JOINT CFO AND I/Q IMBALANCE COMPENSATION FOR THE SC-IFDMA SYSTEM UPLINK

Donatella Darsena

Department of Engineering
Parthenope University,
I-80143, Naples, Italy.

Giacinto Gelli and Francesco Verde

Department of Electrical Engineering
and Information Technology,
University Federico II, I-80125, Naples, Italy.

ABSTRACT

This paper deals with the synthesis of a new reception scheme for the uplink of a single-carrier interleaved frequency-division multiple-access (SC-IFDMA) wireless network. In such networks, the in-phase/quadrature-phase (I/Q) imbalance, introduced at each user transmitter and at the base station receiver, and the carrier frequency offsets (CFOs) between the transmitters and the receiver are sources of severe performance degradation. The proposed receiver, based on the minimum mean-output energy (MMOE) criterion, jointly mitigates CFO and I/Q imbalance effects by processing the received signal and its complex conjugate version. Monte Carlo computer simulations are carried out to assess the effectiveness of the MMOE solution.

1. INTRODUCTION

The single-carrier frequency-division multiple access (SC-FDMA) scheme has been recently proposed for the uplink physical-layer protocol in cellular systems [1, 2, 3], because of its many attractive properties: in particular, its low peak-to-average transmit power ratio (PAPR) relaxes the requirement on the power amplifier of the transmitters. In SC-FDMA system, each user is assigned a subset of orthogonal subcarriers; among all subcarrier allocation schemes, in SC *interleaved* FDMA (SC-IFDMA) the subcarriers are chosen uniformly distributed over the whole bandwidth, which allows every user to efficiently exploit frequency diversity [4].

The performance of SC-IFDMA in the uplink is very sensitive to carrier frequency offsets (CFOs) between transmitters and the receiver, which disrupt subcarrier orthogonality, giving rise to intercarrier interference (ICI) and multiple-access interference (MAI); such a degradation is particularly difficult to compensate for, due to the interleaving between subcarriers of different users [5]. In addition, analog front-end *in-phase/quadrature-phase* (I/Q) imbalances [6] may degrade the performance of SC-IFDMA systems, justifying the synthesis of efficient digital compensation techniques.

Relation to prior work: In recent years, the CFO compensation problem for orthogonal frequency-division multiple-access (OFDMA) has been extensively studied, with particular emphasis on the challenging case of uplink; in particular, [7] represents a comprehensive survey of the latest state-of-the-art OFDMA synchronization and compensation techniques, which, with suitable modifications, can be also applied to the SC-IFDMA scenario. In [8], a time-domain linear CFO compensation algorithm tailored for a SC-IFDMA system is proposed, but the effects of I/Q front-end impairments are neglected. On the other hand, in [6] the effect of transmitter I/Q imbalance on OFDMA and SC-FDMA receivers has been studied, but perfect timing and frequency synchronization is assumed. Although many approaches have been proposed for solving the joint problem of CFO and I/Q imbalance compensation for a single-user OFDM environment (see, for example, [9, 10, 11]), to the best of our knowledge, this challenging problem has not been tackled yet for a multiuser scenario. In [12, 13], we considered the uplink of a SC-IFDMA system affected by timing and frequency errors, transmitter I/Q imbalances, and operating over a highly-dispersive channel, but I/Q imbalance at the receiver is assumed negligible. However, it is difficult to entirely eliminate the I/Q imbalance at the receiver in the analog domain, due to power consumption, size and cost of the devices. In this paper, the problem of joint CFO and transmitter and receiver I/Q imbalance compensation for the uplink of a SC-IFDMA wireless network is tackled; in particular, a *widely-linear* [14, 15, 16] time-variant compensation scheme, based on the minimum mean-output energy (MMOE) criterion, is proposed.

2. THE UPLINK SC-IFDMA SYSTEM MODEL

We consider the uplink of an SC-IFDMA system with M available subcarriers and $K \leq K_{\text{m}}$ active users, which transmit their information-bearing signals to a common base-station (BS). Each user is assigned a disjoint set of $M_{\text{u}} \triangleq$

D. Darsena, G. Gelli, and F. Verde are also with Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT), Research Unit of Napoli. This work was partially supported by the Italian National Projects "Harbour Traffic Optimization System" (HABITAT) and "Servizi per l'Infrastruttura di Rete wIreless Oltre il 3G" (SIRIO).

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M/K_{m} interleaved subcarriers, and both user terminals and the BS are equipped with single-antenna transceivers.

