

# MINIMUM SIDELobe BEAMPATTERN DESIGN FOR MIMO RADAR SYSTEMS: A ROBUST APPROACH

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## ABSTRACT

In this paper, we propose a robust transmit beampattern design for multiple-input multiple-output (MIMO) radar systems. The objective considered here is minimization of the beampattern sidelobes, subject to constraints on the transmit power where the waveform covariance matrix is the optimization variable. Motivated by the fact that the steering vectors are subject to uncertainties in practice, we propose a worst-case robust beampattern design where the uncertainties are parameterized by a deterministic set. We show that the resulting non-convex maximin problem can be translated into a convex problem. We numerically illustrate that the steering vector uncertainty yields a severe degradation in the array performance, i.e., the transmit beampattern. Also, we show that the proposed robust design improves the transmit beampattern by reducing the worst case sidelobe peak levels.

**Index Terms**— Robustness, Waveform design, transmit beampattern, worst-case optimization, sidelobe minimization

## 1. INTRODUCTION

The new generation multiple-input multiple-output (MIMO) radar systems [1–4] have gained a lot of attention recently due to the possibility of designing the transmit beampattern [5–10]. The transmit beampattern design in these MIMO radar systems is mainly based on exploiting the waveform diversity which is a direct result of having different transmitted waveforms. This feature offers more degrees of freedom and flexibility to design a transmit beampattern as close as possible to the desired one while keeping the total transmit power fixed. One common approach is the optimization of the waveform covariance matrix to approximate a desired beampattern and then, designing a set of waveforms in a way that they have the same optimal covariance matrix. Also, to make these waveforms to be a better fit for the real world problem, some practical constraints such as constant modulus or low peak-to-average ratio (PAR) constraints can be added to the waveform design problem [7, 9].

The authors in [5] considered the beampattern matching design problem, i.e., designing a beampattern as close as possible to the desired one via the optimization of the waveform covariance matrix, where a weighted squared error metric is minimized. They further developed their previous work in [9] by considering also a maximum error objective function. In [8], the authors modified the criterion used in [5] by not only considering the beampattern matching design problem but also minimizing the cross-correlation between the signals reflected back from the targets. In the same reference (and also with more details in [7, 10]), the authors formulated another beampattern synthesis problem called minimum sidelobe beampattern design which has also been used to address biomedical problems [11, 12].

However, all of these design criteria and their corresponding solutions are based on the idealistic assumption that the array steering vectors are accurately known. This assumption is not met in many real world situations where there are different sources of uncertainties, e.g., uncertain propagation conditions, inaccurately specified locations or calibration errors. The effect of imperfect prior knowledge of steering vectors on the performance of an array is investigated in [13–16] where the optimal weight vector (beamformer) is obtained by taking the steering vector uncertainty into account. Moreover, the authors in [17, 18] applied the minimax robust frameworks and proposed robust waveforms for MIMO radar systems, where in [17] the uncertain design parameter was the target power spectral density and in [18] the spatial correlation of the target response and the disturbance statistics. All of these studies convey the important conclusion that if the prior information is not perfectly known, the system performance is degraded.

Motivated by these robustness studies, we follow the worst-case robustness methodology to minimize such performance degradation for the transmit beampattern design problem. In this work, we focus on the minimum sidelobe beampattern design problem where our goal is to maximize the difference between the power level at the main target location and the power level at the points in the sidelobe region. We assume that the true steering vectors, i.e., the nominal steering vectors plus the uncertainty vectors, belong to a deterministically bounded set and we find the optimal covariance matrix which results in a robust transmit beampattern design. We formulate this robust design problem as a non-convex maximin optimization problem and show that it can be translated into a single convex semi-definite programming (SDP) problem. We illustrate numerically how the transmit beampattern is degraded if we ignore the existing error in the steering vectors and how the robustness will improve the worst-case performance. We evaluate the performance of the proposed robust design under different power constraints.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

