EFFICIENT JAMMING STRATEGIES ON A MIMO GAUSSIAN CHANNEL WITH KNOWN TARGET SIGNAL COVARIANCE

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ABSTRACT

The problem of jamming on a multiple-input multiple-output (MIMO) Gaussian channel is investigated. We show that the existing result based on the simplification of the system model by neglecting the jamming channel leads to losing important insights regarding the effect of jamming power and jamming channel on the jamming strategy. We find a closed-form optimal solution for the problem under some positive semi-definite condition without considering simplifications in the model. If the condition is not satisfied and the optimal solution may not exist in closed-form, we find a suboptimal solution in closed-form as a close approximation of the optimal solution. Simulation results verify the effectiveness of the proposed solutions.

***Index Terms*—** Jamming, MIMO, closed-form solution

1. INTRODUCTION

The threat of jamming in wireless communications is becoming increasingly significant as wireless communication systems prevail in our everyday life [1]. Thus, it has been studied in many research works and one of the relevant research interests is to investigate the optimal jamming strategy from the perspective of a jammer [2]- [6]. Such perspective helps to reveal the effect of jamming on legitimate communications in the worst case.

When a jammer has multiple antennas, it can maximize the effectiveness of jamming by optimizing its jamming signal. The optimal jamming on multiple-input multiple-output (MIMO) channels is investigated in [7]- [10]. It is shown in [7] that without knowledge of the target signal or its covariance, the jammer can only use basic strategies of allocating power uniformly or maximizing the total power of the interference at the target receiver. In [8], the transmit strategies of a legitimate transmitter and a jammer on a Gaussian MIMO channel are investigated under a game-theoretic modeling with a general utility function. It is assumed that the jammer and the legitimate transmitter have the same level of channel state information (CSI), i.e., both uninformed, both with statistical CSI, or both with exact CSI. The optimal transmitted strategies of the legitimate transmitter and the jammer are represented as solutions to different problems versus different types of CSI. The worst-case jamming on MIMO multiple access and broadcast channels with the covariance of the target signal and all channel information available at the jammer is studied in [9] based on game theory. Some properties of the optimal jamming strategies are characterized through the analysis of the Nash equilibrium of the game. The necessary condition for optimal jamming on MIMO channels with arbitrary inputs when the covariance of the target signal and all channel information are available at the jammer is derived in [10]. For the case of Gaussian target signal, the solution of optimal jamming is given in closed-form. However, it is derived without considering the jamming channel. As a result, the system model is simplified by implicitly assuming that the received jamming signal at the target receiver is exactly the same as the transmitted jamming signal at the jammer.

With the objective of providing a general solution without simplifications of the system model, this work revisits the problem of the optimal jamming on a MIMO Gaussian channel with Gaussian input. The resulting problem becomes more complicated and the method for deriving the optimal solution in [10] no longer applies. It will be shown that, unlike the case without considering the jamming channel, the optimal solution may or may not exist in closed-form depending on the power limit of the jammer. The optimal solution will be given if it exits in closed-form and the solution in [10] will be shown to be a special case of our general solution. It is further shown that the existence of the closed-form optimal solution is not guaranteed in the general case. We then propose a suboptimal solution also in closed-form as an alternative strategy for the jammer so that the complexity of finding the solution remains low. Simulation results will demonstrate that the proposed suboptimal solution is in fact very close (if not equal) to the optimal solution.

