Single-Carrier Modulation with ML Equalization for Large-Scale Antenna Systems over Rician Fading Channels

Yinsheng[†], Zhenhui Tan[†], and Geoffrey Ye Li[‡]
[†]School of EIE, Beijing Jiaotong University, China, email:{09111035,zhhtan}@bjtu.edu.cn
[‡] School of ECE, Georgia Institute of Technology, USA, email:liye@ece.gatech.edu

***Abstract*—In this paper, we investigate *maximum likelihood equalization* (MLE) for a *large-scale antenna* (LSA) system with *single-carrier* (SC) modulation over Rician fading channels. *Orthogonal frequency division multiplexing* (OFDM) is usually used to deal with frequency selectivity of wireless channels. However, for a wireless system with large-scale antennas in a Rayleigh fading channel, by combining the received signals through a *matched filter* (MF), the frequency selective channel can be converted into a frequency flat channel. As a result, SC modulation can be used directly with a simple one-tap equalizer. In a Rician fading channel, however, the *line-of-sight* (LOS) path will cause *mutliuser-interference* (MUI), which cannot be mitigated through MF. As a result, the simple one-tap equalizer leads to an error-floor when the *signal-to-noise ratio* (SNR) is large. In this paper, MLE is used to improve system performance through multiuser detection. From both theoretical analysis and simulation results, the proposed approach can eliminate the error floor and outperform existing approach.**

***Index Terms*—large-scale antenna, single carrier, OFDM, multiuser detection, MLE**

I. Introduction

Multiple-input multiple-output (MIMO) system has been widely studied during the last couple of decades due to its multiplexing and diversity gain and interference suppression capability. The *large-scale antenna* (LSA) system, where the *base station* (BS) is equipped with hundreds of antennas, has gained a lot of attention recently [1]–[4]. Through the employment of an excess number of antennas at the BS, the channel vectors between *user terminal*s (UT) and BS are asymptotically pairwisely orthogonal and the simple *matched filter* (MF) becomes the optimal the detector [5]. Given the number of antennas at the BS, M, it is shown that the transmit power for each UT scales down as $1/M$ or $1/\sqrt{M}$ depending on whether accurate channel parameters are known or not [6]. As a result, the transmit powers of UTs can be arbitrarily small when the number of antennas at the BS goes to infinity. LSA also allows multiusers to be allocated in the same system bandwidth, and thus can increase the spectrum efficiency of the network and energy efficiency of UTs [7].

In traditional MIMO research work, channels are generally assumed to be frequency flat [8] since *orthogonal frequency division multiplexing* (OFDM) is used to convert a frequency selective channel into a group of frequency flat fading channels. Most of existing work in LSA systems considers frequency flat channels [3], [5], [6], which obviously presumes that OFDM is used. In that case, the BS needs hundreds of *discrete Fourier transform* (DFT) to demodulate OFDM signals from each antenna, which is with heavy computational burden. OFDM signal also has a high *peak-to-average power ratio* (PAPR), resulting in low efficiency of the power amplifier. Moreover, *cyclic prefix* (CP) in OFDM wastes some frequency resource. For those reasons, *single-carrier* (SC) modulation has been considered for downlink precoding in LSA systems [7]. Although the approach in [7] is proposed for downlink precoding, it can be easily used for uplink detection due to the duality between downlink and uplink [8]. It is also shown in [9] that the array gain can be still achieved even under the constraint that the per-antenna signal has constant envelope.

In this paper, we investigate *maximum-likelihood equalization* (MLE) in LSA-SC for uplink transmission over a Rician fading channel. Similar to the case in Rayleigh fading channel, the proposed MLE can mitigate the *inter-symbol interference* (ISI) by coherently combining the received signals from different antennas through a MF. As a result, SC modulation can be used directly with a simple one-tap equalizer and OFDM is not necessary. However, in a Rician fading channel, the *line-of-sight* (LOS) component causes MUI in the presence of multiple UTs, which can not be mitigated using MF. In turn, the simple one-tap equalizer leads to an error-floor when the *signal-to-noise ratio* (SNR) is large. In this paper, the proposed MLE suggests a multiuser detection approach, which can cancel the error-floor effect at high SNR. Both theoretical analysis and simulation results show that the proposed MLE can outperform the existing approach [7].