The data block transmitted by the kth user in the nth SC-IFDMA symbol period is $\mathbf{s}_k(n) \in \mathbb{C}^{M_{\text{u}}}$, with $k \in \{1, 2, \ldots, K\}$ and $n \in \mathbb{Z}$. Hereinafter, we assume that **(a1)**: $\mathbf{s}_k(n)$ is a zero-mean circularly-symmetric complex (ZMCSC) random vector, having autocorrelation matrix $\mathbf{R}_{\mathbf{s}_k\mathbf{s}_k} \triangleq \text{E}[\mathbf{s}_k(n)\, \mathbf{s}_k^H(n)] = \sigma_s^2\, \mathbf{I}_{M_{\text{u}}}$, with $\mathbf{s}_{k_1}(n_1)$ statistically independent of $\mathbf{s}_{k_2}(n_2)$ for $k_1 \neq k_2$ or $n_1 \neq n_2$. The vector $\mathbf{s}_k(n)$ is converted into frequency domain through M_{u}-point DFT, thus obtaining $\widetilde{\mathbf{s}}_k(n) \triangleq \mathbf{W}_{\text{dft}}\, \mathbf{s}_k(n)$, where $\mathbf{W}_{\text{dft}} \in \mathbb{C}^{M_{\text{u}} \times M_{\text{u}}}$ denotes the unitary M_{u}-point DFT matrix; the block $\widetilde{\mathbf{s}}_k(n)$ undergoes SC-IFDMA subcarrier mapping, followed by conventional OFDM precoding [17, 18, 19], encompassing M-point inverse DFT (IDFT) and CP insertion:

$$\mathbf{u}_k(n) = \mathbf{T}_{\text{cp}}\, \mathbf{W}_k\, \widetilde{\mathbf{s}}_k(n) \tag{1}$$

where $\mathbf{T}_{\text{cp}} \in \mathbb{R}^{P \times M}$ accounts for the insertion of the CP of length L_{cp}, with $P \triangleq M + L_{\text{cp}}$, whereas $\mathbf{W}_k \in \mathbb{C}^{M \times M_{\text{u}}}$ models the cascade of the interleaved carrier assignment scheme for the kth user and the M-point IDFT [13]. It can be easily verified [22] that $\mathbf{W}_k = \mathbf{\Delta}_k(\mathbf{1}_{K_{\text{m}}} \otimes \mathbf{W}_{\text{dft}}^{-1})$, where $\mathbf{\Delta}_k \triangleq K_{\text{m}}^{-1/2}\, \text{diag}[1, e^{j\frac{2\pi}{M}\bar{\iota}_k}, \ldots, e^{j\frac{2\pi}{M}\bar{\iota}_k(M-1)}]$, with $\bar{\iota}_k \in \{0, 1, \ldots, K_{\text{m}} - 1\}$ representing the index of the first subcarrier allocated to the kth user; consequently, the kth user transmitted block (1) admits the equivalent expression

$$\mathbf{u}_k(n) = \mathbf{T}_{\text{cp}}\, \mathbf{\Delta}_k\, [\mathbf{1}_{K_{\text{m}}} \otimes \mathbf{s}_k(n)] \tag{2}$$

which enlightens the well-known fact [8] that SC-IFDMA subcarrier mapping can be implemented by repeating the block $\mathbf{s}_k(n)$ K_{m} times and multiplying each block by a suitable complex exponential to perform frequency shift.

After digital-to-analog conversion, the analog baseband signal obtained from $\mathbf{u}_k(n)$ is up-converted to radio-frequency using a local oscillator (LO). Ideally, the LO outputs for the in-phase (I) and quadrature-phase (Q) branches should have equal amplitudes and phase difference of $\pi/2$; however, in practice, the matching of I and Q signals is often imperfect, which originates amplitude and phase imbalances between I and Q signals. Adopting the notation of [6], the effect of such I/Q imbalances at the kth transmitter can be equivalently described in the discrete-time baseband model by assuming perfect up-conversion of a distorted version $\widetilde{\mathbf{u}}_k(n)$ of (1), given by

$$\widetilde{\mathbf{u}}_k(n) \triangleq \alpha_k\, \mathbf{u}_k(n) + \beta_k\, \mathbf{u}_k^*(n) \tag{3}$$

where $\alpha_k \triangleq \cos(\Delta\phi_k) + j\Delta a_k \sin(\Delta\phi_k)$ and $\beta_k \triangleq \Delta a_k \cos(\Delta\phi_k) - j\sin(\Delta\phi_k)$, with Δa_k and $\Delta\phi_k$ denoting the amplitude-imbalance and the phase mismatch of the kth user transmitter, respectively.

After propagation over the wireless channel, modeled as a time-invariant finite-impulse response (FIR) filter, whose maximum order L_{max} (including the effects of possible user asynchronisms) does not exceed[1] the CP length, that is, $L_{\text{max}} \leq L_{\text{cp}}$, the received signal at the BS is down-converted to baseband, sampled at rate $1/T_{\text{c}}$, and subject to CP removal to achieve perfect IBI suppression. In the absence of I/Q receiver imbalance, the vector $\mathbf{r}(n) \in \mathbb{C}^M$ of the baseband received samples could be written as

$$\mathbf{r}(n) = \sum_{k=1}^{K} e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\, \mathbf{\Omega}_k\, \widetilde{\mathbf{H}}_k\, \widetilde{\mathbf{u}}_k(n) + \mathbf{w}(n) \tag{4}$$