### 2.1. System model

We consider an array of arbitrary geometry with  $M$  transmit antennas. We also assume that the discrete-time baseband transmitted signals are narrowband and denoted by  $\mathbf{x}(n) = [x_1(n) \ x_2(n) \ \cdots \ x_M(n)]^T$  for  $n = 1, 2, \dots, N$  where  $N$  is the number of samples of each transmitted probing signal. Then, the baseband signal at a generic location  $\theta$  can be stated as

$$y(\theta, n) = \sum_{m=1}^M e^{-j2\pi f_c \tau_m(\theta)} x_m(n), \quad n = 1, \dots, N, \quad (1)$$

where  $f_c$  is the carrier frequency and  $\tau_m(\theta)$  is the relative time delay between the antennas, i.e., the difference between the required time for the signal transmitted by the  $m$ th antenna and the signal transmitted by the reference antenna, to arrive at the target location  $\theta$ . For example, the relative time  $\tau_m(\theta)$ , for a uniform linear array (ULA) equals [19, Chapter 6]

$$\tau_m(\theta) = \frac{(m-1)d \sin(\theta)}{c}, \quad (2)$$

where  $c$  is the waveform speed and  $d$  is the space between antennas. Now, considering the steering vector  $\mathbf{a}(\theta) \in \mathbb{C}^{M \times 1}$

$$\mathbf{a}(\theta) = \begin{bmatrix} e^{j2\pi f_c \tau_1(\theta)} & e^{j2\pi f_c \tau_2(\theta)} & \dots & e^{j2\pi f_c \tau_M(\theta)} \end{bmatrix}^T, \quad (3)$$

the baseband signal at location  $\theta$ ,  $y(\theta, n)$  given in (1), can be rewritten in vector form

$$y(\theta, n) = \mathbf{a}^H(\theta) \mathbf{x}(n), \quad 1, \dots, N. \quad (4)$$

The power of the transmitted signal, i.e., the transmit beampattern, at the target location  $\theta$  is given by

$$p(\theta) = \mathbb{E}\{y(\theta, n)y^H(\theta, n)\} = \mathbf{a}^H(\theta) \mathbf{R} \mathbf{a}(\theta) \quad (5)$$

where  $\mathbf{R} = \mathbb{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\}$  is the covariance matrix of the probing signal  $\mathbf{x}(n)$ .

## 2.2. Problem formulation

We follow the idea of exploiting the resulting diversity of having different waveforms in MIMO radar system to form the beampattern and make it as close as possible to the desired one. Indeed, we control how the power is distributed in space via transmit beampattern design. In this work we focus on applications for which we are interested in minimizing the sidelobe level in a predetermined area.

In [10], the minimum sidelobe beampattern design problem is formulated to satisfy the following goals: (i) maximize the gap between the power (transmit beampattern) at the main location  $\theta_0$  and the power at all the points in the sidelobe region  $\Omega_s$ , (ii) guarantee a certain 3 dB main-beam width. To achieve these goals an optimization problem is formulated as choosing  $\mathbf{R}$  under a power constraint such as to maximize the predefined gap. In general, to model the power constraint over the waveforms, we assume that  $\mathbf{R} \in \mathcal{R}$  where  $\mathcal{R}$  is defined as follows and can be either  $\mathcal{R}_u$  or  $\mathcal{R}_r$  according to the different power constraints:

- Uniform elemental power constraint, i.e.,  $\mathbf{R} \in \mathcal{R}_u$  where  $\mathcal{R}_u \triangleq \{\mathbf{R} \mid \mathbf{R} \succeq 0, R_{ii} = \frac{\gamma}{M}, i = 1, 2, \dots, M\}$
- Relaxed elemental power constraint, i.e.,  $\mathbf{R} \in \mathcal{R}_r$  where  $\mathcal{R}_r = \{\mathbf{R} \mid \mathbf{R} \succeq 0, \text{Tr}\{\mathbf{R}\} = \gamma, (1 - \alpha)\frac{\gamma}{M} \leq R_{ii} \leq (1 + \alpha)\frac{\gamma}{M}, i = 1, 2, \dots, M\}$ ,

where  $R_{ii}$  is the  $i$ th diagonal element of  $\mathbf{R}$ ,  $\gamma$  is the total transmitted power and  $\alpha < 1$  is a positive scalar.

