FEEDBACK BLIND PHASE SYNCHRONIZATION FOR QAM SIGNALS BASED ON CIRCULAR HARMONIC DECOMPOSITION

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ABSTRACT

An algorithm is presented for feedback blind phase synchronization for signals with quadrature amplitude modulation. The algorithm is based on a circular harmonic expansion (CHE) of log-likelihood function (LLF). Retaining one or two most significant terms in this series gives a harmonic or biharmonic circular decomposition of the LLF. Earlier this approach was presented by authors for feedforward blind phase and frequency estimation. In this paper we present application of CHE of LLF to feedback synchronization. The main point is employing the derivative of LLF approximation by CHE as an error signal for phase recovery loop. Optimization of weighting functions is also presented. Simulation results show that optimal biharmonic method has performance close to decision-directed algorithm and approaches modified Cramer-Rao bound. Advantages of the algorithm are wide acquisition range and no necessity of hard decisions that may be absent in soft-output detectors.

Index Terms— Blind estimation, non-data-aided estimation, phase offset, quadrature amplitude modulation, feedback synchronization, closed-loop recovery

1. INTRODUCTION

Phase synchronization procedures are mandatory in the systems performing coherent processing of signals with linear digital modulation. Implementing these procedures it is in some cases necessary to perform blind estimations as the signal does not include special pilot sequences. Phase synchronization methods can be divided into two major classes: feedforward (open-loop) algorithms calculate necessary estimates from the samples of observed signal, and feedback (closed-loop) algorithms constitute recursive tracking systems. For quick initial parameter estimation feedforward methods are usually used. Feedback algorithms are able to track slow parameter fluctuations, so feedforward estimates are used as initial states to launch tracking systems.

Decision-directed (DD) algorithm is often used for feedback synchronization. In [1] it is presented in detail and its characteristics are given. The problem of this algorithm consists in possible convergence to wrong phase values that depend on signal constellation. Modifications of DD algorithm were presented in [2]–[6]. Specifically, in [5], [6] a solution to eliminate wrong phase points is proposed.

Blind (non-data-aided) phase synchronization algorithms for phase-shift-keying signals are considered in [1], [7], and [8]. Some of them represent power-law algorithms, also known as monomial-based Viterbi and Viterbi synchronizers [9]. In [1] the algorithm is based on approximation of LLF. In [7] there are two new algorithms, which are independent of the imperfection of automatic gain control. In [8] maximum likelihood clockless feedback phase recovery is proposed. In [10] two algorithms are analyzed in detail for 16-APSK, they are also based on the power-law algorithms. In [11] non-DD blind carrier recovery algorithm for 16-QAM based on multi-modulus algorithm is presented. Its main advantage is the absence of the decision feedback module and, therefore, low complexity of the algorithm that is important for optical coherent receivers.

In our paper [12] we introduced the idea of using circular harmonic expansion (CHE) of log-likelihood function (LLF) for blind phase offset estimation, later this approach was extended to frequency [13] and joint frequency and phase estimation [14]. These works showed promising results for the use of combination of two circular harmonics of LLF. In [15] we proposed optimization of weighting functions for both harmonic and biharmonic methods. In this paper we consider using CHE of LLF for feedback synchronization.

The paper is organized as follows. In Section 2 problem formulation is stated. In Section 3 CHE and proposed algorithm are considered. In Section 4 optimization of weighting functions is performed. In Section 5 we demonstrate and discuss simulation results.

2. PROBLEM FORMULATION

The phase offset estimation problem can be formulated as follows. Input signal $\{\dot{x}(k)\}$ constitutes a sequence of baud-rate samples after matched filter:

$$\dot{x}(k) = \dot{a}(k)e^{j\varphi_0} + \dot{n}(k), \quad k = 0,...,K-1, \tag{1}$$

where $\dot{a}(k)$ are information symbols uniformly and independently drawn from the modulation constellation $\{\dot{C}_m\}$, $m = 1, \ldots, M$ (M is the constellation size), φ_0 is a

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constant phase offset uniformly distributed over the range 0…2π, $\dot{n}(k)$ are the samples of complex discrete white Gaussian noise with variances of real and imaginary parts equal to σ^2. The signal and noise levels are assumed to be known. Signal-to-noise ratio (SNR) is defined as the ratio between variances of signal and noise components:

$$\mathrm{SNR} = \frac{\overline{\left|\dot{a}(k)\right|^2}}{2\sigma^2} = \frac{1}{2\sigma^2 M}\sum_{m=1}^{M}\left|\dot{C}_m\right|^2. \qquad (2)$$