2. SYSTEM MODEL

A legitimate transmitter with n_{t} antennas sends a signal $\mathbf{s}$ to a receiver with n_{r} antennas. The elements of $\mathbf{s}$ are complex Gaussian with zero mean and covariance $\mathbf{Q}_{\mathrm{s}}$. A jammer with n_{z} antennas attempts to jam the legitimate communication by

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transmitting a jamming signal $\mathbf{z}$ to the receiver. Denote the legitimate channel (from the legitimate transmitter to the receiver) as $\mathbf{H}_{\mathrm{r}}$ and the jamming channel (from the jammer to the receiver) as $\mathbf{H}_{\mathrm{z}}$. In the presence of the jamming signal, the received signal at the legitimate receiver is expressed as

$$\mathbf{y} = \mathbf{H}_{\mathrm{r}}\mathbf{s} + \mathbf{H}_{\mathrm{z}}\mathbf{z} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is the noise at the legitimate receiver with zero mean and covariance $\sigma^2\mathbf{I}$ with $\mathbf{I}$ denoting identity matrix. Note that given the Gaussian channel and Gaussian target signal, the worst-case form of jamming signal is also Gaussian [12]. Denote the covariance of $\mathbf{z}$ as $\mathbf{Q}_{\mathrm{z}}$. Then the information rate of the legitimate communication under jamming is given as [7]

$$R^{\mathrm{J}} = \log|\mathbf{I} + \mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{z}}\mathbf{Q}_{\mathrm{z}}\mathbf{H}_{\mathrm{z}}^{\mathrm{H}} + \sigma^2\mathbf{I})^{-1}| \tag{2}$$

where $|\cdot|$ denotes the determinant of a square matrix and $(\cdot)^{\mathrm{H}}$ stands for the Hermitian transpose. The jammer aims at decreasing the above rate as much as possible given its power limit P_{z}. It is assumed that the jammer has the knowledge of $\mathbf{H}_{\mathrm{r}}$, $\mathbf{H}_{\mathrm{z}}$, and $\mathbf{Q}_{\mathrm{s}}$, but does not know the exact $\mathbf{s}$ (as in [9]-[11]). The jammer can use the available knowledge to find the optimal $\mathbf{Q}_{\mathrm{z}}$ among Hermitian and positive semi-definite (PSD) matrices such that the rate in (2) is minimized. The above assumption makes it possible to investigate the theoretic bound of the worst-case jamming. Note that, practically, jammer may obtain the knowledge of $\mathbf{H}_{\mathrm{r}}$ during the channel feedback between the legitimate transceiver [11].

3. CLOSED-FORM OPTIMAL SOLUTION UNDER PSD CONDITION

The optimal jamming strategy can be found by solving the following problem

$$\min_{\mathbf{Q}_{\mathrm{z}}} \quad R^{\mathrm{J}} \tag{3a}$$

$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}\} \le P_{\mathrm{z}} \tag{3b}$$

where $\mathrm{Tr}\{\cdot\}$ denotes the trace and the PSD constraint $\mathbf{Q}_{\mathrm{z}} \succeq 0$ is omitted for brevity. The solution in [10] is obtained by assuming $\mathbf{H}_{\mathrm{z}} = \mathbf{I}$ and then making the corresponding term in the determinant in (2) diagonal via choosing $\mathbf{Q}_{\mathrm{z}}$. However, it can be seen that the term in the determinant in general cannot be made diagonal via $\mathbf{Q}_{\mathrm{z}}$ when $\mathbf{H}_{\mathrm{z}} \ne \mathbf{I}$. As a result, the method in [10] is invalid here. For finding the solution to this problem, the following two situations need to be considered

- **S1**: The matrix $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$ is positive definite (PD);
- **S2**: The matrix $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$ is PSD but not PD.

In the sequel, the situations **S1** and **S2** are considered and the solutions are found for each of them.

Denote the singular value decomposition (SVD) of $\mathbf{H}_{\mathrm{z}}$ as $\mathbf{H}_{\mathrm{z}} = \mathbf{U}_{\mathrm{z}}\mathbf{\Omega}_{\mathrm{z}}\mathbf{V}_{\mathrm{z}}^{\mathrm{H}}$. Define also $\mathbf{B} \triangleq \mathbf{U}_{\mathrm{z}}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}\mathbf{U}_{\mathrm{z}}$. It is worth noting that $\mathbf{B}$ has the same rank as $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$. For finding the optimal jamming strategy, we first introduce the following lemma.