where, with reference to the kth user, ϵ_k denotes the carrier frequency offset (CFO) normalized to the subcarrier spacing $1/(MT_{\text{c}})$, that is, $|\epsilon_k| \leq 0.5$, $\mathbf{\Omega}_k \triangleq \text{diag}[1, e^{j\frac{2\pi}{M}\epsilon_k}, \ldots, e^{j\frac{2\pi}{M}\epsilon_k(M-1)}] \in \mathbb{C}^{M \times M}$, $\widetilde{\mathbf{H}}_k \in \mathbb{C}^{M \times P}$ is the lower-triangular Toeplitz channel matrix, whereas $\mathbf{w}(n) \in \mathbb{C}^M$ accounts for the thermal noise at the BS and is modeled as **(a2)**: a Gaussian ZMCSC random vector, statistically independent of $\mathbf{s}_k(n)$, with autocorrelation matrix $\mathbf{R}_{\mathbf{ww}} \triangleq \text{E}[\mathbf{w}(n)\, \mathbf{w}^H(n)] = \sigma_w^2\, \mathbf{I}_M$, with $\mathbf{w}(n_1)$ statistically independent of $\mathbf{w}(n_2)$ for $n_1 \neq n_2 \in \mathbb{Z}$.

To take into account I/Q imbalance effects at the receiver, a model similar to (3) can be adopted:

$$\widetilde{\mathbf{r}}(n) = \alpha_{\text{R}}\, \mathbf{r}(n) + \beta_{\text{R}}\, \mathbf{r}^*(n) \tag{5}$$

where $\mathbf{r}(n)$ is given by (4) and α_{R} and β_{R} are defined similarly to α_k and β_k. By substituting (3) and (4) into (5), one has:

$$\begin{aligned}\widetilde{\mathbf{r}}(n) = \sum_{k=1}^{K} \big[&e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\, \alpha_{\text{R}}\, \alpha_k\, \mathbf{\Omega}_k\, \widetilde{\mathbf{H}}_k \\ &+ e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\, \beta_{\text{R}}\, \beta_k^*\, \mathbf{\Omega}_k^*\, \widetilde{\mathbf{H}}_k^*\big]\mathbf{u}_k(n) \\ &+ \sum_{k=1}^{K} \big[e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\, \alpha_{\text{R}}\, \beta_k\, \mathbf{\Omega}_k\, \widetilde{\mathbf{H}}_k \\ &+ e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\beta_{\text{R}}\, \alpha_k^*\, \mathbf{\Omega}_k^*\, \widetilde{\mathbf{H}}_k^*\big]\mathbf{u}_k^*(n) + \mathbf{v}(n)\end{aligned} \tag{6}$$

with $\mathbf{v}(n) \triangleq \alpha_{\text{R}}\, \mathbf{w}(n) + \beta_{\text{R}}\, \mathbf{w}^*(n)$. By taking into account (2) and exploiting the circulant structure of the resulting $\widetilde{\mathbf{H}}_k\, \mathbf{T}_{\text{cp}}$ matrix, it can be shown that $(\widetilde{\mathbf{H}}_k\, \mathbf{T}_{\text{cp}})\, \mathbf{\Delta}_k = \mathbf{\Delta}_k\, \mathbf{H}_k$ and $(\widetilde{\mathbf{H}}_k\, \mathbf{T}_{\text{cp}})\, \mathbf{\Delta}_k^* = \mathbf{\Delta}_k^*\, \mathbf{H}_{\text{mir},k}$, where the $M \times M$ circulant matrices $\mathbf{H}_k$ and $\mathbf{H}_{\text{mir},k}$ can be partitioned as shown by (7) at the top of the next page. As a consequence of the aforementioned rearrangements, by further partitioning the diagonal matrices $\mathbf{\Omega}_k\mathbf{\Delta}_k = \text{diag}[\mathbf{\Omega}_k^{(0)}\mathbf{\Delta}_k^{(0)}, \ldots, \mathbf{\Omega}_k^{(K_{\text{m}}-1)}\mathbf{\Delta}_k^{(K_{\text{m}}-1)}]$ and $\mathbf{\Omega}_k\mathbf{\Delta}_k^* = \text{diag}[\mathbf{\Omega}_k^{(0)}\mathbf{\Delta}_k^{(0)*}, \ldots, \mathbf{\Omega}_k^{(K_{\text{m}}-1)}\mathbf{\Delta}_k^{(K_{\text{m}}-1)*}]$, with $\mathbf{\Omega}_k^{(\ell)}\mathbf{\Delta}_k^{(\ell)} \in \mathbb{C}^{M_{\text{u}} \times M_{\text{u}}}$ and $\mathbf{\Omega}_k^{(\ell)}\mathbf{\Delta}_k^{(\ell)*} \in \mathbb{C}^{M_{\text{u}} \times M_{\text{u}}}$, for $\ell \in \{0, 1, \ldots, K_{\text{m}} - 1\}$, and exploiting the block circulant struc-

[1] When the latter assumption is violated, a channel shortening equalizer can be employed [21] at the receiver.