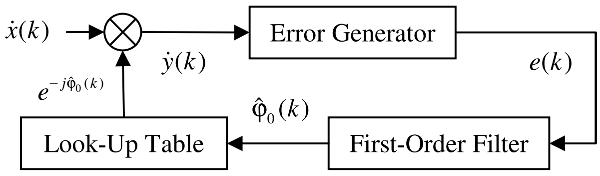

Figure 1. Costas phase recovery loop

The goal is to estimate and correct the phase offset φ_0 in feedback manner under the assumption that information symbols $\dot{a}(k)$ are unknown.

For feedback synchronization Costas loop can be used [1]. The scheme of phase recovery loop is shown in Fig. 1. We will consider the first-order tracking loop that is described by the following recursive equation:

$$\hat{\varphi}_0(k+1) = \hat{\varphi}_0(k) + \gamma\, e(k), \qquad (3)$$

where $e(k)$ and $\hat{\varphi}_0(k)$ are error signal and phase offset estimate on the iteration k, and γ is the step-size parameter.

Performance of Costas phase recovery loop depends on error signal generation and step-size parameter γ, which is related to the equivalent bandwidth of the loop BT by the following formula [1]:

$$\gamma = 4BT/A, \qquad (4)$$

where A is the slope of the S-curve at the origin. S-curve is the dependence of error signal on the phase error and provides information about the loop acquisition capability.

In principle, a signal can be used as an error signal if it satisfies the following requirements: it should have zero expectation in the absence of error, while the derivative of this expectation on error value at this point should be nonzero. Position of the maximum of LLF corresponds to parameter estimate. At the maximum the derivative of LLF is equal to zero. That's why derivative of LLF can be used as an error signal for feedback synchronization [1].

In the following section we briefly describe derivation of error signal.

3. CIRCULAR HARMONIC EXPANSION OF LLF AND PROPOSED ALGHORITHM

LLF for a signal sample $\dot{x}$ is a nonlinear function:

$$\mathrm{LLF}\left(\varphi_0\middle|\dot{x}\right) = \log\left(\frac{1}{2\pi\sigma^2 M}\sum_{m=1}^{M}\exp\left(-\frac{\left|\dot{x}e^{-j\varphi_0} - \dot{C}_m\right|^2}{2\sigma^2}\right)\right). \qquad (5)$$

We can express sample $\dot{x}$ in polar coordinates, i. e. separate its magnitude $r = \left|\dot{x}\right|$ and phase $\varphi = \arg\dot{x}$. As a result, LLF can be expanded in the Fourier series along phase φ that gives its presentation in the form of circular harmonic expansion [16]:

$$\mathrm{LLF}\left(\varphi_0\middle|re^{j\varphi}\right) = \frac{A_0(r)}{2} + \sum_{n=1}^{\infty} A_n(r)\cos\left(n\varphi - n\varphi_0 + \theta_n(r)\right), \qquad (6)$$

where $A_n(r)$ and $\theta_n(r)$ are magnitude and phase of the nth harmonic of the Fourier series:

$$A_n(r)e^{j\theta_n(r)} = \frac{1}{\pi}\int_0^{2\pi}\mathrm{LLF}\left(0\,|\,re^{j\varphi}\right)e^{-jn\varphi}d\varphi. \qquad (7)$$

In the sequel we assume for compactness that $\theta_n(r) = 0$, so that $A_n(r)$ can be negative. We will refer to these alternating versions of $A_n(r)$ as *weighting functions*. It is possible because all complex coefficients (7) for a standard angular position of QAM constellations appear real and $\theta_n(r) = 0$ or π.

Polyharmonic LLF expression (6) contains infinite number of harmonics that leads to the idea of truncating Fourier series, because harmonic magnitudes decrease with their order. As, due to angular symmetry of QAM constellations, only harmonics with n divisible by 4 are nonzero, the resultant formula for LLF after truncating (6) takes the following form:

$$\mathrm{LLF}\left(\varphi_0\middle|\dot{x}\right) = \mathrm{Re}\sum_{n=0}^{N}\dot{F}_{4n}(\dot{x})e^{-j4n\varphi_0}, \qquad (8)$$

where Re denotes the real part of a complex function, N is the number of used harmonics, and

$$\dot{F}_{4n}(\dot{x}) = A_{4n}(r)\exp(j4n\varphi) \qquad (9)$$

is nonlinearly transformed signal sample.

