# SIGNAL MODEL AND DETECTION PERFORMANCE FOR MIMO-OTH RADAR WITH MULTIPATH IONOSPHERIC PROPAGATION AND NON-POINT TARGETS

Qian He\*, Xiaodong Li\*, Zishu He\*, and Rick S. Blum<sup>†</sup>

\*EE Department, Univ. of Electron. Sci. and Tech. of China, Chengdu, Sichuan 611731 China <sup>†</sup>ECE Department, Lehigh University, Bethlehem, PA 18015 USA

# ABSTRACT

Taking into account the existence of multipath ionospheric propagation (MIP), this paper develops the received signal model for a non-point target for multiple-input multipleoutput skywave over-the-horizon (MIMO-OTH) radar for the first time. The model describes the ionospheric state, the number of propagation paths between a radar antenna and the target center, as well as the statistics of the reflection coefficients. It is shown that varying system parameters, such as antenna positions and signal frequencies, can result in causing the model to change from a case with highly correlated reflection coefficients to a case with virtually uncorrelated reflection coefficients. The proposed model is used to solve a target detection problem. It is shown that it is possible to exploit the MIP to improve the detection performance of the MIMO-OTH radar.

Index Terms— Detection, multipath, non-point target.

## 1. INTRODUCTION

Skywave over-the-horizon (OTH) radar employs ionospheric reflection of signals to detect targets beyond visual range. The performance of the OTH radar system relies heavily on the state of the ionosphere. The commonly used models for describing the ionospheric state usually divide the ionosphere into several layers. These models include the international reference ionosphere (IRI) model [1], the Chapman ionosphere model [2], and the multi-quasi-parabolic (MQP) ionospheric model [3], etc. In OTH radar, transmit signals can travel through different ionospheric layers to reach the target depending on their frequencies, and the signal which bounces off the target can also travel through different ionospheric layers before reaching the receivers, leading to multiple propagation paths, a phenomenon called multipath ionospheric propagation (MIP). In traditional OTH radar systems, approaches have been proposed to attempt to eliminate MIP, for example, by selectively choosing the operational frequency, employing a transmit/receive array with higher angular resolution, etc. However, since these approaches may not work perfectly, this paper studies the impact of any remaining MIP.

MIMO radar continues to receive attention [4]-[15]. The application of MIMO techniques to skywave OTH (MIMO-OTH) radar brings new opportunities and challenges to OTH radar system design. MIMO-OTH radar has been shown to be capable of improving the radar resource management flex-ibility and transmit adaptivity [16], reliably estimating the altitude of a maneuvering target [17], and effectively suppressing auroral ionospheric clutter [18], etc. While the existing work on MIMO-OTH radar employ the point target model and mainly focus on beamforming, estimation or waveform design, this work considers a non-point target model and looks at the detection performance of the MIMO-OTH radar.

Considering that one cannot entirely eliminate the MIP in practice, we develop a received signal model which takes account of the MIP for a non-point target for MIMO-OTH radar, where the MQP ionospheric model is employed to capture the ionosphere behavior. The proposed model describes the ionospheric state, the number of propagation paths from a radar antenna to the target center, as well as the reflection coefficients statistics. We show that with the system parameters (such as the antennas positions and the signal frequencies) properly selected, the MIP can be exploited to improve the detection performance.

#### 2. SIGNAL MODEL

In this section, we develop the received signal model of the MIMO-OTH radar and analyze the correlation between the received signals from different propagation paths.

## 2.1. Assumptions and Received Signal Model

Assume the radar antennas and target (if present) lie in a twodimensional Cartesian coordinate system as shown in Fig. 1. Consider a MIMO-OTH radar equipped with M transmit and N receive antennas. The mth,  $m = 1, 2, \dots, M$ , transmit antenna is located at  $(x_{Tm}, y_{Tm})$ . The baseband signal transmitted from  $(x_{Tm}, y_{Tm})$  is  $\sqrt{E/Ms_m}(t)$ , where  $s_m(t)$  is assumed narrowband with normalized energy and E denotes the total transmit energy. The associated carrier frequency and wavelength are  $f_{cm}$  and  $\lambda_m$ , respectively. The nth, n = $1, 2, \dots, N$ , receive antenna is located at  $(x_{Rn}, y_{Rn})$ . A target conforming to the Swerling-I model is considered, which