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_k^{(0)} & \mathbf{H}_k^{(1)} & \ldots & \mathbf{H}_k^{(K_m-2)} & \mathbf{H}_k^{(K_m-1)} \\ \mathbf{H}_k^{(K_m-1)} & \mathbf{H}_k^{(0)} & \ldots & \mathbf{H}_k^{(K_m-3)} & \mathbf{H}_k^{(K_m-2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_k^{(1)} & \mathbf{H}_k^{(2)} & \ldots & \mathbf{H}_k^{(K_m-1)} & \mathbf{H}_k^{(0)} \end{bmatrix} \quad \mathbf{H}_{\text{mir},k} = \begin{bmatrix} \mathbf{H}_{\text{mir},k}^{(0)} & \mathbf{H}_{\text{mir},k}^{(1)} & \ldots & \mathbf{H}_{\text{mir},k}^{(K_m-2)} & \mathbf{H}_{\text{mir},k}^{(K_m-1)} \\ \mathbf{H}_{\text{mir},k}^{(K_m-1)} & \mathbf{H}_{\text{mir},k}^{(0)} & \ldots & \mathbf{H}_{\text{mir},k}^{(K_m-3)} & \mathbf{H}_{\text{mir},k}^{(K_m-2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_{\text{mir},k}^{(1)} & \mathbf{H}_{\text{mir},k}^{(2)} & \ldots & \mathbf{H}_{\text{mir},k}^{(K_m-1)} & \mathbf{H}_{\text{mir},k}^{(0)} \end{bmatrix} \quad (7)$$

ture of the channel matrices given by (7), eq. (6) becomes

$$\begin{aligned} \widetilde{\mathbf{r}}(n) = \sum_{k=1}^{K} \big[& e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \alpha_R \, \alpha_k \, \boldsymbol{\Psi}_k \, \widetilde{\boldsymbol{\mathcal{H}}}_k \\ & + e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \beta_R \, \beta_k^* \, \boldsymbol{\Psi}_{\text{mir},k}^* \, \widetilde{\boldsymbol{\mathcal{H}}}_{\text{mir},k}^* \big] \, \mathbf{s}_k(n) \\ & + \sum_{k=1}^{K} \big[e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \alpha_R \, \beta_k \, \boldsymbol{\Psi}_{\text{mir},k} \, \widetilde{\boldsymbol{\mathcal{H}}}_{\text{mir},k} \\ & + e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \beta_R \alpha_k^* \boldsymbol{\Psi}_k^* \widetilde{\boldsymbol{\mathcal{H}}}_k^* \big] \mathbf{s}_k^*(n) + \mathbf{v}(n) \quad (8) \end{aligned}$$

where, for $\ell \in \{0, 1, \ldots, K_m - 1\}$ and $k \in \{1, 2, \ldots, K\}$, the following matrices have been defined:

$$\begin{aligned} \boldsymbol{\Psi}_k^{(\ell)} &\triangleq \boldsymbol{\Omega}_k^{(\ell)} \, \boldsymbol{\Delta}_k^{(\ell)} \in \mathbb{C}^{M_u \times M_u} \\ \boldsymbol{\Psi}_k &\triangleq [\boldsymbol{\Psi}_k^{(0)}, \boldsymbol{\Psi}_k^{(1)}, \ldots, \boldsymbol{\Psi}_k^{(K_m-1)}]^T \in \mathbb{C}^{M \times M_u} \\ \boldsymbol{\Psi}_{\text{mir},k}^{(\ell)} &\triangleq \boldsymbol{\Omega}_k^{(\ell)} \, \boldsymbol{\Delta}_k^{(\ell)*} \in \mathbb{C}^{M_u \times M_u} \\ \boldsymbol{\Psi}_{\text{mir},k} &\triangleq [\boldsymbol{\Psi}_{\text{mir},k}^{(0)}, \boldsymbol{\Psi}_{\text{mir},k}^{(1)}, \ldots, \boldsymbol{\Psi}_{\text{mir},k}^{(K_m-1)}]^T \in \mathbb{C}^{M \times M_u} \\ \widetilde{\boldsymbol{\mathcal{H}}}_k &\triangleq \sum_{\ell=0}^{K_m-1} \mathbf{H}_k^{(\ell)} \in \mathbb{C}^{M_u \times M_u} \\ \widetilde{\boldsymbol{\mathcal{H}}}_{\text{mir},k} &\triangleq \sum_{\ell=0}^{K_m-1} \mathbf{H}_{\text{mir},k}^{(\ell)} \in \mathbb{C}^{M_u \times M_u} \, . \end{aligned}$$