Lemma 1: Given a Hermitian matrix $\mathbf{J} \succ 0$, the following optimization problem over positive definite matrix $\mathbf{X}$

$$\min_{\mathbf{X}} \quad \log|\mathbf{I} + \mathbf{J}\mathbf{X}^{-1}| \tag{4a}$$

$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{X}\} \le 1 \tag{4b}$$

$$\mathbf{X} \succeq 0 \tag{4c}$$

has the following closed-form solution

$$\mathbf{X} = \mathbf{U}_{\mathbf{J}}\sqrt{\frac{\mathbf{\Lambda}_{\mathbf{J}}}{\lambda} + \frac{\mathbf{\Lambda}_{\mathbf{J}}^2}{4}}\mathbf{U}_{\mathbf{J}}^{\mathrm{H}} - \frac{\mathbf{J}}{2} \tag{5}$$

where $\mathbf{U}_{\mathbf{J}}$ and $\mathbf{\Lambda}_{\mathbf{J}}$ are the eigenvector and eigenvalue matrices, respectively, obtained from the eigenvalue decomposition (EVD) $\mathbf{J} = \mathbf{U}_{\mathbf{J}}\mathbf{\Lambda}_{\mathbf{J}}\mathbf{U}_{\mathbf{J}}^{\mathrm{H}}$, and λ is chosen so that the power constraint (4b) is satisfied with equality.

The proof as well as all other proofs is omitted due to space limitations while they can be found in [13] in details.

Denote the rank of $\mathbf{H}_{\mathrm{z}}$ as r_{z} and assume without loss of generality that the first r_{z} elements on the main diagonal of the matrix $\mathbf{\Omega}_{\mathrm{z}}$ are non-zero. Denote the $r_{\mathrm{z}} \times r_{\mathrm{z}}$ block of $\mathbf{\Omega}_{\mathrm{z}}$ that has these r_{z} diagonal elements as $\mathbf{\Omega}_{\mathrm{z}}^{+}$. Using the definition of $\mathbf{B}$ and the SVD of $\mathbf{H}_{\mathrm{z}}$, the objective function in (2) can be rewritten as

$$R^{\mathrm{J}} = \log|\mathbf{I} + \mathbf{B}(\mathbf{\Omega}_{\mathrm{z}}\hat{\mathbf{Q}}_{\mathrm{z}}\mathbf{\Omega}_{\mathrm{z}}^{\mathrm{H}} + \sigma^2\mathbf{I})^{-1}| \tag{6}$$

where

$$\hat{\mathbf{Q}}_{\mathrm{z}} \triangleq \mathbf{V}_{\mathrm{z}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}\mathbf{V}_{\mathrm{z}}. \tag{7}$$

Let us define the equivalent channel as

$$\tilde{\mathbf{\Omega}}_{\mathrm{z}} \triangleq \begin{array}{c} \\ {\scriptstyle r_{\mathrm{z}}} \\ {\scriptstyle n_{\mathrm{r}}-r_{\mathrm{z}}} \end{array} \begin{array}{c} \begin{array}{cc} {\scriptstyle r_{\mathrm{z}}} & {\scriptstyle n_{\mathrm{r}}-r_{\mathrm{z}}} \end{array} \\ \left[\begin{array}{cc} \mathbf{\Omega}_{\mathrm{z}}^{+} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array}\right] \end{array} \tag{8}$$

where $\mathbf{0}$ represents all-zero matrix, and the equivalent jamming covariance matrix as

$$\tilde{\mathbf{Q}}_{\mathrm{z}} \triangleq \begin{array}{c} \\ {\scriptstyle r_{\mathrm{z}}} \\ {\scriptstyle n_{\mathrm{r}}-r_{\mathrm{z}}} \end{array} \begin{array}{c} \begin{array}{cc} {\scriptstyle r_{\mathrm{z}}} & {\scriptstyle n_{\mathrm{r}}-r_{\mathrm{z}}} \end{array} \\ \left[\begin{array}{cc} \mathbf{Q}_{\mathrm{z}}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array} \tag{9}$$

