$, this update process is reduced further giving an even lower convergence performance.

3. STEADY-STATE MISALIGNMENT ANALYSIS OF CLMS

To analyze the steady-state performance of CLMS, we employ the normalized misalignment given by $\widetilde{\varphi}(n) = \varphi(n)/\|\mathbf{h}(n)\|^2$ where $\varphi(n) = \|\widehat{\mathbf{h}}(n) - \mathbf{h}(n)\|^2$ is the misalignment and $\|\cdot\|$ is the l_2-norm [22]. Defining $\mathbf{v}(n) = \widehat{\mathbf{h}}(n) - \mathbf{h}(n)$ as the $L \times 1$ misalignment vector, $\varphi(n)$ can be expressed as

$$\varphi(n) = \mathbf{v}^T(n)\mathbf{v}(n) = \mathrm{tr}\{\mathbf{v}(n)\mathbf{v}^T(n)\}, \tag{6}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix. The steady-state misalignment η is therefore defined as

$$\eta = \lim_{n\to\infty} E\{\varphi(n)\} = \lim_{n\to\infty} \mathrm{tr}\{\mathbf{Q}(n)\} \tag{7}$$

where $\mathbf{Q}(n) = E\{\mathbf{v}(n)\mathbf{v}^T(n)\}$.

To derive η, we utilize (1), (4), and (5) giving

$$\begin{aligned} \mathbf{v}(n+1) &= \widehat{\mathbf{h}}(n+1) - \mathbf{h}(n+1) \\ &= \widehat{\mathbf{h}}(n) - \mathbf{h}(n) - \mathbf{c}(n) + \mu\widetilde{\mathbf{x}}(n)[d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n)]. \end{aligned} \tag{8}$$

Substituting (2) to (8), we obtain the time-varying difference between the adaptive filter and the unknown system given by

$$\mathbf{v}(n+1) = \mathbf{v}(n) - \mathbf{c}(n) + \mu\widetilde{\mathbf{x}}(n)\{w(n) - \mathbf{x}^T(n)\mathbf{v}(n)\}. \tag{9}$$

Assuming that $\mathbf{c}(n)$ is mutually independent with $\mathbf{x}(n)$ and $\mathbf{v}(n)$, and defining $\mathbf{C} = E\{\mathbf{c}(n)\mathbf{c}^T(n)\}$, we can express (9) as

$$\begin{aligned} \mathbf{Q}(n+1) = \mathbf{Q}(n) + \mathbf{C} &+ \mu^2 E\left\{\widetilde{\mathbf{x}}(n)w^2(n)\widetilde{\mathbf{x}}^T(n)\right\} \\ &- \mu E\left\{\mathbf{v}(n)\left[\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{v}(n)\right]^T\right\} \\ &- \mu E\left\{\left[\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{v}(n)\right]\mathbf{v}^T(n)\right\} \\ &+ \mu^2 E\left\{\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{v}(n)\mathbf{v}^T(n)\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\right\} \end{aligned} \tag{10}$$

Denoting $\sigma_w^2 = E\{w^2(n)\}$ as the noise variance and assuming that $w(n)$ is uncorrelated with $x(n)$, we can express (10), for small values of μ [1], as

$$\begin{aligned} \mathbf{Q}(n+1) = \mathbf{Q}(n) + \mathbf{C} &+ \mu^2\sigma_w^2 E\{\widetilde{\mathbf{x}}(n)\widetilde{\mathbf{x}}^T(n)\} \\ &- \mu E\{\mathbf{v}(n)\mathbf{v}^T(n)\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\} \\ &- \mu E\{\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{v}(n)\mathbf{v}^T(n)\}. \end{aligned} \tag{11}$$

The assumption of small step-size for deriving (11) is reasonable since the maximum value of μ to ensure stability is inversely proportional to the maximum eigenvalue of input correlation matrix [14]. Hence, a small value of $\mu \leq 0.001$ is assumed for a lightly colored input signal vectors of length $L = 1024$. As a result, $\mu^2 < 10^{-6}$ and the last term of (10) is negligible and hence the approximation in (11) is reasonable.