Finally, let $\boldsymbol{\Psi} \triangleq [\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \ldots, \boldsymbol{\Psi}_K] \in \mathbb{C}^{M \times (KM_u)}$ and $\boldsymbol{\Psi}_{\text{mir}} \triangleq [\boldsymbol{\Psi}_{\text{mir},1}, \boldsymbol{\Psi}_{\text{mir},2}, \ldots, \boldsymbol{\Psi}_{\text{mir},K}] \in \mathbb{C}^{M \times (KM_u)}$, the received signal model in the presence of both transmitter and receiver I/Q imbalances as well as CFOs is given by

$$\begin{aligned} \widetilde{\mathbf{r}}(n) = \alpha_R \, \boldsymbol{\Psi} \, \mathbf{a}(n) + \alpha_R \, \boldsymbol{\Psi}_{\text{mir}} \, \mathbf{a}_{\text{mir}}(n) \\ + \beta_R \, \boldsymbol{\Psi}^* \, \mathbf{a}^*(n) + \beta_R \, \boldsymbol{\Psi}_{\text{mir}}^* \, \mathbf{a}_{\text{mir}}^*(n) + \mathbf{v}(n) \quad (9) \end{aligned}$$

where $\mathbf{a}(n) \in \mathbb{C}^{KM_u}$ and $\mathbf{a}_{\text{mir}}(n) \in \mathbb{C}^{KM_u}$ are vertical stackings of $\mathbf{a}_k(n) \triangleq e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \alpha_k \, \widetilde{\boldsymbol{\mathcal{H}}}_k \, \mathbf{s}_k(n) \in \mathbb{C}^{M_u}$ and $\mathbf{a}_{\text{mir},k}(n) \triangleq e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{cp})} \beta_k \, \widetilde{\boldsymbol{\mathcal{H}}}_{\text{mir},k} \, \mathbf{s}_k^*(n)$, respectively, for $k = 1, 2, \ldots, K$. Eq. (9) shows that uncompensated CFOs, as well as I/Q imbalance effects, disrupt the subcarrier orthogonality among users, which gives rise to intercarrier interference (ICI) and multiple-access interference (MAI).

3. CFO AND I/Q IMBALANCE COMPENSATION

Since the combined action of I/Q imbalances at the transmitter and receiver ends, along with CFO effects, may severely degrade the system performance, we propose to employ at the receiver a multiuser CFO compensation scheme with I/Q mitigation capabilities, synthesized under the minimum mean-output-energy (MMOE) criterion.

It can preliminarily be shown that the conjugate autocorrelation matrix $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) \triangleq \text{E}[\widetilde{\mathbf{r}}(n) \, \widetilde{\mathbf{r}}^T(n)]$ of $\widetilde{\mathbf{r}}(n)$ is nonzero and, thus, the received vector $\widetilde{\mathbf{r}}(n)$ turns out to be improper [20]; in this case, *widely-linear* processing schemes [14], which jointly elaborates $\widetilde{\mathbf{r}}(n)$ and its complex conjugate counterpart $\widetilde{\mathbf{r}}^*(n)$, are expected to significantly improve the performance upon linear ones. In addition, it results that both $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) \triangleq \text{E}[\widetilde{\mathbf{r}}(n) \, \widetilde{\mathbf{r}}^H(n)]$ and $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n)$ are time-variant matrices, suggesting that a time-variant processing of the received data should be employed.

Therefore, let $\mathbf{F}(n) \in \mathbb{C}^{(2KM_u) \times (2M)}$, the input-output relationship of the CFO compensation scheme is given by

$$\mathbf{z}(n) = \mathbf{F}(n) \, \widetilde{\mathbf{z}}(n) \quad (10)$$

where $\widetilde{\mathbf{z}}(n) \triangleq [\widetilde{\mathbf{r}}^T(n) \, , \, \widetilde{\mathbf{r}}^H(n)]^T \in \mathbb{C}^{2M}$ is given by

$$\begin{aligned} \widetilde{\mathbf{z}}(n) &= \underbrace{\begin{bmatrix} \alpha_R \, \boldsymbol{\Psi} & \beta_R \, \boldsymbol{\Psi}_{\text{mir}}^* \\ \beta_R^* \, \boldsymbol{\Psi} & \alpha_R^* \, \boldsymbol{\Psi}_{\text{mir}}^* \end{bmatrix}}_{\boldsymbol{\Phi}_0 \in \mathbb{C}^{(2M) \times (2KM_u)}} \underbrace{\begin{bmatrix} \mathbf{a}(n) \\ \mathbf{a}_{\text{mir}}^*(n) \end{bmatrix}}_{\boldsymbol{\xi}_0(n) \in \mathbb{C}^{2KM_u}} \quad (11) \\ &+ \underbrace{\begin{bmatrix} \beta_R \, \boldsymbol{\Psi}^* & \alpha_R \, \boldsymbol{\Psi}_{\text{mir}} \\ \alpha_R^* \, \boldsymbol{\Psi}^* & \beta_R^* \, \boldsymbol{\Psi}_{\text{mir}} \end{bmatrix}}_{\boldsymbol{\Phi}_1 \in \mathbb{C}^{(2M) \times (2KM_u)}} \underbrace{\begin{bmatrix} \mathbf{a}^*(n) \\ \mathbf{a}_{\text{mir}}(n) \end{bmatrix}}_{\boldsymbol{\xi}_1(n) \in \mathbb{C}^{2KM_u}} + \underbrace{\begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}^*(n) \end{bmatrix}}_{\boldsymbol{\eta}(n) \in \mathbb{C}^{2M}} \quad (12) \\ &= \boldsymbol{\Phi}_0 \, \boldsymbol{\xi}_0(n) + \boldsymbol{\Phi}_1 \, \boldsymbol{\xi}_1(n) + \boldsymbol{\eta}(n) \, . \quad (13) \end{aligned}$$