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is assumed to be composed of an infinite number of isotropic and independent scatters that are uniformly distributed over a rectangle with dimension  $\Delta x \times \Delta y$  [12]. The superposition of the reflected signals from all these scatters composes the target echo. Assume the center of the target is  $(x_0, y_0)$  and we denote by U(x, y) the reflection coefficient of the scatter located at  $(x + x_0, y + y_0)$ , where  $-\Delta x/2 \le x \le \Delta x/2$ and  $-\Delta y/2 \le y \le \Delta y/2$ . The reflection coefficients are assumed to be zero-mean white (as we change (x, y)) complex random variables, each with variance  $\mathbb{E}\{|U(x, y)|^2\} =$  $1/(\Delta x \Delta y)$  so that the average energy of each echo is normalized to one, where  $\mathbb{E}\{\cdot\}$  is the expectation operator.

According to electromagnetic theory, the actual propagation path of the radar signal through the ionosphere is a curve, for example the red dotted curve  $\widehat{AF'C}$  in Fig. 1, where  $A(x_{Tm}, y_{Tm})$  represents the location of transmitter *m*, point  $C(x_0, y_0)$  the location of the target center, and *F'* the reflection point of the actual path. The Breit-Tuve theorem [20] proves that the propagation time of the signal traveling along the actual path (say,  $\widehat{AF'C}$ ) is equal to the propagation time of the signal traveling in the air along an equivalent isosceles triangular path (say,  $\overline{AFC}$ ). Throughout the paper, we employ equivalent paths for analysis.

The equivalent propagation of the signal transmitted from transmitter m to receiver n with MIP is illustrated in Fig. 1. Assume the mth transmit signal  $s_m(t)$  reaches the target from  $K_m$  different forward propagation paths due to MIP, where the reflection height of the kth  $(k = 1, \dots, K_m)$  path is  $\hbar_{mk}$ . Assume the signal bounces off from the target and arrives at the nth receiver along  $L_{mn}$  backward propagation paths, where the reflection height of the lth  $(l = 1, \dots, L_{mn})$ backward propagation path is  $h_{mnl}$ . Note that the number of forward and backward paths as well as the reflection heights can be computed based on the MQP model [3] based on the known antenna position, signal frequency, and given cell-under-test. The received signal at receiver n due to the transmission of transmitter m is a superposition of the signals from all  $K_m L_{mn}$  paths, which is

$$r_{mn}(t) = \sum_{l=1}^{L_{mn}} \sum_{k=1}^{K_m} r_{mnkl}(t)$$
(1)

where  $r_{mnkl}(t)$  denotes the received signal from the kth forward propagation path and the *l*th backward propagation path, the clutter-plus-noise free version of which is given by

$$r_{mnkl}(t) = \sqrt{\frac{E}{M}} \int_{x_0 - \frac{\Delta x}{2}}^{x_0 + \frac{\Delta x}{2}} \int_{y_0 - \frac{\Delta y}{2}}^{y_0 + \frac{\Delta y}{2}} s_m[t - \tau_{\text{F}mk}(x_{\text{T}m}, y_{\text{T}m}, \gamma, \beta)] - \tau_{\text{B}mnl}(x_{\text{R}n}, y_{\text{R}n}, \gamma, \beta)] e^{j2\pi f_{mnkl}t} e^{j\varphi_{mnkl}} \times U(\gamma - x_0, \beta - y_0) d\beta d\gamma,$$
(2)

where  $f_{mnkl}$  denotes the Doppler frequency of the target associated with the mnkl-th path and  $\varphi_{mnkl}$  represents the effect of the phase perturbation [19] introduced by the ionosphere reflection which is assumed to be constant during the observation interval.  $\tau_{Fmk}(x_{Tm}, y_{Tm}, \gamma, \beta)$ , the time delay of the signal propagating from the *m*th transmitter to



Fig. 1: Equivalent propagation of MIMO-OTH signals.

the scatter located at  $(\gamma, \beta)$  via the *k*th forward path, and  $\tau_{\text{Bmnl}}(x_{\text{Rn}}, y_{\text{Rn}}, \gamma, \beta)$ , the time delay of the *m*th transmitted signal propagating from the scatter at  $(\gamma, \beta)$  to the *n*th receiver via the *l*th backward path, can be computed via the MQP model [3]. This interesting but complicated computation is documented in [21] but omitted here for brevity.