Similar to [23], assuming that $\mathbf{x}(n)$ and $\mathbf{v}(n)$ are independent, (11) can be simplified as

$$\begin{aligned} \mathbf{Q}(n+1) = \mathbf{Q}(n) + \mathbf{C} &+ \mu^2\sigma_w^2\widetilde{\mathbf{R}} \\ &- \mu E\{\mathbf{v}(n)\mathbf{v}^T(n)\}E\{\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\} \\ &- \mu E\{\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\}E\{\mathbf{v}(n)\mathbf{v}^T(n)\} \end{aligned} \tag{12}$$

where $\widetilde{\mathbf{R}} = E\{\widetilde{\mathbf{x}}(n)\widetilde{\mathbf{x}}^T(n)\}$ is the covariance matrix of $\widetilde{\mathbf{x}}(n)$. Under steady-state condition, we assume that the adaptive algorithm has converged so that $\varphi(n)$ varies to within an expected value giving

$$\lim_{n\to\infty} \mathbf{Q}(n+1) = \lim_{n\to\infty} \mathbf{Q}(n) = \mathbf{Q}. \tag{13}$$

Taking this into account, (12) can then be rewritten for the steady state as

$$\begin{aligned} \mathbf{Q} = \mathbf{Q} + \mathbf{C} &+ \mu^2\sigma_w^2\widetilde{\mathbf{R}} \\ &- \mu\mathbf{Q}E\{\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\} - \mu E\{\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\}\mathbf{Q}, \end{aligned} \tag{14}$$

resulting in

$$\mu^{-1}\mathbf{C} + \mu\sigma_w^2\widetilde{\mathbf{R}} = \mathbf{Q}E\{\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\} + E\{\widetilde{\mathbf{x}}(n)\mathbf{x}^T(n)\}\mathbf{Q}. \tag{15}$$

To further simplify (15), it has been shown in [14] that, for tap-input signal $\mathbf{x}(n)$ with Gaussian distribution,

$$E\{\mathbf{x}(n)\widetilde{\mathbf{x}}^T(n)\} = \frac{\alpha}{\sigma_x}E\{\mathbf{x}(n)\mathbf{x}^T(n)\} = \frac{\alpha}{\sigma_x}\mathbf{R} \tag{16}$$

where $\mathbf{R} = E\{\mathbf{x}(n)\mathbf{x}^T(n)\}$ is the covariance matrix of $\mathbf{x}(n)$ and $\alpha = \sqrt{2/\pi}\exp(-\delta^2/(2\sigma_x^2))$. Substituting (16) into (15), we obtain

$$\frac{\mu\sigma_x}{\alpha}\sigma_w^2\widetilde{\mathbf{R}} + \frac{\sigma_x}{\alpha}\mu^{-1}\mathbf{C} = \mathbf{QR} + \mathbf{RQ}. \tag{17}$$

We note that for input signals which exhibit short-time correlation, $\mathbf{R} \neq \sigma_x^2\mathbf{I}$ and therefore we decompose $\mathbf{R}$ using $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$, where $\mathbf{U}$ is the orthogonal matrix containing the eigenvectors and $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_L\}$ is a matrix of eigenvalues. We then pre- and post-multiply (17) by $\mathbf{U}^T$ and $\mathbf{U}$, respectively, to achieve

$$\frac{\mu\sigma_x}{\alpha}\sigma_w^2\widetilde{\mathbf{R}}' + \frac{\sigma_x}{\alpha}\mu^{-1}\mathbf{C}' = \mathbf{Q}'\mathbf{\Lambda} + \mathbf{\Lambda}\mathbf{Q}' \tag{18}$$

where $\widetilde{\mathbf{R}}' = \mathbf{U}^T\widetilde{\mathbf{R}}\mathbf{U}$, $\mathbf{Q}' = \mathbf{U}^T\mathbf{Q}\mathbf{U}$, and $\mathbf{C}' = \mathbf{U}^T\mathbf{C}\mathbf{U}$. For steady-state misalignment, we note, from (7) and (13), that $\eta = \mathrm{tr}\{\mathbf{Q}\}$. Post-multiplying (18) by $\mathbf{\Lambda}^{-1}$ and taking the trace result in

$$\mathrm{tr}\{\mathbf{Q}'\} = \frac{\mu\sigma_x}{2\alpha}\sigma_w^2\mathrm{tr}\{\widetilde{\mathbf{R}}'\mathbf{\Lambda}^{-1}\} + \frac{\sigma_x}{2\alpha}\mu^{-1}\mathrm{tr}\{\mathbf{C}'\mathbf{\Lambda}^{-1}\}. \tag{19}$$