To compensate for the CFOs of all the users, it would be sufficient to impose the *zero-forcing* (ZF) condition $\mathbf{F}(n) \, \boldsymbol{\Phi}_0 = \mathbf{I}_{2KM_u}$, which admits solution if $\boldsymbol{\Phi}_0$ is full-column rank, i.e., **(c1)**: $2M \geq 2KM_u$ (which is always verified since $K_m \geq K$) and **(c2)**: $\text{rank}(\boldsymbol{\Phi}_0) = 2KM_u$. However, the ZF solution is not unique and thus the remaining degrees of freedom can be exploited to mitigate I/Q imbalance effects. We propose therefore to synthesize $\mathbf{F}(n)$ by minimizing the mean-output-energy at the filter output, subject to the ZF constraint:

$$\mathbf{F}_{\text{mmoe}}(n) = \arg \min_{\mathbf{F}(n)} \text{E}[\|\mathbf{z}(n)\|^2] \quad \text{s.to} \quad \mathbf{F}(n) \, \boldsymbol{\Phi}_0 = \mathbf{I}_{2KM_u} \quad (14)$$

whose solution can be expressed as $\mathbf{F}_{\text{mmoe}}(n) = [\boldsymbol{\Phi}_0^H \mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}^{-1}(n) \, \boldsymbol{\Phi}_0]^{-1} \boldsymbol{\Phi}_0^H \mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}^{-1}(n)$, where $\mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}(n) \triangleq \text{E}[\widetilde{\mathbf{z}}(n) \, \widetilde{\mathbf{z}}^H(n)]$ is the

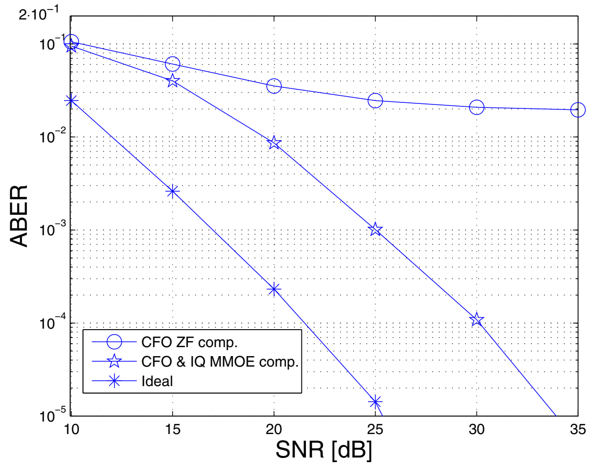

Fig. 1. ABER versus SNR.

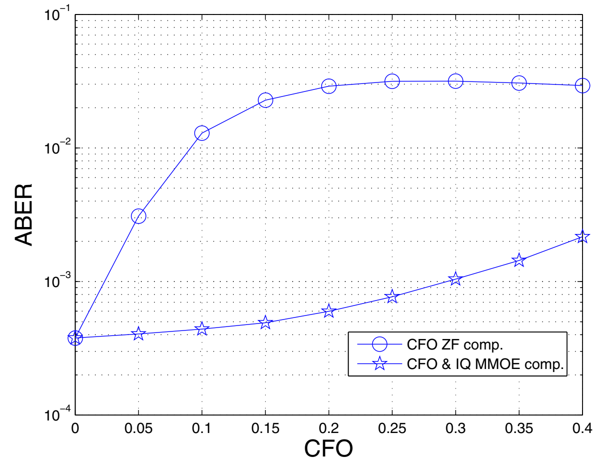

Fig. 2. ABER versus CFO.

autocorrelation matrix of the stacked received vector $\widetilde{\mathbf{z}}(n)$:

$$\mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}(n) = \begin{bmatrix} \mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) & \mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) \\ \mathbf{R}^*_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) & \mathbf{R}^*_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) \end{bmatrix} . \qquad (15)$$