**Lemma 1.** The signal (2) received at receiver n which is initiated by transmitter m and travels along the kth forward (from transmitter m to target center) path and the lth backward (from target center to receiver n) path can be expressed as

$$r_{mnkl}\left(t\right) = \sqrt{\frac{E}{M}} \varepsilon_{mnkl} e^{j2\pi f_{mnkl}t} e^{j\varphi_{mnkl}} e^{-j\phi_{mnkl}} s_m \left(t - \tau_{mn}\right)$$
(3)

where  $\varepsilon_{mnkl} \sim CN(0,1)$  represents the equivalent reflection coefficient with standard complex Gaussian distribution,

$$\phi_{mnkl} = 2\pi f_{cm} \left[ \tau_{Fmk}(x_{Tm}, y_{Tm}, x_0, y_0) - \tau_{Fm1}(x_{Tm}, y_{Tm}, x_0, y_0) + \tau_{Bmnl}(x_{Rn}, y_{Rn}, x_0, y_0) - \tau_{Bmn1}(x_{Rn}, y_{Rn}, x_0, y_0) \right]$$

denotes the phase difference between the signals propagated via the mnkl-th path and the reference path, and

 $\tau_{mn} = \tau_{Fm1} \left( x_{Tm}, y_{Tm}, x_0, y_0 \right) - \tau_{Bmn1} \left( x_m^r, y_m^r, x_0, y_0 \right),$ 

with  $\tau_{Fm1}(x_{Tm}, y_{Tm}, x_0, y_0)$  and  $\tau_{Bmn1}(x_m^r, y_m^r, x_0, y_0)$  denoting the reference time delays corresponding to the forward and backward propagation paths respectively.

Note that proofs for the lemmas and theorems in this paper are provided in [21] but omitted here due to space limitation.

#### 2.2. Correlation Between Received Signals

Based on the received signal model (3), now we discuss the correlation between the reflection coefficients associated with received signals for different paths.

**Theorem 1.** Consider a MIMO-OTH radar system whose received signal can be modeled by (3) in Lemma 1. The reflection coefficients  $\varepsilon_{mnkl}$  and  $\varepsilon_{m'n'k'l'}$  associated with the klmn-th and the k'l'm'n'-th paths are approximately uncorrelated, if at least one of the following inequalities holds

$$\left|\frac{(\hbar_{m'k'}+z_0)(x_{Tm'}-x_0)}{\lambda_{m'}\rho_{m',k'}} + \frac{(\hbar_{m'n'l'}+z_0)(x_{Rn'}-x_0)}{\lambda_{m'}\sigma_{m'n',l'}} - \frac{(\hbar_{mk}+z_0)(x_{Tm}-x_0)}{\lambda_{m}\rho_{m,k}} - \frac{(\hbar_{mnl}+z_0)(x_{Rn}-x_0)}{\lambda_{m}\sigma_{mn,l}}\right| > \frac{1}{\Delta x}$$
(4)

$$\left| \frac{\left( \hbar_{m'k'} + z_0 \right) (y_{Tm'} - y_0)}{\lambda_{m'} \rho_{m',k'}} + \frac{\left( h_{m'n'l'} + z_0 \right) (y_{Rn'} - y_0)}{\lambda_{m'} \sigma_{m'n',l'}} - \frac{\left( \hbar_{mk} + z_0 \right) (y_{Tm} - y_0)}{\lambda_m \rho_{m,k}} - \frac{\left( h_{mnl} + z_0 \right) (y_{Rn} - x_0)}{\lambda_m \sigma_{mn,l}} \right| > \frac{1}{\Delta y} \quad (5)$$

where  $\lambda_m$  is the wavelength of the *m*th transmitted signal;

$$\rho_{m,k} = \left\{ b_m [(\hbar_{mk} + z_0)^2 + z_0^2 - (\hbar_{mk} + z_0)\sqrt{b_m}] \right\}^{1/2}; \quad (6)$$
  
$$\sigma_{mn,l} = \left\{ b_n [(h_{mnl} + z_0)^2 + z_0^2 - (h_{mnl} + z_0)\sqrt{b_n}] \right\}^{1/2}; \quad (7)$$

 $b_m = 4z_0^2 - R_{Dm}^2$ ;  $b_n = 4z_0^2 - R_{Dn}^2$ ;  $z_0$  denotes the earth radius and  $R_{Dm}$  and  $R_{Dn}$  represent the distances from the target center to the mth transmitter and the nth receiver.