where $\mathbf{Q}_{\mathrm{z}}'$ is the part of the matrix to be determined. The equivalent channel $\tilde{\mathbf{\Omega}}_{\mathrm{z}}$ has the size $n_{\mathrm{r}} \times n_{\mathrm{r}}$, and it extends the size of $\mathbf{\Omega}_{\mathrm{z}}$ if $n_{\mathrm{r}} > n_{\mathrm{z}}$ and reduces it if $n_{\mathrm{r}} < n_{\mathrm{z}}$. Correspondingly, the allocation of jamming power in (9) represented by $\mathbf{Q}_{\mathrm{z}}'$ is limited to at most r_{z} dimensions corresponding to the r_{z} non-zero eigenvalues of $\mathbf{\Omega}_{\mathrm{z}}^{+}$. It can be seen that allocating jamming power anywhere else has no effect on the received signal and only leads to jamming power waste. Therefore, the optimal structure of $\hat{\mathbf{Q}}_{\mathrm{z}}$ has to be in the form

$$\hat{\mathbf{Q}}_{\mathrm{z}} = \begin{array}{c} \\ {\scriptstyle n_{\mathrm{r}}} \\ {\scriptstyle n_{\mathrm{z}}-n_{\mathrm{r}}} \end{array} \begin{array}{c} \begin{array}{cc} {\scriptstyle n_{\mathrm{r}}} & {\scriptstyle n_{\mathrm{z}}-n_{\mathrm{r}}} \end{array} \\ \left[\begin{array}{cc} \tilde{\mathbf{Q}}_{\mathrm{z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array} = \begin{array}{c} \\ {\scriptstyle r_{\mathrm{z}}} \\ {\scriptstyle n_{\mathrm{z}}-r_{\mathrm{z}}} \end{array} \begin{array}{c} \begin{array}{cc} {\scriptstyle r_{\mathrm{z}}} & {\scriptstyle n_{\mathrm{z}}-r_{\mathrm{z}}} \end{array} \\ \left[\begin{array}{cc} \mathbf{Q}_{\mathrm{z}}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array}. \tag{10}$$

Given the above definitions, we have the following theorem for the situation **S1**.

Theorem 1: The problem (3) has the following closed-form optimal solution in the situation **S1**

$$\mathbf{Q}_{\rm z}' = \mathbf{U}_{\tilde{\mathbf{A}}}\sqrt{\frac{1}{\lambda}\mathbf{\Lambda}_{\tilde{\mathbf{A}}} + \frac{1}{4}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^2}\mathbf{U}_{\tilde{\mathbf{A}}}^{\rm H} - \mathbf{\Omega}_{\rm z}^{+^{-1}}\left(\frac{1}{2}\tilde{\mathbf{B}} + \sigma^2\mathbf{I}\right)\mathbf{\Omega}_{\rm z}^{+^{-\rm H}} \tag{11}$$

under the condition that the above matrix $\mathbf{Q}_{\rm z}'$ is PSD, where

$$\tilde{\mathbf{B}} \triangleq \mathbf{B}_{11} - \mathbf{B}_{12}(\sigma^2\mathbf{I} + \mathbf{B}_{22})^{-1}\mathbf{B}_{21} \tag{12}$$

with $\mathbf{B}_{11}$, $\mathbf{B}_{12}$, $\mathbf{B}_{21}$, and $\mathbf{B}_{22}$ given by

$$\mathbf{B} = \begin{array}{c c} & \begin{array}{cc} r_{\rm z} & n_{\rm z}-r_{\rm z} \end{array} \\ \begin{array}{c} r_{\rm z} \\ n_{\rm z}-r_{\rm z} \end{array} & \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \end{array} \tag{13}$$

$\mathbf{U}_{\tilde{\mathbf{A}}}$ and $\mathbf{\Lambda}_{\tilde{\mathbf{A}}}$ are obtained from the EVD $\tilde{\mathbf{A}} = \mathbf{U}_{\tilde{\mathbf{A}}}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}\mathbf{U}_{\tilde{\mathbf{A}}}^{\rm H}$ with

$$\tilde{\mathbf{A}} \triangleq \mathbf{\Omega}_{\rm z}^{+^{-1}}\tilde{\mathbf{B}}\mathbf{\Omega}_{\rm z}^{+^{-\rm H}} \tag{14}$$

and λ is chosen such that the jammer's power constraint is satisfied with equality.