After MMOE filtering, having restored subcarrier orthogonality, the user can be separated by picking out the $(2M_{\text{u}})$-dimensional sub-vectors of $\mathbf{z}(n)$. Thus, assuming that the MAI term is negligibly small, the kth-user data vector is

$$\mathbf{q}_k(n) \triangleq \mathbf{R}_k\,\mathbf{z}(n) \approx \begin{bmatrix} e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\alpha_k \widetilde{\boldsymbol{\mathcal{H}}}_k \\ e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\beta_k^* \widetilde{\boldsymbol{\mathcal{H}}}^*_{\text{mir},k} \end{bmatrix} \mathbf{s}_k(n) + \mathbf{R}_k\,\mathbf{F}_{\text{mmoe}}(n)\,\boldsymbol{\eta}(n) \qquad (16)$$

where $\mathbf{R}_k \triangleq \mathbf{1}_2 \otimes [\mathbf{O}_{(k-1)M_{\text{u}}\times M_{\text{u}}}, \mathbf{I}_{M_{\text{u}}}, \mathbf{O}_{(K-k)M_{\text{u}}\times M_{\text{u}}}] \in \mathbb{R}^{(2M_{\text{u}})\times(2KM_{\text{u}})}$, with $\mathbf{1}_2 \triangleq [1,1]^T$ and $\otimes$ denoting the Kronecker product. At this point, the symbol block $\mathbf{s}_k(n)$ transmitted by the kth user is detected according to the minimum mean-square error (MMSE) criterion.

4. SIMULATION RESULTS

We simulate a SC-IFDMA system, employing a total of $M = 64$ QPSK-modulated subcarriers, with a CP of length $L_{\text{cp}} = 16$, and a maximum of $K_{\text{m}} = 4$ users. We assume that only $K = 3$ users are active, whose CFOs are set to $\epsilon_1 = 0.15$, $\epsilon_2 = 0$, and $\epsilon_3 = -0.15$. Each transmitter suffers from severe I/Q imbalance, with $\Delta\phi = \pi/18$ and $\Delta a = 0.5$, which results in a value of the image rejection ratio (IRR) [6] of 5 dB; moreover, the BS is affected by I/Q imbalance, with $\Delta\phi = \pi/20$ and $\Delta a = 0.25$, resulting in IRR $= 10$ dB. The FIR user channels are of maximum order $L_{\text{max}} = L_{\text{cp}}$, whose values are modeled as independent and identically distributed ZMCSC Gaussian random variables with variance $\sigma_h^2(\ell)$, with $\sigma_h^2(\ell) = \sigma_h^2(0)\,\exp(-0.1\,\ell)$, for $\ell \in \{0, 1, \ldots, L_{\text{max}}\}$, where $\sigma_h^2(0)$ ensures a unit average energy for the channel impulse response. The first user is chosen as the desired one; relying on (4), the SNR is defined as SNR $\triangleq$ E$[\|\boldsymbol{\Omega}_1\,\widetilde{\mathbf{H}}_1\,\widetilde{\mathbf{u}}_1(n)\|^2]/E[\|\mathbf{w}(n)\|^2] = \sigma_s^2\,[(|\alpha_1|^2 + |\beta_1|^2)\|\boldsymbol{\Omega}_1\,\widetilde{\mathbf{H}}_1\|^2]/(M\,\sigma_w^2)$. As a figure of merit, we report the average (over all subcarriers) bit-error-rate (ABER) of the proposed MMOE compensation scheme, in comparison with that of two simpler receivers: the first one (referred to as "CFO ZF comp.") performs ZF CFO compensation, by employing a time-invariant matrix $\mathbf{F} = \boldsymbol{\Phi}_0^{-1}$, without taking any countermeasure against I/Q imbalances; the second one (referred to as "Ideal") is a benchmark, working in the absence of CFO and I/Q imbalance impairments.

The results of Fig. 1 clearly show that the "CFO ZF comp." receiver exhibits poor overall performances, since it does not compensate for I/Q imbalance effects, which represent the predominant source of performance degradation; on the contrary, the proposed "CFO & IQ MMOE comp." receiver exhibits the best overall performances, paying a SNR penalty of roughly 8 dB with respect to its ideal counterpart.

In Fig. 2, we report the ABER as a function of the CFO ϵ for SNR = 26 dB; in particular, we set $\epsilon_2 = 0$, whereas $\epsilon_1 = -\epsilon_3 = \epsilon$. Results of Fig. 2 show that the proposed receiver exhibits a remarkable robustness against CFOs variations, assuring satisfactory performances in the whole range of the considered values of ϵ.

5. CONCLUSIONS

In this paper, the problem of joint CFO and transmitter and receiver I/Q imbalance compensation for the uplink of a SC-IFDMA wireless network has been tackled; in particular, a widely-linear time-variant compensation scheme, based on the minimum mean-output energy criterion, has been proposed. Numerical results show that the proposed compensation scheme is able to restore the orthogonality among users, showing remarkable MAI mitigation capabilities.

6. REFERENCES

[1] H. G. Myunga, J. Lim, and D. J. Goodman, "Single carrier FDMA for uplink wireless transmission," *IEEE Veh. Technol. Mag.*, vol. 1, pp. 30–38, Sep. 2006.

