ROBUST RELAY BEAMFORMING FOR MULTIPLE-ANTENNA AMPLIFY-AND-FORWARD RELAY SYSTEM IN THE PRESENCE OF EAVESDROPPER

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ABSTRACT

The problem of robust relay beamforming for the multiple-antenna amplify-and-forward (AF) relay network in the presence of an eavesdropper is studied in the paper, with the partial eavesdropper's channel side information (ECSI) scenario being considered. In our work, the uncertainty of ECSI is modeled by using a bounded region, which imposes independent constraints on the channel gain and direction. We propose a new rank-2 relay beamformer with a special singular value decomposition (SVD) structure, whose optimal solution for the worst-case secrecy rate maximization problem can be derived via simple line searching. Furthermore, the asymptotic optimality of the proposed rank-2 beamformer is proved in the high-relay-power region under certain condition, with the performance of the proposed beamformer being verified by using numerical experiments.

***Index Terms*—** Amplify-and-forward relaying, imperfect channel side information, physical layer security, robust beamforming

1. INTRODUCTION

Being promising in enhancing the security of wireless communication systems, physical (PHY) layer security research has experiencing rapid development in recent years. One of the key concepts in PHY layer security is secrecy capacity, which was first investigated by Wyner [1] and was later studied extensively for various communication systems. Remarkably, analyzing and improving the secrecy rate of cooperative communication systems [2–6] have become part of recent endeavor for expanding the scope of PHY layer security. In secure cooperative communications, much works [7–17] have been dedicated to techniques of secure cooperative beamforming and cooperative jamming (CJ) which are able to strengthen the legitimate channel while suppress the illegitimate channel.

In this paper, we concentrate on robust secure transmission via amplify-and-forward (AF) relay with partial eavesdropper's channel state information (ECSI). With the ECSI uncertainty modeled by a bounded region, the worst-case secrecy rate maximization (WC-SRM) problem is investigated. Unlike previous works on cooperative beamforming for single-antenna relays such as [8–11, 13, 16, 17], beamforming for multiple-antenna relay is addressed. Multiple-antenna relaying was considered in [12] and [15], but for the decode-and-forward protocol. Recently, secure AF protocol was studied in the context of multiple-input and multiple-output (MIMO) systems by [14] and [18]. The analysis of multiplexing and diversity gains for the suboptimal methods is the main concern of [14], and the signal-to-noise ratio (SNR) optimization is the subject of [18]. Therefore, neither work is aiming at secrecy rate optimization.

Different from our previous work [19], where the spherical uncertainty region was used and the rank-1 solution for the WCSRM problem was given, in this work, we adopt the uncertainty region, which puts independent constraints on the channel gain and direction, and design a new rank-2 relay beamformer with a special structure of singular value decomposition (SVD). The proposed uncertainty region separates the uncertainties in channel gain and direction, and can yield closed-form worst-case rate (WCSR) for the rank-2 beamformer considered, which is still unavailable in the case of the spherical region. Therefore, compared with the spherical region, the proposed uncertainty region has a better physical interpretation and allows easier treatment. By reducing the WCSRM problem to a problem with single variable, we derive the optimal solution for the proposed rank-2 beamformer by simple line searching. Further, we discover that if the direction of the legitimate link is outside the direction uncertainty set, the proposed rank-2 beamformer is asymptotically optimal in that its maximum WCSR can approach the rate upper bound as the relay power goes infinity.

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2. SYSTEM MODEL AND PROBLEM FORMULATION

We adopt the same network topology as in [19], which consists of four nodes: the source, Alice, the destination, Bod, the relay, and the eavesdropper, Eve. All nodes have one antenna each, except for the relay, which has M antennas. The channels are flat fading, and there are no direct links among Alice, Bob, and Eve. Making the practical assumption of half-duplexing relay, we have the two-stage procedure for signal transmission.

In the first stage, Alice transmits to the relay the source signal s of unit power. In the second stage, the relay multiplies the received signal by the beamforming matrix $\mathbf{F}$ and sends the processed signal to Bob. Then, the received signal y_b at Bob and the overheard signal y_e at Eve are given by

$$y_b = \mathbf{h}_b^H \mathbf{F}\mathbf{g}s + \mathbf{h}_b^H \mathbf{F}\mathbf{n} + n_b, y_e = \mathbf{h}_e^H \mathbf{F}\mathbf{g}s + \mathbf{h}_e^H \mathbf{F}\mathbf{n} + n_e, \quad (1)$$

where $\mathbf{g}$ is the channel vector between Alice and the relay, $\mathbf{h}_b^H$ and $\mathbf{h}_e^H$ are respectively the channel vectors of the relay-to-Bob link and the relay-to-Eve link, $\mathbf{n}$ is the channel noise of the Alice-to-relay link, and n_b and n_e are the channel noises of the relay-to-Bob link and the relay-to-Eve link, respectively. The entries of $\mathbf{n}$, n_b, and n_e are assumed to be independent complex Gaussian variables from $\mathcal{CN}(0,1)$.