With the given optimal $\mathbf{Q}_{\rm z}'$, the optimal $\mathbf{Q}_{\rm z}$ can be obtained from (7) and (10). In the special case studied in [10], $\mathbf{H}_{\rm z}$ and consequently $\mathbf{U}_{\rm z}$, $\mathbf{\Omega}_{\rm z}$, and $\mathbf{V}_{\rm z}^{\rm H}$ are all equal to $\mathbf{I}$. Therefore, $\tilde{\mathbf{A}}$ and $\mathbf{\Omega}_{\rm z}^+$ simplify to $\tilde{\mathbf{B}}$ and $\mathbf{I}$, respectively. Moreover, it is also assumed that $\mathbf{U}_{\rm z}^{\rm H}\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}\mathbf{U}_{\rm z}$ has full rank, which further simplifies the case so that $\tilde{\mathbf{B}} = \mathbf{B}$. Then it is easy to check that the solution in (11) simplifies to the scalar-form solution in [10].

Theorem 1 gives the closed-form optimal solution of the problem (3) under the condition that $\mathbf{Q}_{\rm z}$, or equivalently, $\mathbf{Q}_{\rm z}'$ given by (11) is PSD. However, it is possible that $\mathbf{Q}_{\rm z}'$ is indefinite. It can happen when the jammer's power limit $P_{\rm z}$ is sufficiently small. It can be seen that $1/\lambda$ decreases when the jammer's power limit becomes smaller. As a result, $\mathbf{Q}_{\rm z}'$ has a larger chance to be indefinite and thereby invalid. For a given power limit $P_{\rm z}$, whether $\mathbf{Q}_{\rm z}'$ in (11) is PSD depends on the channel $\mathbf{H}_{\rm z}$, or essentially, the elements of $\mathbf{\Omega}_{\rm z}^+$. It can be shown that, for a fixed $P_{\rm z}$ and $\mathbf{\Omega}_{\rm z}^+$ such that $\mathbf{Q}_{\rm z}'$ given by (11) is indefinite, there always exists a $\tilde{\mathbf{\Omega}}_{\rm z}^+$ with $\mathrm{Tr}\{\tilde{\mathbf{\Omega}}_{\rm z}^+\} = \mathrm{Tr}\{\mathbf{\Omega}_{\rm z}^+\}$ but different elements, such that $\mathbf{Q}_{\rm z}'$ is PSD if $\mathbf{\Omega}_{\rm z}^+$ is substituted by $\tilde{\mathbf{\Omega}}_{\rm z}^+$. Therefore, the power limit of the jammer as well as the gains of the eigen-channels determine whether or not $\mathbf{Q}_{\rm z}'$ is PSD. The above fact, which reveals the effect of the jamming power limit and the jamming channel on the jammer's strategy, has not been observed before as the jamming channel has been neglected.

For the situation **S2**, the following theorem is in order.