Due to the orthogonal property of $\mathbf{U}$, $\mathrm{tr}\{\widetilde{\mathbf{R}}'\mathbf{\Lambda}^{-1}\} = \mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$, $\mathrm{tr}\{\mathbf{C}'\mathbf{\Lambda}^{-1}\} = \mathrm{tr}\{\mathbf{C}\mathbf{R}^{-1}\}$, and $\mathrm{tr}\{\mathbf{Q}'\} = \mathrm{tr}\{\mathbf{Q}\}$. As a consequence, we arrive at the important relation

$$\eta = \frac{\mu\sigma_x}{2\alpha}\sigma_w^2\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\} + \frac{\sigma_x}{2\alpha}\mu^{-1}\mathrm{tr}\{\mathbf{C}\mathbf{R}^{-1}\}. \tag{20}$$

It can be seen that the first term, which describes the convergence performance, is independent of time variation of $\mathbf{h}(n)$- it completely determines η for a TI system. The second term is therefore an *excess* steady-state misalignment for TV systems and it defines the tracking performance of CLMS.

3.1. Insights Into the Steady-state Misalignment for TI Systems

For a TI system, we note from (20) that η is proportional to μ and hence, with reducing μ, corresponding to a reduction in speed of convergence, η is reduced as expected. In addition, a higher σ_w^2 results in a higher η and a reduction in steady-state performance is exhibited.

To describe the variation of η with δ, we note that the effect of δ in (20) appears in both α and $\widetilde{\mathbf{R}}$. We can see that the number of elements in $\widetilde{\mathbf{x}}(n)$ that are equal to ± 1 decreases with increasing δ. Therefore, when δ increases, $\widetilde{\mathbf{R}} \rightarrow \mathbf{0}_{L\times L}$ and hence, $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\} \rightarrow 0$. On the other hand, α reduces with increasing δ. As a result, the first term of η in (20) may increase or decrease. This implies that variation of η with δ is dependent on the structure of $\mathbf{R}$ and how the terms $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$ and α in (20) decay with δ. Simulations with input signals having various correlation matrix show that for small values of δ, the term $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$ decays more significantly than α with δ. It is therefore expected that the convergence performance of CLMS is higher than SR-LMS.

3.2. Insights Into the Steady-state Misalignment for TV Systems

As can be seen from (20), for a TV system, the second term in (20) is non-zero and hence η increases w.r.t. any variations of the TI system. This term describes the tracking capability of CLMS and we note that with increasing μ, the effect of variation of $\mathbf{h}(n)$ on η reduces. This implies that CLMS can better track the variations of $\mathbf{h}(n)$. We also note from (20) that the second term is inversely proportional to α. Since α reduces with increasing clipping threshold δ, the tracking capability of CLMS reduces with increasing δ. In addition, the tracking capability of SR-LMS (as a special case of CLMS with $\delta = 0$) is better than CLMS (with $\delta > 0$.)

While the convergence performance of CLMS is proportional to σ_w^2, its tracking capability is independent of noise. As a result, when SNR is high in a TV environment, the first term in (20) is small and hence, reducing δ or employing SR-LMS instead of CLMS will achieve good tracking capability and a low η is expected. On the other hand, when the SNR is low, the first term dominates and hence increasing δ will ensure good convergence performance for CLMS resulting in a low η.

To further illustrate the effect of a TV system, we consider, similar to [13][16], the modified first-order Markov process [24] described by

$$\mathbf{h}(n+1) = \xi\mathbf{h}(n) + \sqrt{1-\xi^2}\mathbf{s}(n) \tag{21}$$

where $0 << \xi < 1$ defines the dynamics of the TV system, $\mathbf{s}(n) = [s_0(n), \ldots, s_{L-1}(n)]^T$ is a WGN process with zero mean and variance σ_s^2. According to the definition of $\mathbf{c}(n)$ in (1), we obtain

$$\mathbf{c}(n) = -(1-\xi)\mathbf{h}(n) + \sqrt{1-\xi^2}\mathbf{s}(n). \tag{22}$$