In the partial ECSI case, the channel side information (CSI) of the legitimate links is known by all nodes whereas the exact CSI of the relay-to-Eve link is only available to Eve. To model the uncertainty of ECSI, we assume the unknown $\mathbf{h}_e$ belongs to a bounded uncertainty set $\mathcal{H}$, which is defined by

$$\mathcal{H} = \left\{ \mathbf{h}_e \middle| \mathbf{h}_e = h_e \mathbf{q}_e, h_{el} \le h_e \le h_{eu}, |\mathbf{q}_e^H \mathbf{q}|^2 \ge \delta \right\}, \quad (2)$$

where $h_e = \|\mathbf{h}_e\|$, the unit vector $\mathbf{q}_e = \mathbf{h}_e/h_e$, h_{el} and h_{eu} are respectively the upper and lower bounds of h_e, the unit vector $\mathbf{q}$ is the estimated channel direction, and $\delta \in [0, 1]$.

Note that $\mathcal{H}$ is intimately related to the conic constraint [20] but with extra norm constraints. It can be seen that $\mathbf{q}_e$ falls within the cone whose axis is $\mathbf{q}$ and aperture is $2\arccos\sqrt{\delta}$. In (2), the inequality $h_{el} \le h_e \le h_{eu}$ is the uncertainty in Eve's channel gain, which reflects the distance between the relay and Eve, and the inequality $|\mathbf{q}_e^H \mathbf{q}|^2 \ge \delta$ represents the uncertainty in Eve's channel direction.

With (1) and $\mathcal{H}$ in (2), we have the WCSRM problem for $\mathbf{F}$:

$$\text{(P1)} \quad \max_{\mathbf{F}} R_{wc}(\mathbf{F}), \text{s.t. } p_r(\mathbf{F}) \le P_m, \quad (3)$$

where the WCSR

$$R_{wc}(\mathbf{F}) = \log\left(1 + \gamma_b(\mathbf{F})\right) - \log\left(1 + \gamma_{wc}(\mathbf{F})\right), \quad (4)$$

with the SNR at Bob and the worst-case SNR at Eve given by

$$\gamma_b(\mathbf{F}) = \frac{|\mathbf{h}_b^H \mathbf{F}\mathbf{g}|^2}{1 + \mathbf{h}_b^H \mathbf{F}\mathbf{F}^H \mathbf{h}_b} \text{ and } \gamma_{wc}(\mathbf{F}) = \min_{\mathbf{h}_e \in \mathcal{H}} \frac{|\mathbf{h}_e^H \mathbf{F}\mathbf{g}|^2}{1 + \mathbf{h}_e^H \mathbf{F}\mathbf{F}^H \mathbf{h}_e}, \quad (5)$$

respectively, $p_r(\mathbf{F})$ is the relay power given by

$$p_r(\mathbf{F}) = \text{tr}(\mathbf{F}\mathbf{g}\mathbf{g}\mathbf{F}^H) + \text{tr}(\mathbf{F}\mathbf{F}^H), \quad (6)$$

and P_m is the total relay power.

To facilitate our discussion, we next give the notations that will be used throughout the paper. We define $h_b = \|\mathbf{h}_b\|$, $g = \|\mathbf{g}\|$, $P = P_m/(1 + g^2)$, $\mathbf{v}_1 = \mathbf{g}/g$, the $M \times 2$ matrix $\mathbf{U}_1$ with column vectors as the orthonormal base of span$\{\mathbf{h}_b, \mathbf{q}\}$, and 2×1 unit vectors $\mathbf{e}_b = \mathbf{U}_1^H \mathbf{h}_b/h_b$, $\mathbf{e}_e = \mathbf{U}_1^H \mathbf{h}_e/h_e$, and $\mathbf{e} = \mathbf{U}_1^H \mathbf{q}$. The corresponding orthogonal vectors are $\mathbf{e}_b^\perp$, $\mathbf{e}_e^\perp$, and $\mathbf{e}^\perp$. We let $\alpha = \arccos|\mathbf{e}_b^H \mathbf{e}_e| \in [0, \pi/2]$, $\tau_0 = \arg(\mathbf{e}^{\perp H}\mathbf{e}_b/\mathbf{e}^H \mathbf{e}_b) \in [0, 2\pi]$, and $\beta = \arccos(\sqrt{\delta}) \in [0, \pi/2]$.