[2] 3rd Generation Partnership Project (3GPP) Technical Specification Group Radio Access network: Physical layer aspects for evolved Universal Terrestrial Radio Access (UTRA) Release 7, 3GPP TR 25.814, V7.1.0, Sep. 2006.

[3] H. Ekstrom, A. Furuskar, J. Karlsson, M. Meyer, S. Parkvall, J. Torsner, and M. Wahlqvist, "Technical solutions for the 3G long-term evolution," *IEEE Commun. Mag.*, vol. 44, pp. 38–45, Mar. 2006.

[4] T. Frank, A. Klein, and E. Costa, "IFDMA: a scheme combining the advantages of OFDMA and CDMA," *IEEE Wireless Commun.*, pp. 9–17, June 2007.

[5] A. Wilzeck, Q.-Cai, M.-Schiewer, and T. Kaiser, "Effect of multiple carrier frequency offsets in MIMO SC-FDMA systems," *Proc. International ITG/IEEE Workshop Smart Antennas (WSA 2007)/*, Austria, Feb. 2007.

[6] Y. Yoshida, K. Hayashi, H. Sakai, and W. Bocquet, "Analysis and compensation of transmitter IQ imbalances in OFDMA and SC-FDMA systems", *IEEE Trans. Signal Process.*, vol. 57, pp. 3119–3129, Aug. 2009.

[7] M. Morelli, C.-C.J. Kuo, and M.-O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): a tutorial review," *Proc. IEEE*, vol. 95, pp. 1394–1427, July 2007.

[8] Y. Zhu and K.B. Letaief, "CFO estimation and compensation in SC-IFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3200–3212, Oct. 2010.

[9] D. Tandur and M. Moonen, "Joint adaptive compensation of transmitter and receiver IQ imbalance under carrier frequency offset in OFDM-based systems", *IEEE Trans. Signal Process.*, vol. 55, pp. 5246–5252, Nov. 2007.

[10] J. Tubbax, B. Come, L. Van der Perre, S. Donnay, M. Engels, M. Moonen, and H. De Man "Joint compensation of IQ imbalance and frequency offset in OFDM systems", *Global Telecommunications Conference (GLOBECOM '03)*, Dec. 2003.

[11] I. Barhumi and M. Moonen, "IQ-imbalance compensation for OFDM in the presence of IBI and carrier-frequency offset", *IEEE Trans. Signal Process.*, vol. 55, pp. 256–266, Jan. 2007.

[12] D. Darsena, G. Gelli, and F. Verde, "Joint I/Q imbalances and CFOs compensation for multiuser wireless networks in the presence of IBI," *IEEE International workshop on Measurements and Networking Proceedings*, Oct. 2013.

[13] D. Darsena, G. Gelli, and F. Verde, "Joint blind channel shortening and compensation of transmitter I/Q imbalances and CFOs for uplink SC-IFDMA systems," *Physical Communication*, Oct. 2013.

[14] B. Picinbono and P. Chevalier, "Widely linear estimation with complex data", *IEEE Trans. Signal Process.*, vol. 43, pp. 2030–2033, Aug. 1995.

[15] D. Darsena, G. Gelli, L. Paura, and F. Verde, "Widely linear equalization and blind channel identification for interference-contaminated multicarrier systems", *IEEE Trans. Signal Process.*, vol. 53, pp. 1163–1177, Mar. 2005.

[16] D. Darsena, G. Gelli, L. Paura, and F. Verde, "Subspace-based blind channel identification of SISO-FIR systems with improper random inputs", *Signal Processing*, vol. 84, pp. 2021–2039, Nov. 2004.

[17] D. Darsena, "Successive narrowband interference cancellation for OFDM systems", *IEEE Communications Letters*, vol. 11, pp. 73–75, Jan. 2007.

[18] D. Darsena and F. Verde, "Successive NBI cancellation using soft decisions for OFDM systems", *IEEE Signal Processing Letters*, vol. 15, pp. 873–876, 2008.

[19] D. Darsena, G. Gelli, and F. Verde, "Perfect symbol recovery and NBI suppression in MIMO-OFDM systems", *Electronics Letters*, vol. 50, pp. 225–227, Jan. 2014.

[20] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. Inf. Theory*, vol. 39, pp. 1293–1302, July 1993.

[21] D. Darsena, G. Gelli, L. Paura, and F. Verde, "Blind channel shortening for space-time-frequency block coded MIMO-OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 1022–1033, Mar. 2012.

[22] D. Darsena, G. Gelli, L. Paura, and F. Verde, "Blind channel shortening for asynchronous SC-IFDMA systems with CFOs," *IEEE Trans. Wireless Commun.*, vol. 12, pp. 5529–5543, Nov. 2013.

[23] A. Ben-Israel and T. N. E. Greville, *Generalized Inverses*. New York: Springer-Verlag, 2002.