Assuming $E\{\mathbf{h}(n)\} = 0$ and substituting (22) to (9),

$$\eta \approx \frac{\mu\sigma_x}{2\alpha}\sigma_w^2 \mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\} + \frac{\sigma_x}{\mu\alpha}(1-\xi)\sigma_s^2 \mathrm{tr}\{\mathbf{R}^{-1}\}, \tag{23}$$

where the approximation is due to the dependency of $\mathbf{c}(n)$ on $\mathbf{v}(n)$. However, for $\xi \approx 1$, which is practically reasonable to describe the variation of an acoustic room impulse response [13], theoretical derivation of η using (23) is almost accurate.

For the case where the input signal is i.i.d. Gaussian, $\widetilde{\mathbf{R}} = \widetilde{\sigma}_x^2\mathbf{I}$ where $\widetilde{\sigma}_x^2 = \mathrm{erfc}\left\{\delta/(\sigma_x\sqrt{2})\right\}$ [16] and hence, (23) can be simplified, for i.i.d. input signals to

$$\eta \approx \frac{\mu}{2\alpha}\frac{\sigma_w^2\widetilde{\sigma}_x^2}{\sigma_x}L + \frac{1}{\mu\alpha}\frac{\sigma_s^2}{\sigma_x}(1-\xi)L \tag{24}$$

Equation (24) is a simplified version of η derived in [16] for i.i.d. inputs which is valid for small step-size.

3.3. Derivation of the Optimal Step-size for CLMS

As evident from (20), there is a trade-off between tracking and convergence performance of CLMS due to step-size μ. Employing (20), we further proceed to derive an optimal step-size for which CLMS achieves its lowest possible η. Differentiating (20) with respect to μ and equating the resultant equation to zero,

$$\mu_{\mathrm{opt}} \approx \sqrt{\frac{\mathrm{tr}\{\mathbf{C}\mathbf{R}^{-1}\}}{\sigma_w^2\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}}}. \tag{25}$$

Therefore when $\mathbf{C} \rightarrow \mathbf{0}$, which corresponds to a slowly varying system, $\mu_{\mathrm{opt}} \rightarrow 0$. In addition, $\mathrm{tr}\{\mathbf{C}\mathbf{R}^{-1}\}$ and $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$ in (25) depend on the structure of $\mathbf{R}$ which, in turn, is dependent on the input signal correlation. For the special case where the variation of $\mathbf{h}(n)$ is represented by (21), we achieve, through the use of (25),

$$\mu_{\mathrm{opt}} \approx \sqrt{\frac{2(1-\xi)\sigma_s^2\mathrm{tr}\{\mathbf{R^{-1}}\}}{\sigma_w^2\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}}}. \tag{26}$$

Note that with increasing δ, the value of $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$ reduces and hence μ_{opt} increases to achieve good tracking capability. It is also important to note that while $\mathrm{tr}\{\widetilde{\mathbf{R}}\mathbf{R}^{-1}\}$ may approach zero, μ_{opt} may not approach infinity and it should be kept small since its value must be bounded by the stability of convergence. Therefore the modified μ_{opt} can be given by

$$\mu_{\mathrm{opt,modified}} = \min\{\mu_{\mathrm{opt}}, \mu_{\max}\} \tag{27}$$

where, as derived in [14], $\mu_{\max} = \alpha/(\sigma_x\lambda_{\max})$ such that $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{R}$.

4. SIMULATION RESULTS AND FURTHER DISCUSSION

For simulations, we consider $\mathbf{h}(n)$ defined by (21) where $\mathbf{h}(0)$ is a random sequence of length $L = 256$ samples with normal distribution. A Gaussian colored signal $x(n)$ is obtained by filtering a WGN through a lowpass FIR filter with coefficients $[0.3574,\ 0.9,\ 0.3574]$ chosen to generate a speech-like spectrum [25]. A sampling rate of $f_s = 8000$ Hz is considered throughout the simulations.