We rewrite the optimization problem in [10, Section 13.2.4] where we replace the 3 dB main-beam width constraint with a condition by which we guarantee a certain power level  $P_L$  for points  $\theta_i, i = 1, 2$  within the mainlobe region. Note that the 3 dB main-beam width constraint can be easily realized by setting  $P_L = 0.5$ . Let  $t$  denote the gap between the power at  $\theta_0$  and the power at the

points  $\mu_i$  in the sidelobe region  $\Omega_s$ . This optimization problem similar to [7] turns into

$$\begin{aligned} \max_{\mathbf{R} \in \mathcal{R}, t} \quad & t \\ \text{s.t.} \quad & \mathbf{a}^H(\theta_0) \mathbf{R} \mathbf{a}(\theta_0) - \mathbf{a}^H(\mu_i) \mathbf{R} \mathbf{a}(\mu_i) \geq t, \quad \forall \mu_i \in \Omega_s \\ & (P_L - \delta) \mathbf{a}^H(\theta_0) \mathbf{R} \mathbf{a}(\theta_0) \leq \mathbf{a}^H(\theta_i) \mathbf{R} \mathbf{a}(\theta_i) \\ & \mathbf{a}^H(\theta_i) \mathbf{R} \mathbf{a}(\theta_i) \leq (P_L + \delta) \mathbf{a}^H(\theta_0) \mathbf{R} \mathbf{a}(\theta_0), i = 1, 2. \end{aligned} \quad (6)$$

In (6),  $\Omega_s$  denotes a discrete set of grid points  $\mu_i$  representing the sidelobe region, and  $\delta$  is a small value. This is an SDP problem which can be solved efficiently in polynomial time using any SDP solver, e.g., CVX [20, 21].

However, the whole problem formulation above and consequently its resulting solution, i.e., the optimal covariance matrix, is a function of the steering vector  $\mathbf{a}(\theta)$ . Note that  $\mathbf{a}(\theta)$  is a known function of  $\theta$  if there is a perfect calibration of the transmit array. The steering vector uncertainties may arise from uncertain propagation conditions, estimation errors, imprecise array modeling, element-position perturbations, or other calibration errors, etc [13–16]. Therefore, such uncertainty cannot be avoided in practice and it causes beampattern degradation. To address this problem and take the uncertainty into account, we exploit a worst-case robust optimization framework where a certain level of performance is guaranteed to be achieved for all steering vectors which belong to a predefined deterministic uncertainty set. Specifically, we propose a worst-case robust problem formulation to handle uncertainties in the steering vectors of the sidelobe points  $\mu_i$  and the main location  $\theta_0$ . For tractability reasons, we do not robustify the main-beam width constraints in (6) and for notational simplicity, we assume that these convex constraints are incorporated into the set  $\mathcal{R}$ .

Let  $\mathbf{a}(\theta) = \hat{\mathbf{a}}(\theta) + \tilde{\mathbf{a}}(\theta)$  where  $\hat{\mathbf{a}}(\theta)$  is the nominal estimate (prior information) of the steering vector and  $\tilde{\mathbf{a}}(\theta)$  is the perturbation vector which belongs to the deterministic set  $\mathcal{E}_\theta$  for any  $\theta$ . This uncertainty set is defined as  $\mathcal{E}_\theta \triangleq \{\tilde{\mathbf{a}}(\theta) \mid \|\tilde{\mathbf{a}}(\theta)\|_{\mathbf{W}}^2 = \tilde{\mathbf{a}}(\theta)^H \mathbf{W} \tilde{\mathbf{a}}(\theta) \leq \epsilon_\theta\}$  where  $\mathbf{W}$  is an  $M \times M$  diagonal weight matrix with positive elements. This assumption implies that one is free to weight element-wise uncertainties.