Algorithms for feedforward phase estimation based on Fourier series truncation were studied in our previous works [12]–[14]. Now we apply this approach for feedback phase estimation.

Derivative of LLF (8) along the phase φ_0 is (term with $n = 0$ does not depend on phase, so its derivative is zero)

$$\frac{\partial\,\mathrm{LLF}(\varphi_0|\dot{x})}{\partial\varphi_0} = \mathrm{Im}\sum_{n=1}^{N}4n\dot{F}_{4n}(\dot{x})e^{-j4n\varphi_0}, \qquad (10)$$

where Im denotes the imaginary part of a complex function.

Using (10) for signal after phase correction ($\dot{y}(k)$ in Fig. 1), we get the following formula for error signal of feedback synchronization system:

$$e(k) = \mathrm{Im}\sum_{n=1}^{N}n\dot{F}_{4n}\left(\dot{y}(k)\right). \qquad (11)$$

Truncated Fourier series gives global approximation which minimizes integral squared error, while to reduce loop noise it is necessary to use a local approximation of LLF in the vicinity of its maximum. Such optimization of weighting functions for harmonic and biharmonic methods was proposed in [15] and in the next section is briefly considered for feedback synchronization.

4. OPTIMIZATION OF WEIGHTING FUNCTIONS BY MINIMIZING ESTIMATION VARIANCE

As shown in [17], estimation variance can be obtained by quadratic approximation (i.e., truncating the Taylor series) of LLF in the vicinity of the true value. In [18] we showed that to minimize phase offset estimation variance and find optimal weighting functions it is necessary to solve the following optimization problem:

$$\overline{X_1^2} \big/ \left(\overline{X_2}\right)^2 \to \min_{A_n(r)}, \tag{12}$$

where X_1 and X_2 are the first and second derivatives of LLF, respectively. But in the case of feedback synchronization X_1 corresponds to error signal, and the mean value of X_2 — to the S-curve slope. Thus, in (12) we minimize estimation variance of the loop.

Optimal weighting functions were obtained in [15] for harmonic method ($N = 1$):

$$A_4(r) = -D_4(r) \big/ \left(2N_4(r)\right) \tag{13}$$

and for biharmonic method ($N = 2$):

$$A_4(r) = \frac{N_{48}(r)D_8(r) - 2N_8(r)D_4(r)}{4N_4(r)N_8(r) - N_{48}^2(r)}, \tag{14}$$

$$A_8(r) = \frac{N_{48}(r)D_4(r) - 2N_4(r)D_8(r)}{4N_4(r)N_8(r) - N_{48}^2(r)}, \tag{15}$$

where

$$N_4(r) = 16\int_0^{2\pi} \sin^2(4\varphi)\, p(r,\varphi)d\varphi, \tag{16}$$

$$D_4(r) = -16\int_0^{2\pi} \cos(4\varphi)\, p(r,\varphi)d\varphi, \tag{17}$$

$$N_{48}(r) = 64\int_0^{2\pi} \sin(4\varphi)\sin(8\varphi)\, p(r,\varphi)d\varphi, \tag{18}$$

$$N_8(r) = 64\int_0^{2\pi} \sin^2(8\varphi)\, p(r,\varphi)d\varphi, \tag{19}$$

$$D_8(r) = -64\int_0^{2\pi} \cos(8\varphi)\, p(r,\varphi)d\varphi. \tag{20}$$

Averaging in (16)–(20) is made over PDF of the signal sample $\dot{x}$, that can be written in polar coordinates:

$$p(r,\varphi) = \frac{r}{2\pi\sigma^2 M}\sum_{m=1}^{M}\exp\left(-\frac{r^2 + r_m^{\ 2} - 2rr_m\cos(\varphi - \varphi_m)}{2\sigma^2}\right), \tag{21}$$

where $r_m = \left|\dot{C}_m\right|$ and $\varphi_m = \arg \dot{C}_m$.

In the next section we demonstrate simulation results with optimal weighting functions.