3. ROBUST RELAY BEAMFORMERS

In this section, we investigate the WCSRM problem (P1) and design the robust relay beamformer under partial ECSI. Note that the WCSR in (4) is determined by the worst-case SNR at Eve which from (2) and (5) can be expressed as

$$\begin{aligned} \gamma_{wc}(\mathbf{F}) &= \max_{\substack{h_{el} \le h_e \le h_{eu}, \\ \|\mathbf{q}_e\|^2 = 1, |\mathbf{q}_e^H \mathbf{q}|^2 \ge \delta}} \frac{h_e^2 |\mathbf{q}_e^H \mathbf{F}\mathbf{g}|^2}{1 + h_e^2 \|\mathbf{F}^H \mathbf{q}_e\|^2} \\ &= \max_{\|\mathbf{q}_e\|^2 = 1, |\mathbf{q}_e^H \mathbf{q}|^2 \ge \delta} \frac{h_{eu}^2 |\mathbf{q}_e^H \mathbf{F}\mathbf{g}|^2}{1 + h_{eu}^2 \|\mathbf{F}^H \mathbf{q}_e\|^2}. \end{aligned} \quad (7)$$

It can be seen that the optimization for $\gamma_{wc}(\mathbf{F})$ is a fractional quadratically constrained quadratic problem (QCQP) [21], whose optimal solution can be obtained numerically. However, tractable WCSRM problem requires closed-form $\gamma_{wc}(\mathbf{F})$, which, unfortunately, is still out of reach for general relay beamformers. Therefore, we settle with suboptimal relay beamformers with simple structure and propose a new rank-2 beamformer with special SVD structure (Rank-2 SSVD relay beamformer).

Definition 1 (Rank-2 SSVD Relay Beamformer)**.** *Given $\mathbf{U}_1$, $\mathbf{v}_1$ defined in Section 2, and $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_1^\perp]$ with unit vector $\mathbf{v}_1^\perp$ orthogonal to $\mathbf{v}_1$, the rank-2 relay beamformer with special SVD structure is defined as $\mathbf{F}_{2D} = \mathbf{U}_1\mathbf{E}\mathbf{D}\mathbf{V}^H$ where the 2×2 unitary matrix $\mathbf{E} = [\mathbf{e}_x, \mathbf{e}_x^\perp]$ with unit vectors $\mathbf{e}_x$ and $\mathbf{e}_x^\perp$, and the 2×2 diagonal matrix $\mathbf{D} = diag\{\sigma_1, \sigma_2\}$ with $\sigma_k \ge 0$ $(k = 1, 2)$.*

$\mathbf{F}_{2D}$ has two parts: $\sigma_1 \mathbf{U}_1 \mathbf{e}_x \mathbf{v}_1^H$ and $\sigma_2 \mathbf{U}_1 \mathbf{e}_x^\perp \mathbf{v}_1^{\perp H}$. The first part is a MF beamformer which spans the signal space and is for useful signal transmission, while the second, orthogonal to the first part, is for noise injection by amplifying the received channel noise. The design of $\mathbf{F}_{2D}$ involves the optimization of σ_k $(k = 1, 2)$ and the forwarding vector $\mathbf{e}_x$.

Before we start our discussion, we present the following lemma which will be useful in our later derivations.

Lemma 1. *Define the function $R(u)$ with variable $u \ge 0$ as*

$$R(u) = \log\left(1 + \frac{h_1 u}{1 + h_2 u}\right) - \log\left(1 + \frac{h_3 u}{1 + h_4 u}\right). \quad (8)$$

Suppose $h_1 \ge 0$, $h_2 > 0$, $h_3 \ge 0$, and $h_4 > 0$.

1. If (i) $h_2 < h_4$ and $h_1/h_2 \le h_3/h_4$, or (ii) $h_2 \ge h_4$ and $h_1 \le h_3$, $R(u) \le 0$ for $\forall u \ge 0$. Equivalently, to ensure $R(u) > 0$ for some $u \ge 0$, it must hold that (i) $h_2 < h_4$ and $h_1/h_2 > h_3/h_4$, or (ii) $h_2 \ge h_4$ and $h_1 > h_3$.

2. If $h_3 = 0$, $R(u)$ is increasing in u.

3. If $h_3 > 0$ and assuming the conditions in 1) for $R(u) > 0$ hold, $R(u)$ has the following properties.

(a) If (i) $h_2 < h_4$ and $h_1 > h_3 h_2/h_4$, or (ii) $h_2 \ge h_4$ and $h_1 \ge h_3 h_2/((h_4/h_2)(h_3 + h_4) - h_3)$ and $h_1 > h_3$, the function $R(u)$ is increasing in u satisfying $R(u) > 0$.