Theorem 2: The problem (3) has the following closed-form optimal solution in the situation **S2**

$$\mathbf{Q}_{\rm z}' = \mathbf{U}_{\tilde{\mathbf{A}}1}\sqrt{\frac{1}{\lambda}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^+ + \frac{1}{4}\mathbf{\Lambda}_{\tilde{\mathbf{A}}^+}^2}\mathbf{U}_{\tilde{\mathbf{A}}1}^{\rm H} - \frac{1}{2}\mathbf{U}_{\tilde{\mathbf{A}}1}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^+\mathbf{U}_{\tilde{\mathbf{A}}1}^{\rm H} - \sigma^2\mathbf{\Omega}_{\rm z}^{+^{-1}}\mathbf{\Omega}_{\rm z}^{+^{-\rm H}} \tag{15}$$

under the condition that the above $\mathbf{Q}_{\rm z}'$ is PSD, where $\mathbf{U}_{\tilde{\mathbf{A}}1}$ and $\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^+$ are obtained from the following EVD

$$\tilde{\mathbf{A}} = \mathbf{U}_{\tilde{\mathbf{A}}}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}\mathbf{U}_{\tilde{\mathbf{A}}}^{\rm H} = \begin{array}{c} \begin{array}{cc} r_{\tilde{\mathbf{A}}} & r_{\rm z}-r_{\tilde{\mathbf{A}}} \end{array} \\ \begin{bmatrix} \mathbf{U}_{\tilde{\mathbf{A}}1} & \mathbf{U}_{\tilde{\mathbf{A}}2} \end{bmatrix} \end{array} \begin{bmatrix} \mathbf{\Lambda}_{\tilde{\mathbf{A}}}^+ & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\tilde{\mathbf{A}}1}^{\rm H} \\ \mathbf{U}_{\tilde{\mathbf{A}}2}^{\rm H} \end{bmatrix} \tag{16}$$

with $r_{\tilde{\mathbf{A}}}$ denoting the rank of $\tilde{\mathbf{A}}$.

With the given optimal $\mathbf{Q}_{\rm z}'$, the optimal $\mathbf{Q}_{\rm z}$ can be obtained from the definitions (7) and (10).

It can be shown that if $\tilde{\mathbf{A}}$ has full rank, then (15) becomes equivalent to (11). Similar to the situation **S1**, $\mathbf{Q}_{\rm z}$ given by (15) can be indefinite depending on the jammer's power limit $P_{\rm z}$ and the jamming channel $\mathbf{\Omega}_{\rm z}^+$. To tackle this problem, in the next section we find a suboptimal solution of the problem (3) as a close approximation of the optimal solution for the case when $\mathbf{Q}_{\rm z}'$ given in (11) or (15) is indefinite.

4. SUBOPTIMAL JAMMING STRATEGY IN CLOSED-FORM

In order to solve the problem when the closed-form in (11) or (15) is invalid, two approaches can be used. The first one is to find the optimal solution numerically. However, due to the complexity consideration, it is preferred if a suboptimal solution can be found in a closed-form. Such suboptimal jamming strategy is characterized in the following theorem.

Theorem 3: A suboptimal closed-form solution closely approximating the optimal solution to the considered problem (3) in the situation **S1** is given as

$$\mathbf{Q}_{\rm z}' = \mathbf{U}_{\tilde{\mathbf{A}}}\sqrt{\frac{1}{\tilde{\lambda}}\mathbf{\Lambda}_{\tilde{\mathbf{A}}} + \frac{1}{4}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^2}\mathbf{U}_{\tilde{\mathbf{A}}}^{\rm H} - \frac{1}{2}\tilde{\mathbf{A}} + (\tilde{\epsilon} - 1)\mathbf{D}_0 \tag{17}$$

where $\mathbf{D}_0 \triangleq \sigma^2\mathbf{\Omega}_{\rm z}^{+^{-1}}\mathbf{\Omega}_{\rm z}^{+^{-\rm H}}$, and $\tilde{\epsilon}$ and $\tilde{\lambda}$ are the optimal solutions to the problem

$$\min_{\epsilon,\lambda} \quad \epsilon \tag{18a}$$

$$\text{s.t.} \quad \mathbf{U}_{\tilde{\mathbf{A}}}\sqrt{\frac{1}{\lambda}\mathbf{\Lambda}_{\tilde{\mathbf{A}}} + \frac{1}{4}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^2}\mathbf{U}_{\tilde{\mathbf{A}}}^{\rm H} - \frac{1}{2}\tilde{\mathbf{A}} + (\epsilon - 1)\mathbf{D}_0 \succeq 0 \tag{18b}$$