The rest of this paper is organized as follows. The system model of a LSA-SC system is presented in Section II. In Section III, the MLE is presented as well as some discussions. We analyze performance for the proposed LSA-SC system in Section IV. Simulation results are shown in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

We assume that each UT has only one antenna. Denote $h_k[l,m]$ to be the l-th discrete path from the k-th UT to the m-th receive antenna at the BS. In a Rician fading channel, the first path contains a LOS component and a Rayleigh fading component while the other paths contain only the Rayleigh

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

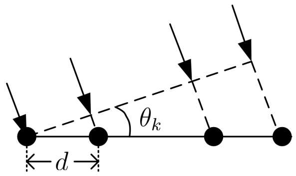


Fig. 1. LSA system using Linear antenna array.

fading components, that is [8]

$$h_k[l,m] = \mu_k[m]\delta[l] + \widetilde{h}_k[l,m], \tag{1}$$

where $\delta[\cdot]$ is the Kronecker impulse function, $\mu_k[m]$ denotes the LOS component and $\widetilde{h}_k[l,m] \sim \mathcal{CN}(0,\sigma_l^2)$, with σ_l^2 being the power of the l-th path and $\sum_{l=0}^{L}\sigma_l^2 = \sigma^2$, denotes the Rayleigh fading component. Similar to [7], we further assume that the Rayleigh fading components for different k's, different l's and different m's are independent, that is

$$\mathrm{E}(\widetilde{h}_{k_0}^*[l_0,m_0]\widetilde{h}_k[l,m]) = \sigma_l^2\delta[l_0-l]\delta[m_0-m]\delta[k_0-k], \tag{2}$$

In the presence of the LOS component, channel response, $h_k[l,m]$, has a non-zero mean. Therefore, we have

$$\begin{aligned}\mathrm{E}(h_{k_0}^*[l_0,m_0]h_k[l,m]) = \mu_{k_0}^*[m_0]\mu_k[m]\delta[l_0]\delta[l]+ \\ \sigma_l^2\delta[l_0-l]\delta[m_0-m]\delta[k_0-k].\end{aligned} \tag{3}$$

Denote $\kappa = |\mu_k[m]|^2/\sigma^2$ as the Rician factor. If the overall channel power is normalized, then the LOS component, $\mu_k[m]$, can be represented by [8]

$$\mu_k[m] = \sqrt{\frac{\kappa}{\kappa+1}}e^{j\varphi_k}e^{j\phi_k[m]}, \tag{4}$$

where φ_k is a uniform distributed phase offset for the k-th UT, $\phi_k[m]$ denotes the extra phase on the m-th antenna caused by wave propagation among antennas, and it depends on the placement of antennas and the arrival angle of signal.

Without loss of generality, we consider a linear antenna array in this paper, where the antennas are placed along a straight line, as shown in Fig. 1. From the figure, the extra phase can be represented by

$$\phi_k[m] = (m-1)2\pi\widetilde{d}\sin\theta_k, \tag{5}$$

where θ_k is the arrival angle of the k-th UT, $\widetilde{d} = d/\lambda$ is the normalized distance with d and λ being the distance between adjacent antennas and the wave length, respectively.