Then the worst-case robust beampattern design is formulated as

$$\begin{aligned} \max_{\mathbf{R}} \min_{\tilde{\mathbf{a}}(\theta_0), \tilde{\mathbf{a}}(\mu_i), i} \quad & g(\theta_0, \mu_i, \mathbf{R}) \\ \text{s.t.} \quad & \mathbf{R} \in \mathcal{R}, \tilde{\mathbf{a}}(\theta_0) \in \mathcal{E}_{\theta_0}, \tilde{\mathbf{a}}(\mu_i) \in \mathcal{E}_{\mu_i}, \forall \mu_i \in \Omega_s, \end{aligned} \quad (7)$$

where the objective function  $g(\theta_0, \mu_i, \mathbf{R})$  equals

$$[\hat{\mathbf{a}}(\theta_0) + \tilde{\mathbf{a}}(\theta_0)]^H \mathbf{R} [\hat{\mathbf{a}}(\theta_0) + \tilde{\mathbf{a}}(\theta_0)] - [\hat{\mathbf{a}}(\mu_i) + \tilde{\mathbf{a}}(\mu_i)]^H \mathbf{R} [\hat{\mathbf{a}}(\mu_i) + \tilde{\mathbf{a}}(\mu_i)]. \quad (8)$$

In general,  $\epsilon_{\mu_i}$  can be chosen to be either a constant for all  $\mu_i \in \Omega_s$  or a function of  $\mu_i$ , i.e.,  $\epsilon_{\mu_i} = f(\mu_i)$ , for each  $\mu_i \in \Omega_s$ . The only limitation related to the function  $f(\cdot)$  is that it should provide a positive value for at least one of the points in the sidelobe region, i.e., there should be a  $\mu'$  for which  $f(\mu') > 0$ .

## 3. ROBUST TRANSMIT BEAMPATTERN DESIGN

The maximin optimization problem (7) is not jointly convex in its variables. Specifically, the inner minimization is not convex in  $\tilde{\mathbf{a}}(\mu_i)$  for a given  $\mathbf{R}$ . In the following theorem, we reformulate the robust beampattern design problem as a convex SDP problem.

**Theorem 1.** Let  $\mathbf{R} \in \mathcal{R}$ ,  $\tilde{\mathbf{a}}(\theta_0) \in \mathcal{E}_{\theta_0}$  and  $\tilde{\mathbf{a}}(\mu_i) \in \mathcal{E}_{\mu_i}$ . The worst-case robust beampattern is given as a solution to the following SDP problem

$$\begin{aligned} & \max_{\mathbf{R} \in \mathcal{R}; t; \beta_{i,1} \geq 0; \beta_{i,2} \geq 0} t \\ & \text{s.t.} \begin{bmatrix} \mathbf{Q} + \beta_{i,1} \mathbf{W}_1 + \beta_{i,2} \mathbf{W}_2 & \mathbf{Q} \tilde{\mathbf{b}} \\ \tilde{\mathbf{b}}^H \mathbf{Q} & \tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}} - t - \beta_{i,1} \epsilon_{\theta_0} - \beta_{i,2} \epsilon_{\mu_i} \end{bmatrix} \succeq 0, \forall \mu_i, \end{aligned} \quad (9)$$

where  $\tilde{\mathbf{b}} = [\hat{\mathbf{a}}^H(\theta_0) \quad \hat{\mathbf{a}}^H(\mu_i)]^H$  and the Hermitian matrices  $\mathbf{Q}$ ,  $\mathbf{W}_1$  and  $\mathbf{W}_2$  are defined as  $\mathbf{Q} \triangleq \begin{bmatrix} \mathbf{R} & \mathbf{0}_M \\ \mathbf{0}_M & -\mathbf{R} \end{bmatrix}$ ,  $\mathbf{W}_1 \triangleq \begin{bmatrix} \mathbf{W} & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{0}_M \end{bmatrix}$  and  $\mathbf{W}_2 \triangleq \begin{bmatrix} \mathbf{0}_M & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{W} \end{bmatrix}$ , respectively.