$$\mathrm{Tr}\left\{\sqrt{\frac{1}{\lambda}\mathbf{\Lambda}_{\tilde{\mathbf{A}}} + \frac{1}{4}\mathbf{\Lambda}_{\tilde{\mathbf{A}}}^2} - \frac{1}{2}\tilde{\mathbf{A}} + (\epsilon - 1)\mathbf{D}_0\right\} = P_{\rm z} \tag{18c}$$

$$0 \le \epsilon \le 1 \tag{18d}$$

$$\lambda > 0. \tag{18e}$$

It is worth mentioning that the constraints (18b)-(18e) specify a non-empty feasible set.

The above suboptimal solution given by (17) is proposed based on the following reasons. First and most important, it can be shown that $\mathbf{Q}_{\rm z}'$ given by the above suboptimal solution is the same as the $\mathbf{Q}_{\rm z}'$ given by (11) when the latter one is PSD (and consequently $\tilde{\epsilon} = 0$). Therefore, the use of (17) is sufficient for calculating the jamming strategy in all cases because (17) gives the optimal solution (11) when (11) is PSD

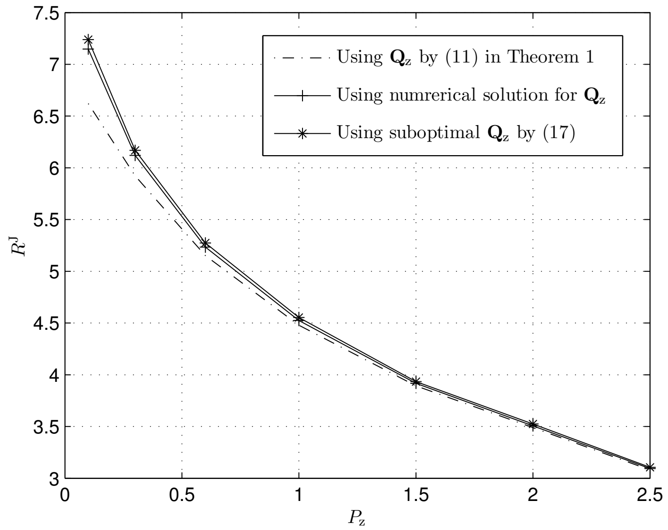

Fig. 1. Comparison of R^{J} versus P_{z} with $\mathbf{Q}_{\mathrm{z}}$ given by (11), the optimal numerical solution, and (17), respectively.

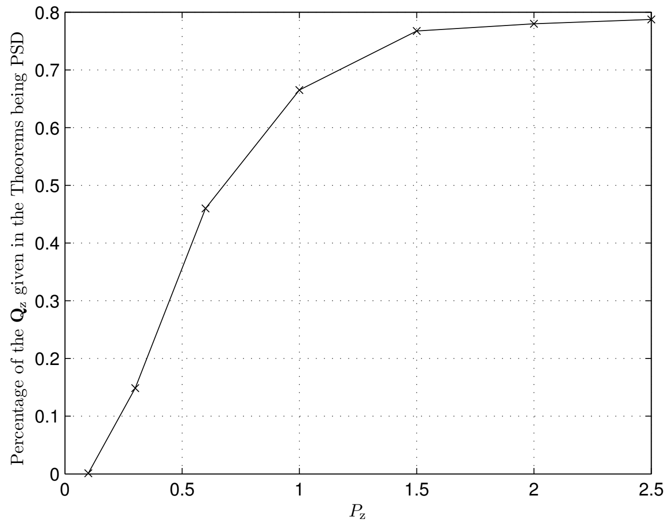

Fig. 2. Percentage that $\mathbf{Q}_{\mathrm{z}}$ given by (11) is PSD versus P_{z}.

and gives the suboptimal solution otherwise. Second, as will be shown in simulations, the above suboptimal solution given by (17) is in fact very close to the optimal one found numerically. Third, compared to the numerical solution, the suboptimal solution given by (17) can be obtained with negligible complexity since the parameters $\tilde{\epsilon}$ and $\tilde{\lambda}$ can be obtained by a simple bisectional search.