We first investigate the steady-state normalized misalignment of CLMS for $0.99999 \leq \xi \leq 1$. In this illustrative example, we used $\delta = 0.4\sigma_x$, $\mu = 0.001$ and SNR=30 dB. Figure 2 (a) shows $\widetilde{\varphi}(n)$ along with its corresponding $\widetilde{\eta} = \eta/\|\mathbf{h}(n)\|^2$ computed using (23). The horizontal solid lines denote the analytically determined $\widetilde{\eta}$ while the dashed lines denote the simulated steady-state of $\widetilde{\varphi}(n)$ computed by averaging over 10 s after the algorithm converged to its steady-state. As can be seen, with increasing ξ, the steady-state misalignment improves. We also note that $\widetilde{\eta}$ approximates the simulated values of $\widetilde{\varphi}(n)$ well- hence verifying our analysis of CLMS for both TI ($\xi = 1$) and TV ($\xi < 1$) systems.

We next evaluate the theoretical results of the steady-state misalignment for $10 \leq \mathrm{SNR} \leq 40$ dB when $\xi = 1$, $\mu = 0.001$, and $\delta = 0.3\sigma_x$. Figure 2 (b) shows the variation of $\widetilde{\varphi}(n)$ along with its corresponding values of $\widetilde{\eta}$ for different SNRs. As can be seen, the steady-state values of $\widetilde{\varphi}(n)$ reduces with increasing SNR in accordance with (23), i.e., a lower σ_w^2 results in a lower steady-state misalignment. As can be seen from Fig. 2 (b), $\widetilde{\eta}$ sufficiently approximates the corresponding steady-state values of $\widetilde{\varphi}(n)$.

To evaluate the variation of steady-state misalignment of CLMS with respect to $0 \leq \delta \leq 2\sigma_x$, we consider both TI and TV systems

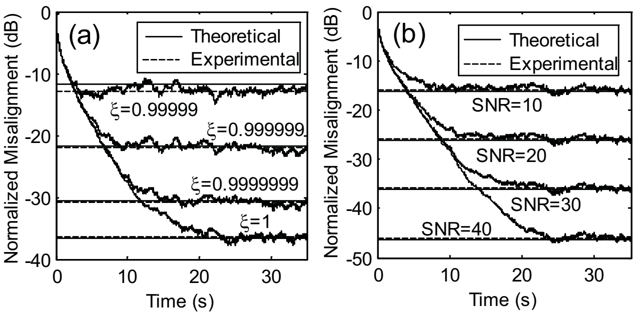

Fig. 2. Normalized misalignment of CLMS with a correlated Gaussian input, (a) for different values of ξ with SNR=30 dB, (b) for different values of SNR for a TI system.

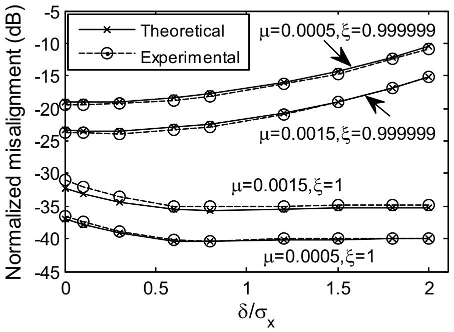

Fig. 3. Normalized misalignment of CLMS with respect to δ/σ_x when $\mu = 0.5 \times 10^{-3}$ and $\mu = 0.15 \times 10^{-2}$ for TI and TV systems with correlated Gaussian input.

with $\xi = 1$ and 0.999999, respectively. Figure 3 shows the normalized steady-state misalignment of CLMS against δ/σ_x for step-sizes $\mu = 0.5 \times 10^{-3}$ and $\mu = 0.15 \times 10^{-2}$ when SNR=30 dB. These results are obtained by averaging over five independent trials. The solid lines illustrate the analytically determined values of $\widetilde{\eta}$ while results obtained by simulation are shown by dashed lines. For a TI system, the amount of normalized steady-state misalignment reduces with increasing δ; an approximately 4 dB improvement when δ increases from 0 to $2\sigma_x$. On the other hand, the normalized steady-state misalignment increases by approximately 9 dB for a TV system. This result demonstrates that the tracking capability of CLMS improves with reducing δ while the steady-state convergence capability (defined as the amount of steady-state misalignment for a TI system) decreases. Therefore there is a trade-off between tracking and convergence performance for CLMS.