*Proof:* First, we define  $\tilde{\mathbf{b}} = [\hat{\mathbf{a}}^H(\theta_0) \quad \hat{\mathbf{a}}^H(\mu_i)]^H$ . Given  $\tilde{\mathbf{b}}$  and  $\tilde{\mathbf{b}}$ ,  $g(\theta_0, \mu_i, \mathbf{R})$  in (7) can be stated as

$$g(\theta_0, \mu_i, \mathbf{R}) = [\tilde{\mathbf{b}} + \tilde{\mathbf{b}}]^H \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & -\mathbf{R} \end{bmatrix} [\tilde{\mathbf{b}} + \tilde{\mathbf{b}}].$$

Then, considering the definition of  $\mathbf{Q}$ ,  $\mathbf{W}_1$  and  $\mathbf{W}_2$ , the optimization problem (7) can be equivalently rewritten as

$$\begin{aligned} & \max_{\mathbf{R} \in \mathcal{R}, t} t \\ & \text{s.t.} \quad \tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}} + 2\text{Re}\{\tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}}\} + \tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}} - t \geq 0, \\ & \quad \tilde{\mathbf{b}}^H \mathbf{W}_1 \tilde{\mathbf{b}} \leq \epsilon_{\theta_0}, \\ & \quad \tilde{\mathbf{b}}^H \mathbf{W}_2 \tilde{\mathbf{b}} \leq \epsilon_{\mu_i}, \forall \mu_i \in \Omega_S. \end{aligned} \quad (10)$$

Here, we use the following lemma to reformulate the optimization problem (10).

**Lemma 1.** (*S-Procedure* [22]): Let  $f_k(\mathbf{x}) : \mathbb{C}^n \rightarrow \mathbb{R}$ ,  $k = 0, 1, 2$ , be defined as  $f_k(\mathbf{x}) = \mathbf{x}^H \mathbf{A}_k \mathbf{x} + 2\text{Re}\{\mathbf{b}_k^H \mathbf{x}\} + c_k$ , where  $\mathbf{A}_k = \mathbf{A}_k^H \in \mathbb{C}^{n \times n}$ ,  $\mathbf{b}_k \in \mathbb{C}^n$ , and  $c_k \in \mathbb{R}$ . Then, the statement (implication)  $f_0(\mathbf{x}) \geq 0$  for all  $\mathbf{x} \in \mathbb{C}^n$  such that  $f_k(\mathbf{x}) \geq 0$ ,  $k = 1, 2$  holds if and only if there exists  $\beta_1, \beta_2 \geq 0$  such that

$$\begin{bmatrix} \mathbf{A}_0 & \mathbf{b}_0 \\ \mathbf{b}_0^T & c_0 \end{bmatrix} - \beta_1 \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^T & c_1 \end{bmatrix} - \beta_2 \begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^T & c_2 \end{bmatrix} \succeq 0,$$

if there exists a point  $\hat{\mathbf{x}}$  with  $f_k(\hat{\mathbf{x}}) > 0$ ,  $k = 1, 2$ .

The constraints in the optimization problem (10) can be rewritten as

$$\begin{aligned} f_0(\tilde{\mathbf{b}}) &= \tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}} + 2\text{Re}\{\tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}}\} + \tilde{\mathbf{b}}^H \mathbf{Q} \tilde{\mathbf{b}} - t \geq 0 \\ f_1(\tilde{\mathbf{b}}) &= -\tilde{\mathbf{b}}^H \mathbf{W}_1 \tilde{\mathbf{b}} + \epsilon_{\theta_0} \geq 0 \\ f_2(\tilde{\mathbf{b}}) &= -\tilde{\mathbf{b}}^H \mathbf{W}_2 \tilde{\mathbf{b}} + \epsilon_{\mu_i} \geq 0, \forall \mu_i \in \Omega_S \end{aligned}$$

Now, according to the *S-Procedure* lemma, these three quadratic constraints can be satisfied simultaneously if we find  $\beta_{i,1}, \beta_{i,2} \geq 0$  for which the aforementioned linear matrix inequality (LMI) holds. Thus, the problem boils down to the SDP problem (9) with  $(2M + 1) \times (2M + 1)$  LMIs as the constraints (one LMI for each point  $\mu_i$  in the sidelobe region).  $\square$