(b) If (i) $h_2 \ge h_4$ and $h_3 < h_1 < h_3 h_2/h_4$, or (ii) $h_4 < h_2 < h_4(1 + h_4/h_3)$ and $h_3 h_2/h_4 \le h_1 < h_3 h_2/((h_4/h_2)(h_3 + h_4) - h_3)$, or (iii) $h_2 \ge h_4(1 + h_4/h_3)$ and $h_1 \ge h_3 h_2/h_4$, the function $R(u)$ has unique maximum at

$$u^{opt} = \frac{\sqrt{h_1 h_3 (h_2 - h_4)(h_1 + h_2 - h_3 - h_4)} + (h_1 h_4 - h_2 h_3)}{(h_2^2 h_3 - h_1 h_4^2) + h_1 h_3 (h_2 - h_4)}. \quad (9)$$

Proof. Due to space limitation, the proof is omitted and the detailed derivation will be given in [22]. ☐

Now we start to derive the worst-case SNR at Eve for the beamformer $\mathbf{F}_{2D}$. First, we have

$$\begin{aligned} &\gamma_{wc}(\mathbf{F}_{2D}) \\ &= \max_{\|\mathbf{q}_e\|^2 = 1, |\mathbf{q}_e^H \mathbf{q}|^2 \ge \delta} \frac{g^2 h_{eu}^2 \sigma_1^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x|^2}{1 + h_{eu}^2 \sigma_1^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x|^2 + h_{eu}^2 \sigma_2^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x^\perp|^2} \\ &\overset{(a)}{=} \max_{\substack{|\mathbf{q}_e^H \mathbf{U}_1 \mathbf{U}_1^H \mathbf{q}|^2 \ge \delta \\ \|\mathbf{U}_1^H \mathbf{q}_e\|^2 + \|\mathbf{U}_1^{\perp H} \mathbf{q}_e\|^2 = 1}} \frac{g^2 h_{eu}^2 \sigma_1^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x|^2}{1 + h_{eu}^2 \sigma_1^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x|^2 + h_{eu}^2 \sigma_2^2 |\mathbf{q}_e^H \mathbf{U}_1 \mathbf{e}_x^\perp|^2} \\ &\overset{(b)}{=} \max_{|\mathbf{e}_e^H \mathbf{e}|^2 \ge \delta} \frac{g^2 h_{eu}^2 \sigma_1^2 |\mathbf{e}_e^H \mathbf{e}_x|^2}{1 + h_{eu}^2 \sigma_1^2 |\mathbf{e}_e^H \mathbf{e}_x|^2 + h_{eu}^2 \sigma_2^2 |\mathbf{e}_e^H \mathbf{e}_x^\perp|^2}, \end{aligned} \quad (10)$$

where the $M \times (M-2)$ matrix $\mathbf{U}_1^\perp$ is such that $\mathbf{U}_1^H \mathbf{U}_1^\perp = \mathbf{O}$ and $\mathbf{U}_1^{\perp H} \mathbf{U}_1^\perp = \mathbf{I}$, (a) is from $\mathbf{U}_1 \mathbf{U}_1^H + \mathbf{U}_1^\perp \mathbf{U}_1^{\perp H} = \mathbf{I}$ and $\mathbf{q}^H \mathbf{U}_1^\perp = \mathbf{0}$, and (b) is from the observation that the objective function in (10) is increasing in the norm of $\mathbf{U}_1^H \mathbf{q}_e$.

Then, denote $\mathbf{e}_e = \mathbf{e}_x \cos\xi + \mathbf{e}_x^\perp e^{j\zeta} \sin\xi$, $\eta = \arccos|\mathbf{e}^H \mathbf{e}_x| = \arcsin|\mathbf{e}^H \mathbf{e}_x^\perp|$, and $\kappa = \arg(\mathbf{e}_x^{\perp H}\mathbf{e}/\mathbf{e}_x^H \mathbf{e})$, where $\xi, \eta \in [0, \pi/2]$

and $\zeta, \kappa \in [0, 2\pi]$. Note that we assume the projection of $\mathbf{e}_e$ on $\mathbf{e}_x$ to be positive since the objective function in (10) in invariant to its phase. With the above notations, the optimization in (10) can be recast as

$$\begin{aligned} &\max_{\xi,\zeta} \frac{g^2 h_{eu}^2 \sigma_1^2 \cos^2 \xi}{1 + h_{eu}^2 \sigma_1^2 \cos^2 \xi + h_{eu}^2 \sigma_2^2 \sin^2 \xi}, \\ &\text{s.t. } |\mathbf{e}_e^H \mathbf{e}|^2 = |\cos\eta \cos\xi + e^{j(\kappa - \zeta)} \sin\eta \sin\xi|^2 \geq \delta. \end{aligned} \tag{11}$$

Optimizing ζ yields $\zeta^{\text{opt}} = \kappa$ and gives the worst-case SNR at Eve as function depending solely on $\boldsymbol{\sigma} = [\sigma_1, \sigma_2]$ and η:

$$\begin{aligned} \gamma_{wc}(\boldsymbol{\sigma}, \eta) = &\max_{\xi} \frac{g^2 h_{eu}^2 \sigma_1^2 \cos^2 \xi}{1 + h_{eu}^2 \sigma_1^2 \cos^2 \xi + h_{eu}^2 \sigma_2^2 \sin^2 \xi} \\ &\text{s.t. } \cos(\eta - \xi) \geq \cos\beta, 0 \leq \xi \leq \pi/2. \end{aligned} \tag{12}$$