5. SIMULATION RESULTS

To estimate accuracy of the proposed algorithms with optimized weighting functions, computer simulation was performed. We used observation of $K = 1000$ symbols, to measure phase estimation variance the results were averaged over observation length and 100 Monte-Carlo trials. Initial phase φ_0 was zero (to avoid transient processes in the loop). The equivalent bandwidth of the loop BT was chosen so that the equivalent observation length for feedforward estimation is 100 symbols [1]. The slope of the S-curve at the origin was numerically estimated by simulation at infinite SNR.

The resultant dependences of estimation variance on SNR for standard 16-QAM square and 32-QAM cross constellations are shown in Fig. 2 and Fig. 3. The curves are presented for the following algorithms:

- “CHE1, ideal”— harmonic method with optimal SNR-dependent weighting function (13).
- “CHE2, ideal”— biharmonic method with optimal SNR-dependent weighting functions (14)–(15).
- “CHE1, fixed”— harmonic method with optimal SNR-fixed weighting function.
- “CHE2, fixed”— biharmonic method with optimal SNR-fixed weighting functions.
- “DD”— DD algorithm.
- “MCRB”— modified Cramer-Rao bound (MCRB) for phase estimations [1].

From the figures it can be seen that biharmonic method with optimal SNR-dependent weighting functions gives results close to DD algorithm and approaches MCRB while harmonic method demonstrates performance loss about 3 dB for 16-QAM and up to 9 dB for 32-QAM. For low SNR values our curves are lower than MCRB. The main reason is a periodic nature of estimated parameter and natural “wrapping” of the estimation error to the range $-\pi/4 \ldots +\pi/4$. This wrapping is not accounted for in MCRB. Upper limit of wrapped phase estimation variance is equal to the variance of uniform distribution for the range mentioned above: $(\pi/2)^2/12 = \pi^2/48 \approx 0{,}21$ rad^2.

Also in Fig. 2, 3 the results are presented for simplified implementation of harmonic and biharmonic methods with SNR-fixed weighting functions. They demonstrate that using weighting functions calculated for a certain moderate SNR values (for example, 20 dB for 16-QAM and 25 dB for 32-QAM) does not lead to any notable performance loss. Using weighting functions calculated for higher SNR values leads to performance loss at low SNR values, and vice versa. In practical implementation it is recommended to use weighting functions calculated for expected SNR.

In Fig. 4, 5 S-curves are presented for considered algorithms. Analysis of S-curves gives information about loop acquisition capability. In particular, its nulls with positive slope represent stable equilibrium points while those with negative slope are unstable points. So, acquisition range for DD algorithm is about ±0,3 rad for 16-QAM and 0,2 rad for 32-QAM, while for CHE1 and CHE2 it is the whole symmetry range (±π/4) for both constellations. Fig. 6, where transient processes for 32-QAM are presented, confirms this conclusion. It is seen that DD algorithm converges to wrong phase value, and dispersion (estimation