Using Theorem 1, the infinite number of constraints in (10) turn to a single SDP constraint for each point  $\mu_i$  in (9). Note that after the optimal  $\mathbf{R}$  has been determined, one can synthesize a waveform

signal  $\mathbf{x}(n)$  that has the same covariance matrix as  $\mathbf{R}$ . One simple approach is  $\mathbf{x}(n) = \mathbf{R}^{1/2} \mathbf{w}(n)$  where  $\mathbf{w}(n)$  is a sequence of i.i.d. random vectors with mean zero and covariance matrix  $\mathbf{I}$ . However, as we mentioned in Section 1, more practical constraints, e.g., constant modulus or low PAR, can be added to the waveform synthesis problem. For detailed discussion see [7, 9] [10, Chapter 14].

In the next section, first we observe what the cost of uncertainty is in terms of the loss in the performance, and then we see how the proposed robust scheme improves the transmit beampattern design. Finally the role of different covariance matrix sets, i.e., the different power constraints, are investigated.

#### 4. NUMERICAL RESULTS

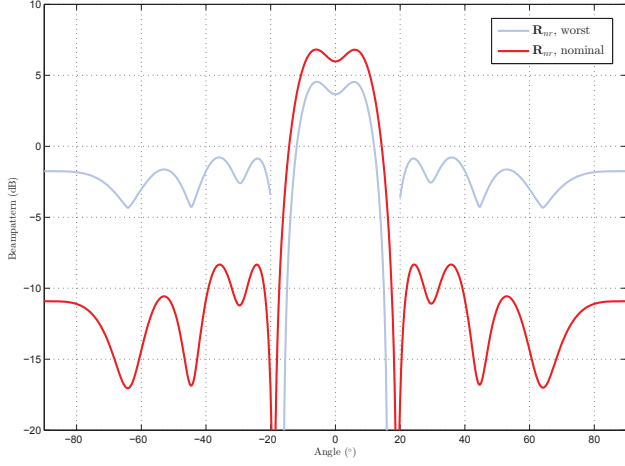
In this section, we illustrate the performance of the proposed robust beampattern design. We consider a uniform linear array with  $M = 10$  number of antennas and half wave-length antenna spacing. The main-beam is centered at  $\theta_0 = 0^\circ$  and the sidelobe region is  $\Omega_S = [-90^\circ, -20^\circ] \cup [20^\circ, 90^\circ]$ . We consider a grid size of  $0.3^\circ$  to represent each point  $\mu_i \in \Omega_S$ . The total power  $\gamma$  is set to 1. Also, we assume that  $\epsilon_{\theta_0} = \epsilon_{\mu_i} = \epsilon$  for all  $\mu_i \in \Omega_S$  and  $\mathbf{W} = \mathbf{I}_M$ . The size of the uncertainty set  $\epsilon$  is assumed to be 5% of  $\|\hat{\mathbf{a}}(\theta)\|^2 = M$  which equals 0.5.

For reference, the optimal covariance matrix  $\mathbf{R}_{nr}$ , when no uncertainty is taken into account, is obtained by solving problem (6) using the nominal steering vectors  $\hat{\mathbf{a}}(\theta)$ . The optimal robust covariance matrix, denoted  $\mathbf{R}^*$ , is obtained by solving (9). Recall that the set  $\mathcal{R}$  includes the main-beam width constraint as described in (6) using nominal steering vectors to control the mainlobe width, otherwise the width of the mainlobe will increase [14]. Two points  $\theta_1 = -10^\circ$  and  $\theta_2 = 10^\circ$  are chosen in the mainlobe region for which the power is guaranteed to be as high as 90% of the power at  $\theta_0$ , i.e.,  $P_L = 0.9$ .