The closed-from suboptimal solution for the situation **S2** can be obtained similarly and is neglected here.

5. SIMULATIONS

In this simulation, we compare the rates of the legitimate communication under jamming when the jammer's strategy $\mathbf{Q}_{\mathrm{z}}$ is given by (i) the expression in (11), (ii) the optimal solution obtained numerically, and (iii) the approximation in (17), respectively.

The specific setup of this simulation is as follows. The number of antennas at the legitimate transmitter and receiver are set to be 4 and 3, respectively, while the number of antennas at the jammer is 5. The power limit for the legitimate transmitter is 3 and the power allocation at the legitimate transmitter is based on waterfilling. The noise variance σ^2 is set to be 1. The elements of the target signal $\mathbf{s}$ and the channels $\mathbf{H}_{\mathrm{r}}$ and $\mathbf{H}_{\mathrm{z}}$ are generated from complex Gaussian distribution with zero mean and unit variance. As a result $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$ is always PD, which leads to situation **S1**. We use 800 channel realizations and calculate the average R^{J} versus the power limit of the jammer P_{z}.

Fig. 1 shows the average R^{J} with $\mathbf{Q}_{\mathrm{z}}$ obtained using the three aforementioned methods. Three observations can be made from this figure. First, there is a gap between the average R^{J} with $\mathbf{Q}_{\mathrm{z}}$ given by (11) and the average R^{J} with the optimal $\mathbf{Q}_{\mathrm{z}}$ found numerically when P_{z} is small. The gap exists because $\mathbf{Q}_{\mathrm{z}}$ given by (11) is not always PSD and when it is not PSD, it no longer gives the optimal solution of the problem. Second, the gap between the average R^{J} with $\mathbf{Q}_{\mathrm{z}}$ obtained numerically and the average R^{J} given by the suboptimal $\mathbf{Q}_{\mathrm{z}}$ in (17) is very small. It verifies that the proposed suboptimal solution is in fact very close to the optimal solution of the considered problem. Third, the three curves of average R^{J} converge when P_{z} increases.

Fig. 2 shows the percentage that $\mathbf{Q}_{\mathrm{z}}$ given by (11) is PSD in all 800 channel realizations. It verifies the aforementioned fact that $\mathbf{Q}_{\mathrm{z}}$ given by (11) can be indefinite when the jammer's power limit $\mathbf{P}_{\mathrm{z}}$ is small. Even when $\mathbf{P}_{\mathrm{z}}$ is larger (above 2), there remains a 20% chance that $\mathbf{Q}_{\mathrm{z}}$ given by (11) is indefinite. It verifies the other fact that whether $\mathbf{Q}_{\mathrm{z}}$ given by (11) is PSD also depends on the jamming channel.

Using the observations from the two figures, it can be seen that the suboptimal solution given by (17) is a very good approximation of the optimal jamming strategy since it is very close to the optimal one when $\mathbf{Q}_{\mathrm{z}}$ given by (11) is indefinite while it becomes optimal when $\mathbf{Q}_{\mathrm{z}}$ given by (11) is PSD.

6. CONCLUSION

The general solution to the problem of jamming on a MIMO Gaussian channel with Gaussian input is found under the PSD condition. The effect of jamming power and jamming channel on the optimal jamming strategy is analyzed. For the case that the PSD condition is not satisfied, a suboptimal solution in closed-form is obtained as an approximation of the optimal solution. Simulation results demonstrate the optimal solution and the suboptimal solution versus the power limit of the jammer and show that the proposed suboptimal solution is very close to the optimal one.

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