Define $\mathbf{h}_k[l] = (h_k[l,1], h_k[l,2], \cdots, h_k[l,M])^{\mathrm{T}} \in \mathcal{C}^{M\times 1}$ as a channel vector. Then, (1) can be rewritten in vector form as

$$\mathbf{h}_k[l] = \boldsymbol{\mu}_k\delta[l] + \widetilde{\mathbf{h}}_k[l], \tag{6}$$

where $\boldsymbol{\mu}_k = (\mu_k[1], \mu_k[2], \cdots, \mu_K[M])^{\mathrm{T}}$ and $\widetilde{\mathbf{h}}_k[l] = (\widetilde{h}_k[1], \widetilde{h}_k[2], \cdots, \widetilde{h}_k[M])^{\mathrm{T}}$ are the LOS component and the Rayleigh fading component vectors, respectively.

When the number of antennas is large enough, from (3)

$$\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{h}_k[l] \approx \frac{1}{M}\boldsymbol{\mu}_{k_0}^{\mathrm{H}}\boldsymbol{\mu}_k\delta[l_0]\delta[l] + \sigma_l^2\delta[l_0-l]\delta[k_0-k]. \tag{7}$$

Denote $\mathbf{y}[n] = (y[n,1], y[n,2], \cdots, y[n,M])^{\mathrm{T}}$ to be the receive signal vector, where $y[n,m]$ is the n-th sample of the received signal at the m-th antenna. Then,

$$\mathbf{y}[n] = \sum_{l=0}^{L}\sum_{k=1}^{K}\mathbf{h}_k[l]x_k[n-l] + \mathbf{w}[n] = \sum_{l=0}^{L}\mathbf{H}[l]\mathbf{x}[n-l] + \mathbf{w}[n], \tag{8}$$

where $\mathbf{x}[n] = (x_1[n], x_2[n], \cdots, x_K[n])^{\mathrm{T}}$ is the transmitted signal vector with $x_k[n]$ being the symbol from the k-th UT, $\mathbf{w}[n] = (w[n,1], w[n,2], \cdots, w[n,M])^{\mathrm{T}}$ is the noise vector with $w[n,m]$ being the additive noise at the m-th antenna with zero mean and $\mathrm{E}(|w[n,m]|^2) = \sigma_w^2$, and $\mathbf{H}[l] = (\mathbf{h}_1[l], \mathbf{h}_2[l], \cdots, \mathbf{h}_K[l]) \in \mathcal{C}^{M\times K}$ is the overall channel matrix.

III. Maximum Likelihood Equalization

In general, the more received signal vectors are used to estimate a transmitted signal vector, the better the estimation performance. However, if the channel has $L+1$ paths, the transmit vector, $\mathbf{x}[n]$, can only affect the following L received vectors. Therefore, it is good enough to estimate $\mathbf{x}[n]$ using $\mathbf{y}[n], \mathbf{y}[n+1], \cdots, \mathbf{y}[n+L]$. Define $\boldsymbol{\mathcal{Y}} \in \mathcal{C}^{[(L+1)M]\times 1}$, $\boldsymbol{\mathcal{X}} \in \mathcal{C}^{[(2L+1)M]\times 1}$, and $\boldsymbol{\mathcal{W}} \in \mathcal{C}^{[(L+1)M]\times 1}$ as the overall received vector, the overall transmit vector, and the overall additive noise, respectively, that is

$$\boldsymbol{\mathcal{Y}} = \begin{pmatrix}\mathbf{y}[n+L] \\ \vdots \\ \mathbf{y}[n]\end{pmatrix}, \boldsymbol{\mathcal{X}} = \begin{pmatrix}\mathbf{x}[n+L] \\ \vdots \\ \mathbf{x}[n-L]\end{pmatrix}, \boldsymbol{\mathcal{W}} = \begin{pmatrix}\mathbf{w}[n+L] \\ \vdots \\ \mathbf{w}[n]\end{pmatrix}.$$

Define $\boldsymbol{\mathcal{H}} \in \mathcal{C}^{[(L+1)M]\times[(2L+1)K]}$ to be an overall channel matrix, that is

$$\boldsymbol{\mathcal{H}} = \begin{pmatrix}\mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[L] & & & \\ & \mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[L] & & \\ & & \ddots & & & \ddots & \\ & & & \mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[L]\end{pmatrix}.$$