From Theorem 1, the correlation between the reflection coefficients depends on several system parameters such as the antenna positions, target location, the target size, and the frequency of the transmitted signals. The conditions given can provide guidance on how to design a MIMO-OTH radar system to provide a set of approximately uncorrelated or correlated reflection coefficients.

Example: Consider the case where the ionosphere is composed of two layers, layer E and layer F. For layer E, the semi-thickness is 15km, the maximum electron density  $0.17 \times 10^{12}/m^3$  occurs at height 115km. For layer F, the semi-thickness is 100km, the maximum electron density  $2 \times 10^{12}/m^3$  occurs at height 310km. Suppose a target of length  $\Delta x = 400m$  and  $\Delta y = 30m$  is centered at (950, 0)km. If the *n*th receiver is located at (100, 0)km, the mth transmitter is located at (0,0) and the mth transmitted signal has a frequency of 20MHz, then there are 2 forward propagation paths and 2 backward propagation paths which give a total of 4 paths. It can be verified that in this case, the reflection coefficients associated with any two different paths are approximately uncorrelated. However, the change of certain system parameters can invalidate the test for approximately uncorrelated reflection coefficients. For example, say the signal frequency is changed to 18MHz or the transmitter is moved to (50, 0)km. These changes will lead to correlated reflection coefficients if all other system parameters are fixed.

# 3. MIMO-OTH RADAR TARGET DETECTION

## **3.1. Optimum Detector**

Based on the received signal model developed, we study the detection performance of the MIMO-OTH radar in this section. To reduce complexity in Sections 3 and 4, we assume the receive antennas are appropriately placed to get complete correlation. This simplifying assumption is easily extended. Further assume that the receive antennas are uniformly linearly spaced with spacing  $d_r$ . Thus  $\varepsilon_{mnkl} = \varepsilon_{mkl}$ ,  $f_{mnkl} = f_{mkl}$ ,  $\varphi_{mnkl} = \varphi_{mkl}$ ,  $\tau_{mn} = \tau_m$ ,  $L_{mn} = L_m$  for all n, and  $\phi_{mnkl} = \phi_{m1kl} + (n-1)2\pi f_{cm} d_r \cos \theta_{ml}/c$ , where  $\theta_{ml}$  is the elevation angle (with respect to the horizon) of the signal corresponding to the *m*th transmitter and the *l*th backward path. Then, the signals received at receiver *n* can be expressed as

$$r_{n}(t) = \sqrt{E/M} \sum_{m=1}^{M} \sum_{l=1}^{L_{m}} \sum_{k=1}^{K_{m}} \varepsilon_{mkl} e^{j2\pi f_{mkl}t} e^{j\varphi_{mkl}} e^{-j\phi_{m1kl}} \\ \times e^{-j(n-1)2\pi f_{cm}d_{r}\cos\theta_{ml}/c} s_{m}(t-\tau_{m}) + w_{n}(t)$$
(8)

Assume the transmitted signals  $s_m(t), m = 1, \dots, M$ are approximately mutually orthogonal for any delay and Doppler of interest. Passing the received signal (8) to a bank of filters, matched to the set of transmitted signals from all of the transmit antennas, the output of matched filter matched to  $s_m(t), m = 1, \dots, M$  is

$$\tilde{r}_{mn} = \sqrt{E/M} \mathbf{a}_{mn}^T \boldsymbol{\varepsilon}_m + w_{mn}$$

where  $\varepsilon_m = [\varepsilon_{m11}, \varepsilon_{m12}, ..., \varepsilon_{mL_mK_m}]^T$ ,  $\mathbf{a}_{mn} = [a_{mn11}, a_{mn12}, ..., a_{mnL_mK_m}]^T$ ,  $a_{mnlk} = e^{-j(n-1)2\pi f_{cm}d_r \cos\theta_{m1}/c} e^{j\varphi_{mkl}}e^{-j\phi_{m1kl}}$ ,  $k = 1, ..., K_m$ ,  $l = 1, ..., L_m$ , and  $w_{nm}$  represents the clutter-plus-noise after matched filtering.