In addition, from Fig. 3, a reduction in μ from 0.15×10^{-2} to 0.5×10^{-3} causes approximately 5 dB increase in normalized steady-state misalignment for a TV system while a 5 dB improvement in steady-state misalignment is observed for a TI system. Therefore, similar to the effect of δ, there is a trade-off between the tracking and convergence performance of CLMS with respect to the value of μ for CLMS. We also note that while the steady-state misalignment increases with μ for a TI system as expected, there exists an optimal μ for a TV system. This optimal value may be approximated using (27). For the simulation conducted above, we achieve $\mu_{\mathrm{opt}} \approx 1.8 \times 10^{-3}$ while the simulated result shows that $\mu_{\mathrm{opt}} \approx 1.6 \times 10^{-3}$. The difference between the steady-state values of $\widetilde{\varphi}(n)$ resulted from these two values of μ is 0.14 dB, which is

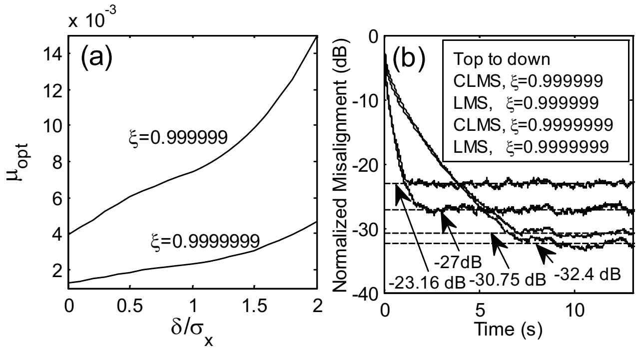

Fig. 4. (a) Variation of μ_{opt} computed using (26) against δ/σ_x when $\xi = 0.9999999$, (b) Variation of η for CLMS and LMS when $\xi = 0.999999$ and 0.9999999.

negligible in practical applications.

We now evaluate the variation of μ_{opt} with δ for the TV system defined by (21). We confine $0 \le \delta \le 2\sigma_x$ such that $\mu_{\mathrm{opt}} < \mu_{\max}$ and hence, $\mu_{\mathrm{opt,modified}} = \mu_{\mathrm{opt}}$. Figure 4 (a) shows the variation of μ_{opt}, computed using (26), with δ/σ_x when $\xi = 0.999999$ and 0.9999999. As can be seen, a higher δ results in a higher μ_{opt}. In addition, a higher degree of time variation of the unknown system (with a smaller ξ) results in a higher value for μ. Both of the above results are in conjunction with the simulations.

Figure 4 (b) compares the variation of η in CLMS and LMS for TV system. We consider $\mu = \mu_{\mathrm{opt}}$ for CLMS and the value of μ for LMS is considered so that to LMS achieves the same convergence rate as that of CLMS for TV system with $\xi = 0.999999$ and $\xi = 0.9999999$. In this simulation, we have used $L = 256$, $\delta = 0.6$ and SNR=30 dB. The value of step-size is $\mu = 0.0062337$ for CLMS and $\mu = 0.0016$ for LMS when $\xi = 0.999999$. In addition, $\mu = 0.0019558$ for CLMS and $\mu = 0.0006$ for LMS when $\xi = 0.9999999$. Results are obtained by averaging over five independent runs. As can be seen, the steady-state misalignment performance of LMS is 3.8 dB and 1.6 dB lower than CLMS for $\xi = 0.999999$ and $\xi = 0.9999999$, respectively. This good performance is achieved at the cost of increase in computational complexity of LMS. The reduction of complexity in CLMS is in the coefficient update process (4). Since input elements are now 0, 1 and -1, the number of multiplications is L times less than that of LMS at each iteration. On the other hand, the number of additions is reduced depending on the number of elements of input vectors having values less than δ. In the current simulation with Gaussian distribution and $\delta = 0.6\sigma_x$, about 50% of additions is eliminated at each iteration.

5. CONCLUSION

The steady-state misalignment error of CLMS for system identification is analyzed for correlated Gaussian input signals for both time-invariant and time-varying systems. We showed that the tracking capability of CLMS improves with reducing clipping threshold while the convergence capability of CLMS decreases. In high SNR environments, a low clipping threshold will achieve good convergence and tracking capabilities. We then derived an optimal step-size of CLMS to achieve the minimum possible steady-state misalignment and showed that CLMS performance using optimal step-size is comparable with that of the LMS algorithm, while we achieve a considerably less computational load in CLMS.

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