Evidently, the objective function in (12) is decreasing ξ. Therefore, if $\cos\eta \geq \cos\beta$ or $\eta \leq \beta$, $\xi^{\text{opt}} = 0$, and if $\cos\eta < \cos\beta$ or $\eta > \beta$, ξ^{opt} should satisfy $\cos(\xi^{\text{opt}} - \eta) = \cos\beta$ which gives $\xi^{\text{opt}} = \eta - \beta$. Finally substituting the optimal ξ^{opt} into the objective function yields the worst-case SNR at Eve as

$$\gamma_{wc}(\boldsymbol{\sigma}, \eta) = \begin{cases} \frac{g^2 h_{eu}^2 \sigma_1^2}{1 + h_{eu}^2 \sigma_1^2}, \text{ if } 0 \leq \eta \leq \beta, \\ \frac{g^2 h_{eu}^2 \sigma_1^2 \cos^2(\eta - \beta)}{1 + h_{eu}^2 \sigma_1^2 \cos^2(\eta - \beta) + h_{eu}^2 \sigma_2^2 \sin^2(\eta - \beta)}, \\ \text{if } \beta < \eta \leq \pi/2. \end{cases} \tag{13}$$

For the SNR as Bob, with simple derivation we have from (5) and Definition 1 that

$$\gamma_b(\mathbf{F}_{2D}) = \frac{g^2 h_b^2 \sigma_1^2 |\mathbf{e}_b^H \mathbf{e}_x|^2}{1 + h_b^2 |\mathbf{e}_b^H \mathbf{e}_x|^2 + h_b^2 |\mathbf{e}_b^H \mathbf{e}_x^\perp|^2}. \tag{14}$$

Express $\mathbf{e}_x$ and $\mathbf{e}_x^\perp$ as $\mathbf{e}_x = \mathbf{e}\cos\eta + \mathbf{e}^\perp e^{j\tau} \sin\eta$ and $\mathbf{e}_x^\perp = e^{-j\kappa}\left(\mathbf{e}\sin\eta - \mathbf{e}^\perp e^{j\tau}\cos\eta\right)$, where $\eta \in [0, \pi]$ and $\tau \in [0, 2\pi]$. Note that the projection of $\mathbf{e}_x$ on $\mathbf{e}$ is set to be positive since both γ_b and γ_{wc} are invariant to its phase. The term $e^{-j\kappa}$ in the expression of $\mathbf{e}_x^\perp$ is from $\kappa = \arg(\mathbf{e}_x^{\perp H}\mathbf{e}/\mathbf{e}_x^H \mathbf{e})$. Then, $|\mathbf{e}_b^H \mathbf{e}_x| = |\cos\alpha \cos\eta + e^{j(\tau - \tau_0)} \sin\alpha \sin\eta|$ and $|\mathbf{e}_b^H \mathbf{e}_x^\perp| = |\cos\alpha \sin\eta - e^{j(\tau - \tau_0)} \sin\alpha \cos\eta|$. As $\gamma_b(\mathbf{F}_{2D})$ is increasing in $|\mathbf{e}_b^H \mathbf{e}_x|$ and decreasing in $|\mathbf{e}_b^H \mathbf{e}_x^\perp|$ from (14), $\gamma_b(\mathbf{F}_{2D})$ is maximized by $\tau^{\text{opt}} = \tau_0$ for fixed $\boldsymbol{\sigma}$ and η, which gives the SNR at Bob in terms of $\boldsymbol{\sigma}$ and η:

$$\gamma_b(\boldsymbol{\sigma}, \eta) = \frac{g^2 h_b^2 \sigma_1^2 \cos^2(\eta - \alpha)}{1 + h_b^2 \sigma_1^2 \cos^2(\eta - \alpha) + h_b^2 \sigma_2^2 \sin^2(\eta - \alpha)}. \tag{15}$$

From (13) and (15), the WCSR of $\mathbf{F}_{2D}$ is expressed as the function of $\boldsymbol{\sigma}$ and η as

$$R_{wc}(\boldsymbol{\sigma}, \eta) = \log\left(1 + \gamma_b(\boldsymbol{\sigma}, \eta)\right) - \log\left(1 + \gamma_{wc}(\boldsymbol{\sigma}, \eta)\right). \tag{16}$$

Furthermore, knowing that $p_r(\mathbf{F}_{2D}) = (1 + g^2)\sigma_1^2 + \sigma_2^2$, we have the WCSRM problem for $\mathbf{F}_{2D}$ as

$$\max_{(1+g^2)\sigma_1^2 + \sigma_2^2 \leq P_m, \eta \in [0, \pi/2]} R_{wc}(\boldsymbol{\sigma}, \eta). \tag{17}$$

As no closed-form solution exists for joint $\boldsymbol{\sigma}$ and η optimization for (17), we consider first finding $\boldsymbol{\sigma}^{\text{opt}}$ for fixed η and then solving the optimal η^{opt} by line search. For fixed $\eta \in [0, \pi/2]$, there are two cases: 1) $\eta \leq \beta$ and 2) $\eta > \beta$.