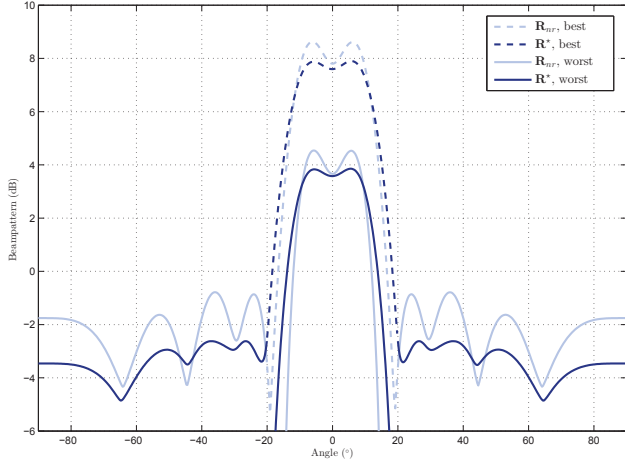
We are interested to study the worst-case performance, which can be thought of as a lower bound to the achievable performances of all steering vectors which belong to the deterministic uncertainty set. The worst-case steering vectors for  $\theta_0$  and  $\mu_i$ s are obtained from the solution to the inner minimization problem in (7), using the fixed covariance matrices  $\mathbf{R}^*$  and  $\mathbf{R}_{nr}$ , respectively. Note that the inner minimization problem can be decoupled with respect to the variables  $\theta_0$  and  $\mu_i$ . Therefore, we first solve the convex minimization problem with respect to  $\theta_0$ , i.e.,  $\min_{\|\tilde{\mathbf{a}}(\theta_0)\|^2 \leq \epsilon} [\hat{\mathbf{a}}(\theta_0) + \tilde{\mathbf{a}}(\theta_0)]^H \mathbf{R} [\hat{\mathbf{a}}(\theta_0) + \tilde{\mathbf{a}}(\theta_0)]$ , using CVX [20, 21]. Then, we find the local optimum for the non-convex maximization problem with respect to each  $\mu_i \in \Omega_S$ , i.e.,  $\max_{\|\tilde{\mathbf{a}}(\mu_i)\|^2 \leq \epsilon} [\hat{\mathbf{a}}(\mu_i) + \tilde{\mathbf{a}}(\mu_i)]^H \mathbf{R} [\hat{\mathbf{a}}(\mu_i) + \tilde{\mathbf{a}}(\mu_i)]$ , using non-linear programming method `fmincon` with interior-point algorithm in MATLAB with random initialization. We treat the points in the mainlobe region similar to  $\theta_0$  to find the worst-case steering vectors, i.e., a minimization problem with the same uncertainty size is solved for each point implying that the worst steering vectors are the ones which yield the minimum beampattern.

According to the different choices of steering vectors, we introduce the following two different scenarios: *worst* and *nominal*, which are exploited to illustrate the performance of our proposed robust design. The first scenario, *worst*, represents the case where the steering vectors  $\mathbf{a}(\theta) = \hat{\mathbf{a}}(\theta) + \tilde{\mathbf{a}}(\theta)$  are used. While for the second scenario, *nominal*, we assume that the steering vectors are accurately known, i.e.,  $\tilde{\mathbf{a}}(\theta) \equiv \mathbf{0}$ .

In Fig. 1, in order to show how steering vector uncertainty will affect the performance, we compare the transmit beampattern of the



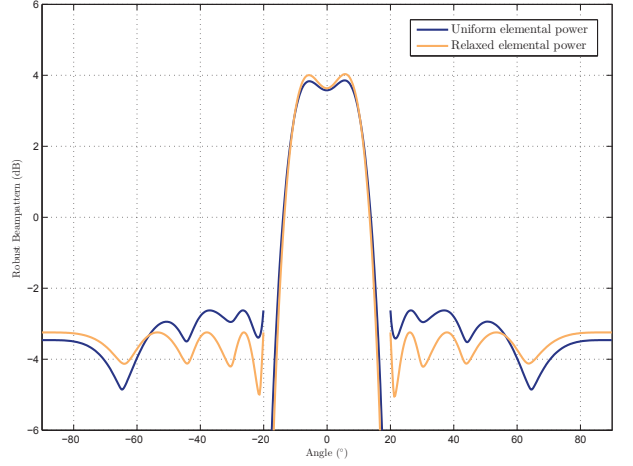
**Fig. 1.** Nominal and worst-case performance of the minimum sidelobe beampattern design using  $\mathbf{R}_{nr}$ , under the uniform elemental power constraint.