Then, from (8), we can obtain

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{X}} + \boldsymbol{\mathcal{W}}. \tag{9}$$

With additive white Gaussian noise, ML estimation of $\mathbf{x}[n]$ can be obtained by minimizing

$$L(\mathbf{x}[n]) = (\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{X}})^{\mathrm{H}}(\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{X}}). \tag{10}$$

Then, the ML estimation of $\mathbf{x}[n]$ can be obtained by solving

$$\frac{\partial L(\mathbf{x}[n])}{\partial \mathbf{x}^*[n]} = \mathbf{0}. \tag{11}$$

Direct calculation of (11) yields

$$\mathbf{H}^{\mathrm{H}}\boldsymbol{\mathcal{Y}} = \mathbf{H}^{\mathrm{H}}\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{X}}, \tag{12}$$

where $\mathbf{H} = \left(\mathbf{H}^{\mathrm{T}}[L], \cdots, \mathbf{H}^{\mathrm{T}}[0]\right)^{\mathrm{T}} \in \mathcal{C}^{[(L+1)M]\times K}$.

In above, the derivation is similar to that in the traditional MIMO system [8]. However, in a LSA system, (12) can be further simplified. When the antenna number is large enough, from (7), we have

$$\frac{1}{M}\mathbf{H}^{\mathrm{H}}[l_0]\mathbf{H}[l] \approx \frac{1}{M}\mathbf{\Phi}_K^{\mathrm{H}}\mathbf{\Phi}_K\delta[l_0]\delta[l] + \sigma_l^2\delta[l_0-l]\mathbf{I}, \quad (13)$$

where $\mathbf{\Phi}_K = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \cdots, \boldsymbol{\mu}_K) \in \mathcal{C}^{M\times K}$. From (13), (12) can be simplified as

$$\mathbf{H}^{\mathrm{H}}\boldsymbol{\mathcal{Y}} \approx \mathbf{H}^{\mathrm{H}}\mathbf{H}\mathbf{x}[n] \approx (\mathbf{\Phi}_K^{\mathrm{H}}\mathbf{\Phi}_K + M\sigma^2\mathbf{I})\mathbf{x}[n]. \quad (14)$$

As a result, the ML estimation of $\mathbf{x}[n]$ can be given by

$$\widehat{\mathbf{x}}[n] \approx \mathbf{A}_K^{-1}\frac{1}{M}\mathbf{H}^{\mathrm{H}}\boldsymbol{\mathcal{Y}} = \mathbf{A}_K^{-1}\widetilde{\mathbf{x}}[n], \quad (15)$$

where $\mathbf{A}_K$ is a coefficient matrix defined by

$$\mathbf{A}_K = \frac{1}{M}\mathbf{\Phi}_K^{\mathrm{H}}\mathbf{\Phi}_K + \sigma^2\mathbf{I}, \quad (16)$$

and

$$\widetilde{\mathbf{x}}[n] = \frac{1}{M}\mathbf{H}^{\mathrm{H}}\boldsymbol{\mathcal{Y}}, \quad (17)$$

can be regarded as a single user detection using MF. From (15), the ML estimation can be obtained by linear combination of the MF-based single-user detection using coefficient matrix $\mathbf{A}_K$. Note that the coefficient matrix, $\mathbf{A}_K$, depends on the average of channel rather than the instantaneous response.

We have developed MLE for LSA over Rician fading channels. When there is no LOS component, the coefficient matrix, $\mathbf{A}_K$, becomes an identity matrix, and thus the derived MLE suggests a simple MF operation as shown in (17). In this case, the proposed approach is actually the same as that in [7]. Although the work in [7] is proposed for donwlink precoding, it can be easily used for uplink demodulation according to the duality between downlink and uplink [8]. When the number of antennas is very large, the channel vectors for different UTs and different paths are asymptotically orthogonal, and we can thus extract a particular path for a particular UT using MF. As a result, the frequency selective channel is converted into a frequency flat channel for each UT, and thus a simple one-tap equalizer can be used to demodulate the received signal.