1) In the case $\eta \leq \beta$, from both (13) and (15), it can be inferred that $R_{wc}(\boldsymbol{\sigma}, \eta)$ is decreasing in σ_2 so $\sigma_2^{\text{opt}} = 0$. Taking $\sigma_2^{\text{opt}} = 0$ yields the problem

$$\max_{\sigma_1^2 \leq P} \log\left(1 + \frac{g^2 h_b^2 \sigma_1^2 \cos^2(\eta - \alpha)}{1 + h_b^2 \sigma_1^2 \cos^2(\eta - \alpha)}\right) - \log\left(1 + \frac{g^2 h_{eu}^2 \sigma_1^2}{1 + h_{eu}^2 \sigma_1^2}\right) \tag{18}$$

which is in the same form as the problem (56) of [19]. Therefore, the method for solving the problem in (56) of [19] can be applied here and yields $\sigma_1^{\text{opt}} = 0$ for $h_b \cos(\eta - \alpha) \leq h_{eu}$, and

$$\sigma_1^{\text{opt}} = \begin{cases} \sqrt{P}, \text{if } P_m < \frac{\sqrt{1+g^2}}{h_b \cos(\eta - \alpha) h_{eu}}, \\ \frac{1}{\left((1+g^2) h_b^2 h_{eu}^2 \cos^2(\eta - \alpha)\right)^{1/4}}, \text{if } P_m \geq \frac{\sqrt{1+g^2}}{h_b \cos(\eta - \alpha) h_{eu}}, \end{cases} \tag{19}$$

for $h_b \cos(\eta - \alpha) > h_{eu}$.

2) In the case $\eta > \beta$, we rewrite the problem in (17) is

$$\begin{aligned} \max_{\boldsymbol{\sigma}} &\log\left(1 + \frac{g^2 h_{b1} \sigma_1^2}{1 + h_{b1}\sigma_1^2 + h_{b2}\sigma_2^2}\right) \\ &- \log\left(1 + \frac{g^2 h_{e1} \sigma_1^2}{1 + h_{e1}\sigma_1^2 + h_{e2}\sigma_2^2}\right) \\ &\text{s.t. } (1 + g^2)\sigma_1^2 + \sigma_2^2 \leq P_m, \end{aligned} \tag{20}$$

where $h_{b1} = h_b^2 \cos^2(\eta - \alpha)$, $h_{b2} = h_b^2 \sin^2(\eta - \alpha)$, $h_{e1} = h_{eu}^2 \cos^2(\eta - \beta)$, and $h_{e2} = h_{eu}^2 \sin^2(\eta - \beta)$. To solve (20), we let $t = \sigma_1^2/(1 + h_{b2}\sigma_2^2) \Rightarrow \sigma_1^2 = t(1 + h_{b2}\sigma_2^2)$ and get

$$\begin{aligned} &\max_{t, \sigma_2^2} \log\left(1 + \frac{g^2 h_{b1} t}{1 + h_{b1} t}\right) - \log\left(1 + \frac{g^2 h_{e1} t}{h_{e1} t + \frac{1 + h_{e2}\sigma_2^2}{1 + h_{b2}\sigma_2^2}}\right) \\ &\text{s.t. } (1 + g^2)(1 + h_{e1}\sigma_2^2) t + \sigma_2^2 \leq P_m, t, \sigma_2 \geq 0. \end{aligned} \tag{21}$$

Again, there are two cases: a) $h_{e2} \leq h_{b2}$ and b) $h_{e2} < h_{b2}$.

a) If $h_{e2} \leq h_{b2}$, $(1 + h_{e2}\sigma_2^2)/(1 + h_{b2}\sigma_2^2)$ is decreasing in σ_2 so $\sigma_2^{\text{opt}} = 0$ which produces $\mathbf{F}_{2D}$ of rank-1. As in the case of $\eta \leq \beta$, resorting to the result for (56) in [19], we have that $\sigma_1^{\text{opt}} = t^{\text{opt}} = 0$ if $h_{b1} \leq h_{e1}$, and

$$\sigma_1^{\text{opt}} = t^{\text{opt}} = \begin{cases} \sqrt{P}, & \text{if } P_m \leq \sqrt{\frac{1+g^2}{h_{b1} h_{e1}}}, \\ \frac{1}{\left((1+g^2) h_{b1} h_{e1}\right)^{1/4}}, & \text{if } P_m > \sqrt{\frac{1+g^2}{h_{b1} h_{e1}}}, \end{cases} \tag{22}$$

otherwise.