Arrange the outputs of the *m*th matched filter at the *N* receivers into a vector  $\tilde{r}_m$ , which is

$$\tilde{\mathbf{r}}_m = \left[\tilde{r}_{m1}, ..., \tilde{r}_{mN}\right]^T = \mathbf{A}_m \boldsymbol{\varepsilon}_m + \mathbf{w}_m \tag{9}$$

where  $\mathbf{A}_m = [\mathbf{a}_{m1}, ..., \mathbf{a}_{mN}]^T$  and  $\mathbf{w}_m = [w_{m1}, ..., w_{mN}]^T$ . The output of the M matched filters at the N receivers can be expressed as

$$\tilde{\mathbf{r}} = [\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_2, ..., \tilde{\mathbf{r}}_M]^T = \sqrt{E/M} \mathbf{A} \boldsymbol{\varepsilon} + \mathbf{w}$$
 (10)

where  $\mathbf{A} = Diag\{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_M\}$ , with  $Diag\{\cdot\}$  denoting the block diagonal operator,  $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1^T, \boldsymbol{\varepsilon}_2^T, ..., \boldsymbol{\varepsilon}_M^T]^T$  is the reflection coefficient vector which is zero mean complex Gaussian distributed as per Lemma 1, and  $\mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T, ..., \mathbf{w}_M^T]^T$ denotes the clutter-plus-noise vector which obeys a complex Gaussian distribution with mean zero and nonsingular covariance matrix  $\mathbf{R}_{\mathbf{w}}$ . The clutter-plus-noise is assumed to be unrelated (independent) to all other random parameters in the observed signals.

The MIMO-OTH radar hypothesis testing problem chooses between the target presence hypothesis  $H_1$  and the target absence hypothesis  $H_0$  where

$$H_1: \quad \tilde{\mathbf{r}} = \sqrt{E/M} \mathbf{A} \boldsymbol{\varepsilon} + \mathbf{w} , \qquad (11)$$
$$H_0: \quad \tilde{\mathbf{r}} = \mathbf{w}$$

The optimal Neyman-Pearson (NP) test is [21]

$$T = \tilde{\mathbf{r}}^H \mathbf{R}_{\mathbf{w}}^{-1} \mathbf{A} \mathbf{P}^{-1} \mathbf{A}^H \mathbf{R}_{\mathbf{w}}^{-1} \tilde{\mathbf{r}} \gtrsim_{H_0}^{H_1} \delta, \qquad (12)$$

where the threshold  $\delta$  is determined by the false alarm probability,  $\mathbf{P} = E/M\mathbf{A}^H\mathbf{R}_{\mathbf{w}}^{-1}\mathbf{A} + \mathbf{R}^{-1}$ , and  $\mathbf{R}$  is the nonsingular covariance matrix of vector  $\boldsymbol{\varepsilon}$ .

## 3.2. Diversity Gain

Next we discuss the detection performance of the MIMO-OTH radar. Diversity gain, defined as the negative of the slope of the miss probability versus signal-to-clutter-plus-noise ratio (SCNR) for the high SCNR region when a logarithmic scale is employed for both axes [13, 14], is used to explain the probability of detection performance observed in the next section.

**Theorem 2.** Considering a MIMO-OTH radar with M transmit antennas and N receive antennas appropriately placed, assume each transmitted signal  $s_m(t), m = 1, ..., M$ , induces  $L_m K_m$  echoes due to MIP and the receive signal can be modeled by (10). The diversity gain of the radar system, denoted by g, satisfies

$$g \le \min\{MN, \sum_{m=1}^{M} L_m\}$$
(13)

and the maximum diversity gain suggested in (13) is achievable under certain conditions (see numerical examples for more discussion).

Theorem 2 indicates that for a MIMO-OTH radar with appropriately placed antennas, increasing the number of transmit antennas, the number of receive antennas, or the number of backward propagation paths is favorable for improving the maximum achievable diversity gain, while the number of forward propagation paths has no effect on the diversity gain. It is worth noting that when the number of backward propagation paths is large enough such that  $\sum_{m=1}^{M} L_m \ge MN$ , the MIMO-OTH radar can obtain a diversity of MN even if all its antennas are closely spaced. Intuitively, although the closely spaced antennas seem unable to fully exploit the spacial diversity, the multiple paths introduced by MIP leads to a sufficient number of independent channels in order to fill the gap. Thus the existence of MIP can help in increasing the diversity gain of a MIMO-OTH radar which can provide more favorable probability of detection performance as shown in the next section.