For example, if we only use one path in (15), say $l = 0$, the MLE in (15) is immediately reduced to a simple one-tap equalizer, that is

$$\widehat{x}_k[n] = \frac{\mathbf{h}_k^{\mathrm{H}}[0]\mathbf{y}[n]}{M\sigma^2}. \quad (18)$$

In the presence of the LOS component in Rician channels, the coefficient matrix, $\mathbf{A}_K$, is not diagonal any more. It indicates that the MUI caused by the LOS component will not vanish, which is different from the Rayleigh fading channels. As a result, the derived MLE suggests a multiuser detection to avoid MUI, as shown in (15). Note that when there is only a single user, (15) can also be reduced to a one-tap equalizer. When $K = 1$, (15) is reduced to

$$\widehat{x}_1[n] = \frac{\mathbf{h}_1^{\mathrm{H}}[0]\mathbf{y}[n]}{\boldsymbol{\mu}_1^{\mathrm{H}}\boldsymbol{\mu}_1 + M\sigma^2}, \quad (19)$$

if we use only the first path for MF operation.

From above, the LSA-SC system can always provide an equivalent frequency flat channel over a Rayleigh fading channel no matter whether for single user case or multiple user case. For single user over a Rician fading channel, such an equivalent frequency flat channel still exists. However, for multiple user in Rician channel, a multiple user detection is required to avoid MUI caused by the LOS components.

IV. Asymptotical Analysis

In the above, the proposed MLE is derived under the assumption that the number of antenna is large enough. In practical systems, the number of antenna is always finite. In this case, we have

$$\begin{aligned}
\widetilde{x}_{k_0}[n] = & \underbrace{\sum_{l_0=0}^{L}\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{h}_{k_0}[l_0]x_{k_0}[n]}_{\text{desired signal}} + \underbrace{\sum_{l_0=0}^{L}\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{w}[n+l_0]}_{\text{additive noise}} \\
& + \underbrace{\sum_{l\neq l_0}\sum_{l_0=0}^{L}\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{h}_{k_0}[l]x_{k_0}[n+l_0-l]}_{\text{ISI}} \\
& + \underbrace{\sum_{k\neq k_0}\sum_{l_0=0}^{L}\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{h}_{k}[l_0]x_{k}[n]}_{\text{synchronous MUI}} \\
& + \underbrace{\sum_{l\neq l_0}\sum_{k\neq k_0}\sum_{l_0=0}^{L}\frac{1}{M}\mathbf{h}_{k_0}^{\mathrm{H}}[l_0]\mathbf{h}[l]x_{k}[n+l_0-l]}_{\text{asynchronous MUI}}, \quad (20)
\end{aligned}$$

by substituting (8) into the left-hand-side of (17). From (20), besides the additive noise, the MF operation results in three extra interference terms. The first term is the ISI caused by the transmitted symbols at other time instances of the same UT, the second term is the synchronous MUI caused by the transmitted symbols from other UTs at the same time instance, and the third term is the asynchronous MUI caused by the transmitted symbols from other UTs at different time instances. Actually, the asynchronous MUI can be also considered as ISI caused by other UTs.