b) If $h_{e2} > h_{b2}$, $(1 + h_{e2}\sigma_2^2)/(1 + h_{b2}\sigma_2^2)$ is increasing in σ_2 so the relay power constraint is active for both problems in (21) and (20) when the optimum is achieved. Then, by plugging into (20) the variable changes

$$\sigma_1^2 = \frac{P_m \tilde{\sigma}_1^2}{(1 + g^2)\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}, \sigma_2^2 = \frac{P_m \tilde{\sigma}_1^2}{(1 + g^2)\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}, u = \frac{\tilde{\sigma}_1^2}{\tilde{\sigma}_2^2}, \tag{23}$$

we can rewrite the problem in (20) as an unconstrained optimization problem with single variable $u \geq 0$ and the objective function

$$R_{wc}(u) = \log\left(1 + \frac{h_1 u}{1 + h_2 u}\right) - \log\left(1 + \frac{h_3 u}{1 + h_4 u}\right) \tag{24}$$

which is exactly $R(u)$ in (8) but has redefined coefficients as $h_1 = g^2 h_{b1}/(1 + h_{b2} P_m)$, $h_2 = (1 + g^2 + h_{b1} P_m)/(1 + h_{b2} P_m)$, $h_3 = g^2 h_{e1}/(1 + h_{e2} P_m)$, and $h_4 = (1 + g^2 + h_{e1} P_m)/(1 + h_{e2} P_m)$.

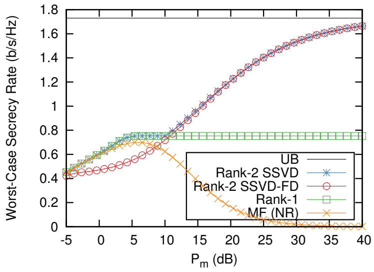

Fig. 1. Worse-case secrecy rate versus P_m. $h_{eu}^2 = 5$ dB, $\alpha = \pi/6$, and $\delta = 0.9$.

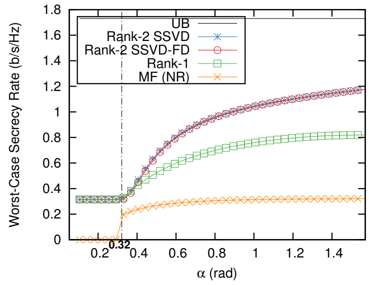

Fig. 2. Worst-case secrecy rate versus α. $P_m = 15$ dB, $h_{eu}^2 = 5$ dB, $\delta = 0.9$ and $\beta = \arccos\sqrt{\delta} \approx 0.32$.

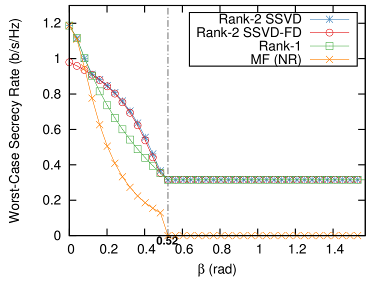

Fig. 3. Worst-case secrecy rate versus β. $P_m = 15$ dB, $h_{eu}^2 = 5$ dB, and $\alpha = \pi/6 \approx 0.52$.

Thus, maximization $R_{wc}(u)$ in (24) over u can be solved by applying Lemma 1. With u^{opt} obtained, $\boldsymbol{\sigma}^{\text{opt}}$ is calculated from (23) by enforcing constraint (6). Note that Lemma 1 also includes the possibility of $R_{wc}(u)$ increasing in u, which indicates that $R_{wc}(\boldsymbol{\sigma}, \eta)$ is increasing in σ_1 and thus the problem in (20) has $\sigma_1^{\text{opt}} = \sqrt{P}$ and $\sigma_2^{\text{opt}} = 0$ as its maximum point.

After the above steps for $\boldsymbol{\sigma}^{\text{opt}}$ in the two cases of $\eta \le \beta$ and $\eta > \beta$, we can find η^{opt} using some standard line searching method. Next we fix $\eta = \alpha$ and obtain the Rank-2 SSVD beamformer with fixed beamforming direction (Rank-2 SSVD-FD) defined by $\mathbf{F}_{2D\alpha} = \mathbf{U}_1[\mathbf{e}_b, \mathbf{e}_b^{\perp}]\mathbf{D}\mathbf{V}^H$, which is a special case of $\mathbf{F}_{2D}$. Obviously, $\mathbf{F}_{2D\alpha}^{\text{opt}}$ can be calculated similarly as $\mathbf{F}_{2D}^{\text{opt}}$ by simply fixing $\eta = \alpha$. Note that for $\alpha > \beta$

$$R_{wc}(\mathbf{F}_{2D\alpha}) = \log\left(1 + \frac{g^2 h_b^2 \sigma_1^2}{1 + h_b^2 \sigma_1^2}\right) - \log\left(1 + \frac{g^2 h_{eu}^2 \sigma_1^2 \cos^2(\alpha - \beta)}{1 + h_{eu}^2 \sigma_1^2 \cos^2(\alpha - \beta) + h_{eu}^2 \sigma_2^2 \sin^2(\alpha - \beta)}\right). \tag{25}$$