**Fig. 2.** Performance comparison of the different minimum sidelobe beampatterns, i.e., using  $\mathbf{R}^*$  and  $\mathbf{R}_{nr}$ , under the uniform elemental power constraint.

nominal scenario when there is no uncertainty with that of the worst scenario for the non-robust optimal covariance matrix  $\mathbf{R}_{nr}$ . This comparison is performed under the uniform elemental power constraint, i.e.,  $\mathcal{R} = \mathcal{R}_u$ . The latter scenario represents the realistic situation where the optimal covariance matrix  $\mathbf{R}_{nr}$  is obtained by using the prior knowledge, i.e., nominal steering vectors, but the performance evaluation is done by using the true steering vectors different from the nominal ones. As can be seen from Fig. 1, deviations from the nominal steering vectors may severely degrade performance. With only 5% errors of the steering vectors, the peak sidelobe levels differ by approximately 7.5 dB.

To illustrate the gain we acquire by taking robustness into account, we plot the transmit beampattern under the uniform elemental power constraint, i.e.,  $\mathcal{R} = \mathcal{R}_u$ , in Fig. 2 where we compare the results provided using the robust optimal covariance matrix  $\mathbf{R}^*$  and the non-robust optimal covariance matrix  $\mathbf{R}_{nr}$ . First, we compare their worst-case performance, i.e., the worst scenario. It can be concluded from Fig. 2 that using the robust worst-case design frame-



**Fig. 3.** Robust minimum sidelobe beampattern design under different power constraints: uniform elemental and relaxed elemental power constraints.

work makes the gap between the peak sidelobe level and the power at  $\theta_0$  be 1.8 dB larger than the gap provided by the non-robust case. This gap for the robust design approximately equals 6.1 dB whereas it is 4.3 dB for the non-robust case. In addition to comparing their worst-case performance, we have also considered the best possible performance for the points in the mainlobe region to show how uncertainty may increase the transmit beampattern. Note that the best possible minimum power level for the sidelobe points can be very low.

Fig. 3 illustrates the effect of different power constraints on the robust transmit beampattern, i.e., using  $\mathbf{R}^*$  for the worst scenario. The different constraints are the relaxed elemental power constraint, i.e.,  $\mathcal{R} = \mathcal{R}_r$  where  $\alpha = 0.2$  and the uniform elemental power constraint, i.e.,  $\mathcal{R} = \mathcal{R}_u$ . In Fig. 3, we observe that for the proposed robust design, the gap between the power level at  $\theta_0$  and the peak sidelobe level is increased when we use the relaxed elemental power constraint thanks to the extra flexibility. Indeed, this behavior has also been reported in [10] for the minimum sidelobe beampattern design, i.e., using  $\mathbf{R}_{nr}$  for the nominal scenario.

## 5. CONCLUSION

We have considered the sidelobe minimization beampattern design problem for MIMO radar communication systems. In contrast to prior work, we consider uncertainties in the steering vectors. These uncertainties may arise from miscalibration, estimation errors, etc. We have proposed a worst-case robust design of the beampattern, taking into account uncertainties in a deterministic set. The resulting non-convex optimization problem has been recast into a set of tractable convex problems for each point in the sidelobe region, using the  $\mathcal{S}$ -Procedure Lemma. We illustrated that even moderate deviations from the nominal steering vectors may result in severe degradation of performance. Taking this uncertainty into account and using the robust design, we showed that a noticeable reduction of the sidelobe peaks is possible. We also illustrated the potential performance gains from allowing some variations in the element-wise power levels.

## 6. REFERENCES

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