Denote $P_{\mathrm{ISI}}, P_{\mathrm{SMUI}}, P_{\mathrm{AMUI}}$ as the powers of the ISI, synchronous MUI, and asynchronous MUI, respectively. Assuming that the transmitted symbols, $x_k[n]$'s, are independent for different k's and different n's, from (20), we have

$$\begin{aligned}
P_{\mathrm{ISI}} = & \frac{2\kappa}{M(\kappa+1)}\left(\frac{1}{\kappa+1} - \sigma_0^2\right) \\
& + \frac{1}{M}\left[\left(\frac{1}{\kappa+1}\right)^2 - \sum_{l_0=0}^{L}\sigma_{l_0}^4\right], \quad (21)
\end{aligned}$$

$$P_{\mathrm{AMUI}} = \frac{2(K-1)\kappa}{M(\kappa+1)}\left(\frac{1}{\kappa+1} - \sigma_0^2\right) + \frac{K-1}{M}\left[\left(\frac{1}{\kappa+1}\right)^2 - \sum_{l_0=0}^{L}\sigma_{l_0}^4\right], \tag{22}$$

$$P_{\mathrm{SMUI}} = \sum_{k\neq k_0}[\mathbf{A}_K]_{(k_0,k)}[\mathbf{A}_K]_{(k,k_0)} + \frac{2(K-1)\kappa\sigma_0^2}{M(\kappa+1)} + \frac{K-1}{M}\sum_{l_0=0}^{L}\sigma_{l_0}^4. \tag{23}$$

More detailed derivation is not presented here due to page limit. From (21) to (22), $P_{\mathrm{ISI}}, P_{\mathrm{SMUI}}, P_{\mathrm{AMUI}}$ decrease as the increasing of antenna number. This coincides with the principle of LSA [4]. We also note that P_{AMUI} is $K-1$ times of P_{ISI} since the asynchronous MUI can be also regarded as the ISI caused by other UTs. When $M \to \infty$, we have

$$\lim_{M\to\infty} P_{\mathrm{ISI}} = 0, \tag{24}$$

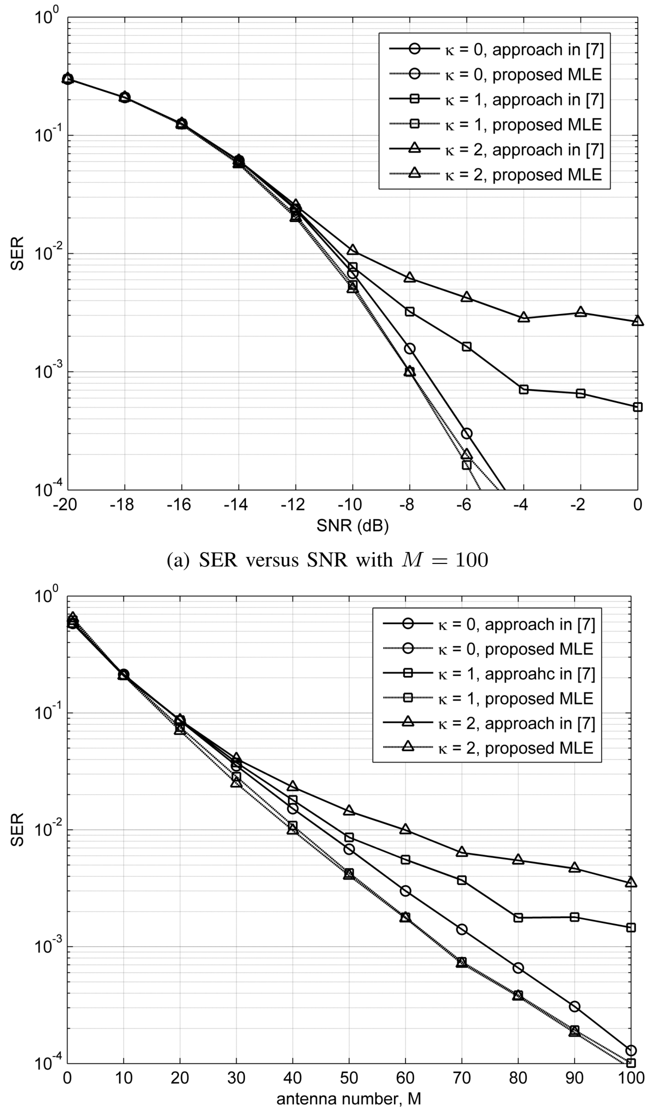
$$\lim_{M\to\infty} P_{\mathrm{AMUI}} = 0, \tag{25}$$

$$\lim_{M\to\infty} P_{\mathrm{SMUI}} = \sum_{k\neq k_0}[\mathbf{A}_K]_{(k_0,k)}[\mathbf{A}_K]_{(k,k_0)}. \tag{26}$$