It can be seen that if $\sigma_1^2/\sigma_2^2 \to 0$ as $P_m \to \infty$, $R_{wc}(\mathbf{F}_{2D\alpha}) \approx \log(1 + g^2)$ which is the upper bound of the maximum WCSR. Therefore, the $\mathbf{F}_{2D\alpha}$ with $\boldsymbol{\sigma}$ growing as such is optimal in the asymptotic sense. As $R_{wc}(\mathbf{F}_{2D}^{\text{opt}}) \ge R_{wc}(\mathbf{F}_{2D\alpha}^{\text{opt}}) \ge R_{wc}(\mathbf{F}_{2D\alpha})$, we have the follow proposition from the discussion above.

Proposition 1. *If $\alpha > \beta$, $\mathbf{F}_{2D}^{opt}$ and $\mathbf{F}_{2D\alpha}^{opt}$ are asymptotically optimal for the WCSRM problem (P1) in that $\lim_{P_m \to \infty} R_{wc}(\mathbf{F}_{2D}^{opt}) = \lim_{P_m \to \infty} R_{wc}(\mathbf{F}_{2D\alpha}^{opt}) = \log(1 + g^2)$.*

4. NUMERICAL RESULTS

As experiment setups, the relay antenna number $M = 4$, $g^2 = 10$ dB, $h_b^2 = 10$ dB, $h_{eu}^2 = 5$ dB, $\mathbf{v}_1 = [0.064 + 0.420j, -0.348 + 0.443j, 0.195 + 0.481j, -0.386 + 0.289]^T$, $\mathbf{q}_b = \mathbf{h}_b/h_b = [-0.669 + 0.258j, -0.263 + 0.168j, 0.424 + 0.311j, -0.335 + 0.024j]$, and $\mathbf{q} = \mathbf{q}_b \cos\alpha + e^{j\pi/4}\mathbf{q}_b^{\perp} \sin\alpha$ with $\mathbf{q}_e^{\perp} = [-0.301 - 0.081j, 0.943 + 0.009j, 0.048 + 0.084j, -0.058 - 0.021j]$ orthogonal to $\mathbf{q}_b$. Aside from the WCSRs of the proposed beamformers, the WCSRs of the optimal rank-1 beamformer given in [22] for (P1) with set $\mathcal{H}$, and the MF solution in Theorem 5 of [19] for the perfect CSI case[1], which is referred to as the non-robust MF beamformer (MF (NR)), are presented as comparisons. The rate upper bound (UB) $\log_2(1 + g^2)$ is also provided when necessary.

[1]We use h_{eu} as Eve's channel gain to compute this beamformer since we are only concerned with the impact of direction uncertainty in the experiment.

Fig . 1 gives the curves of the WCSR versus P_m. P_m is from -5 to 40 dB, $h_{eu}^2 = 5$ dB, $\alpha = \pi/6$, and $\delta = 0.9$. Fig. 1 shows that in the high P_m region, the rank-2 solutions have similar WCSRs and are superior in that their WCSRs approach the rate upper bound as is prescribed by Proposition 1. The non-robust MF beamformer, although has moderate rate loss when P_m is low, become much inferior as P_m grows.

The impacts of α and β are shown in Figs. 2 and 3, respectively. In Fig. 2, α is from $\pi/60$ to $\pi/2$ with $\beta = \arccos\sqrt{0.9} \approx 0.32$. In Fig. 3, β is from 0 to $\pi/2$ with $\alpha = \pi/6 \approx 0.52$. In both figures, $P_m = 15$ dB and $h_{eu}^2 = 5$ dB. Fig. 2 shows that as α increases, the WCSRs of all solutions are improved but with the non-robust MF beamformer as the worst. Especially as α moves beyond β, the rank-2 beamformers enjoy a larger performance gain which reflects the benefit brought by noise amplification. In Fig. 3, the rate curves are similar to those in Fig. 2 but have a reversed trend, and again the non-robust MF beamformer become much worst for large β.

To sum up, the Rank-2 SSVD beamformer has the best performances in all figures. In addition, it is noteworthy that in the cases of low P_m (Fig. 1) and $\alpha \le \beta$ (Figs. 2 and 3), the WCSRs of the rank-1 and Rank-2 SSVD solutions are almost identical. Such phenomenon will be fully explained in [22].

5. CONCLUSION

The secure relay beamforming problem has been studied for the multiple-antenna AF relay network with partial ECSI. With the ECSI uncertainty set which separates the uncertainties in channel gain and direction, we have solved the WCSRM problem for a class of rank-2 beamformers with special SVD structure, which is asymptotically optimal for large relay power under mild assumption, Numerical experiments verified the effectiveness of the proposed beamformers.

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