# 4. NUMERICAL EXAMPLES

This section studies the detection performance of the MIMO-OTH radar via numerical investigations. Assume the MIMO-OTH radar has M transmit antennas centered at (0,0)km. The signals transmitted from different transmit antennas are approximately orthogonal with respect to each other. The N receive antennas centered at (100, 0)km are assumed to be properly spaced as per Section 3 just to simplify matters. Suppose a target, if present, is located at (1500, 0)km. The reflection coefficients in  $\varepsilon$  are independent standard complex Gaussian random variables. The phase perturbations  $\varphi_{mkl}$ are assumed uniformly distributed in  $[0, 2\pi]$ . Assume the clutter-plus-noise vector w has a diagonal covariance matrix  $\mathbf{R}_{\mathbf{w}} = \sigma_w^2 \mathbf{I}$ , where **I** is the identity matrix. The optimum Neyman-Pearson detector in (12) is employed and the false alarm probability is set to  $P_{fa} = 10^{-3}$ . The simulation results are obtained from 100000 Monte-Carlo experiments.

For some MIMO-OTH radar systems with different configurations  $(M \times N)$ , the miss detection probability  $P_M$  is plotted versus the SCNR in Fig. 2. The SCNR is defined as  $10 \log[(E/M)/\sigma_w^2]$ . It is observed that for each case, the  $P_M$  decreases with the increase of SCNR as expected. For the  $1 \times 2$  configuration with  $L_1 = K_1 = 1$ , it is seen that the  $P_M$  are decreased by approximately a decade when the SCNR is increased by 10 dB. Therefore the slope of the miss probability versus SCNR, if plotted on a log-log scale, is approximately 1. This verifies the conclusion in Theorem 2, which predicts that the maximum diversity gain is  $g = \min\{MN, \sum_{m=1}^{M} L_m\} = \min\{1, 2\} = 1$ . Of notice is that the maximum diversity gain is achieved in our examples since the reflection coefficients in  $\varepsilon$  are statistically independent [21].



Fig. 2: Miss detection probability versus SCNR curves.

For the  $2 \times 2$  configurations, without MIP such that  $L_m = K_m = 1$  for m = 1 and 2, it is observed that increasing SCNR by 10 dB decreases the  $P_M$  by approximately two decades. Hence the slope of the decrease of the  $P_M$  versus SCNR on a log-log scale is g = 2. This again verifies the prediction in Theorem 2 that  $g = \min\{MN, \sum_{m=1}^{M} L_m\} = \min\{4, 2\} =$ 2. When  $K_1 = 1$ ,  $K_2 = 1$ ,  $L_1 = 2$ ,  $L_2 = 1$ , it is seen that the diversity gain is g = 3, which agrees with Theorem 2 where  $g = \min\{MN, \sum_{m=1}^{M} L_m\} = \min\{4, 3\} = 3$ . Comparing with the case without MIP, it is obvious that increasing the number of froward propagation paths improves the detection performance. When  $K_1 = 1$ ,  $K_2 = 1$ ,  $L_1 = 3$ ,  $L_2 = 2$ , it is seen that the diversity gain is g = 4. This again agrees with Theorem 2 where  $g = \min\{MN, \sum_{m=1}^{M} L_m\} = \min\{4, 5\} = 4$  and shows that increasing the number of multipaths can improve the detection performance.

# 5. CONCLUSIONS

MIMO-OTH radar equipped with M transmit antennas and N receive antennas was considered. Taking into account the existence of MIP by using the MQP ionospheric model, the received signal model of MIMO-OTH radar has been developed for non-point targets. The model describes the ionospheric state, the number of propagation paths, and the statistics of the reflection coefficients. Conditions for determining whether the reflection coefficients associated with different propagation paths are approximately uncorrelated have been provided. The model we developed is employed to solve a detection problem. Under the assumptions of orthogonal transmitted signals and complex Gaussian clutter-plus-noise, the optimum detector has been derived. The diversity gain for MIMO-OTH radar target detection was shown to be upper bounded by  $\min\{MN, \sum_{m=1}^{M} L_m\}$ . It was demonstrated that when MIP exists, it can be exploited to improve the detection performance by carefully configuring the system parameters of the MIMO-OTH radar.

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