The channel vectors for different UTs and different arrival paths approach orthogonal when $M \to \infty$. Therefore, the interference powers vanish correspondingly. In the presence of the LOS component, however, the synchronous MUI cannot be mitigated due to the presence of the LOS component even if the antenna number is very large. As a result, multiuser detection has to be adopted as shown in above.

V. Simulation Results

In this section, simulation results are provided to demonstrate the performance improvement of the proposed MLE over Rician fading channels. In the simulation, we consider a SC transmission with *quadrature-phase-shift-keying* (QPSK) modulation. The antennas are linearly placed as shown in Fig. 1. Four users share the same spectrum through spatial multiplexing. A discrete multipath channel with 20 taps is assumed for each receive antenna. There is a LOS component for the first arrival path and the strength of the LOS component is determined by the Rician factor, κ. For comparison, the approach in [7] is also included. Note that although the approach in [7] is proposed to downlink precoding, it can be easily used for uplink decoding due to the duality between downlink and uplink.



(a) SER versus SNR with $M = 100$

(b) SER versus antenna number M when SNR = -5 dB.

Fig. 2. Numerical results for the proposed MLE and existing approach.

Fig. 2 (a) shows the *symbol-error-ratio* (SER) versus SNR. From the figure, there is an error floor for the approach in [7] due to the presence of LOS component. With a large Rician factor, the LOS component is strong and thus the interference is obvious. Using the proposed MLE, the data symbols for different UTs can be detected jointly. As a result, the error floor can be mitigated, resulting a significant performance improvement at large SNRs.

The SER versus the number of antennas, M, is presented in Fig. 2 (b). Using the approach in [7], the SER cannot be further improved as the increase of antenna number, M since the power of overall interferences is limited by the synchronous MUI when M is large enough, as demonstrated in (26). However, with multiuser detection using the proposed MLE, there is no such restriction and thus performance can be improved.

VI. Conclusions

In this paper, we have investigated the MLE for uplink transmission in LSA-SC system over the Rician fading channels. In this case, MUI caused by the LOS component cannot be canceled by increasing the number of antennas, leading to an error floor when using single user detection approach. Joint multiuser detection for all the users is necessary to mitigate the error floor.

References

[1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[2] F. Rusek, D. Perrsson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up mimo: opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[3] E. G. Larsson, F. Tufvesson, O. Edfors, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *submitted to IEEE Commun. Mag.*, 2011.

[4] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "Signal processing in massive MIMO systems," *submitted to Journal on Selected Topics in Signal Processing*.

[5] J. Hoyidis, S. ten Brink, and M. Debbah, "Massive MIMO in UL/DL of cellular networks: How many antennas do we need?" *IEEE J. Sel. Area Commun.*, vol. 31, pp. 160–171, Jan. 2013.

[6] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser mimo systems," *submitted to IEEE Trans. Commun.*, arXiv:1112.3810 2011.

[7] A. Pitarokoilis, S. K. Mohammed, and E. G. Larsson, "On the optimality of single-carrier transmisson in large-scale antenna systems," *IEEE Wireless Commun. Lett.*, vol. 1, no. 4, Apr. 2012.

[8] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[9] S. K. Mohammed and E. G. Larsson, "Per-antenna constant envelope precoding for large multi-user mimo systems," *IEEE Trans. Commun.*, vol. 61, no. 3, pp. 1059–1071, Mar. 2013.