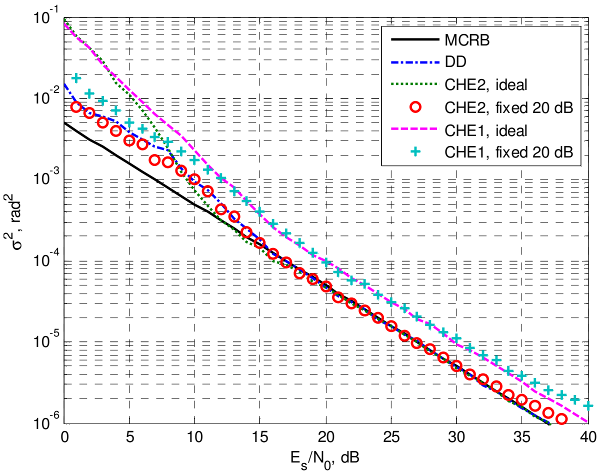

Figure 2. Phase estimation variance for 16-QAM

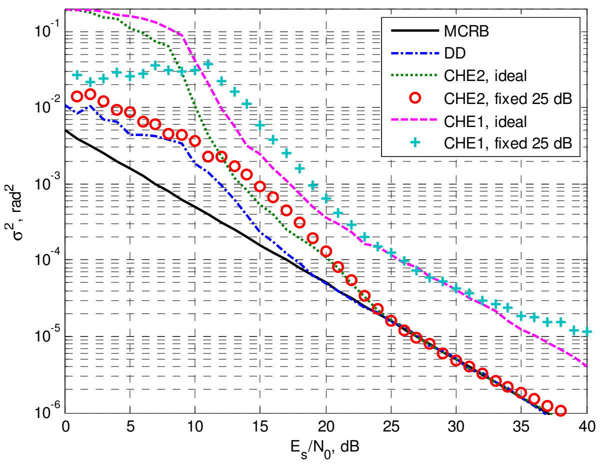

Figure 3. Phase estimation variance for 32-QAM

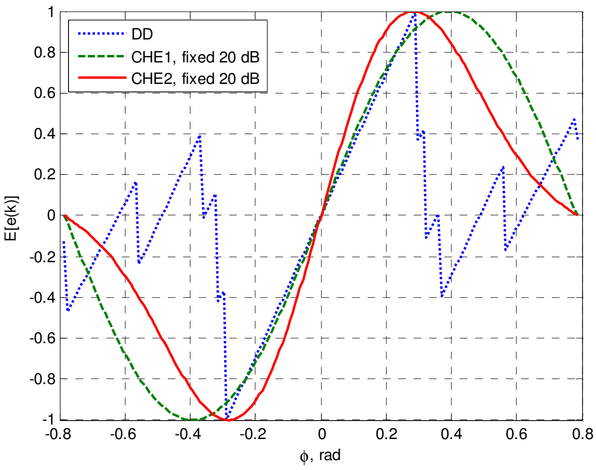

Figure 4. S-curves for 16-QAM (noise-free signal)

variance) for CHE2 is lower than for CHE1. It should be noted that for CHE1 the absence of wrong stable equilibrium points is guaranteed, while for CHE2 it depends on weighting functions, as error signal is generated by summing two harmonics.

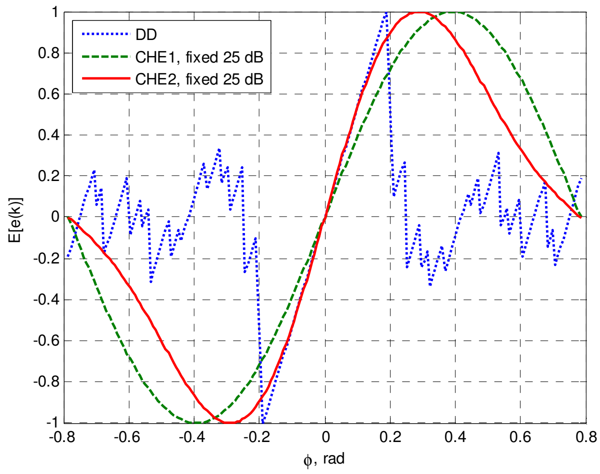

Figure 5. S-curves for 32-QAM (noise-free signal)

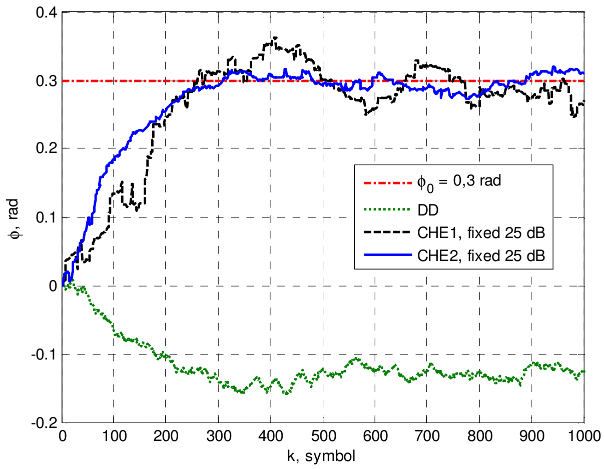

Figure 6. Transient processes for 32-QAM and SNR = 20 dB

6. CONCLUSION

Proposed algorithm of feedback synchronization demonstrates good performance. In particular, optimal biharmonic method has estimation variance close to DD algorithm and approaches MCRB. In comparison with DD algorithm our algorithm possesses wider acquisition range (for harmonic method, absence of wrong stable equilibrium points is guaranteed). Also, it does not require hard decisions that may be absent in soft-output detectors.

In the same manner CHE of LLF can be applied to feedback frequency synchronization by using second-order tracking loop [1] instead of first-order filter in Fig. 1.

CHE of LLF can be easily applied to any amplitude-phase shift keying modulation. The difference consists only in symmetry of constellation, i.e. in the numbers of used harmonics.

The challenging direction of investigation is the application of the proposed approach to carrier recovery over wireless channels.

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