MIMO RADAR FILTERBANK DESIGN FOR INTERFERENCE MITIGATION

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ABSTRACT

MIMO radars transmit multiple waveforms simultaneously. As the number of waveforms used increases, the cross-correlation between the waveforms also tends to increase if the transmission time or bandwidth is not increased. By using a bank of mismatched filters at the receivers, it is possible to decrease the peak cross-correlation and autocorrelation sidelobe levels of the used waveforms. Furthermore, interference power can also be significantly reduced at the same time. We propose a filterbank design for the MIMO radar receiver based on minimizing the interference power at the receiver while controlling peak sidelobe and cross-correlation values, resulting in a convex optimization problem.

***Index Terms*—** MIMO radar, mismatched filter, filter design, filterbanks, interference, interference mitigation

1. INTRODUCTION

A MIMO radar is a radar concept in which multiple transmitters transmit different waveforms simultaneously. The MIMO radar system can be either distributed, so that the transmitters and receivers are distributed over an area, or colocated, in which case the transmitters and receivers are positioned within the same location. The mismatched[1] filterbank can be applied to both MIMO radar designs.

Optimal operation of the MIMO radar requires that the waveforms can be separated at the receiver, which typically requires that the waveforms are orthogonal. However, it is not possible to have waveforms that are orthogonal for all time delays and Doppler shifts[2]. Optimization of transmitted waveforms for MIMO radar has been studied in many papers in an attempt to obtain waveforms as close to orthogonal as possible. Despite numerous optimization approaches[3–5], achieving low peak sidelobe (PSL) and peak cross-correlation (PCC) levels seems difficult. However, using a mismatched filterbank at the receiver allows reduction of the PSL and PCC further.

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It is known that the matched filter that correlates the transmitted waveform with itself maximizes the SNR in additive white Gaussian noise. The peak autocorrelation sidelobe of the matched filter is determined by the transmitted waveform. To improve the orthogonality of the signals at the receiver, it is possible to use mismatched filters[1]. Mismatched filters have been traditionally used in radars to reduce the sidelobes at the cost of a decrease in the SNR[1]. Mismatched filterbank design for MIMO radars was considered in [6] for limiting the peak autocorrelation sidelobe and cross-correlation levels. Mismatched filtering has also been studied for PSL and integrated sidelobe level reduction in [7] and clutter rejection with a binary sequence in [8].

The drawback of using a mismatched filter is the reduction of SNR at the filter output. Therefore, a constraint on the SNR loss was used in [6]. In this paper, we propose optimizing the MIMO radar receiver filters taking into account both interference and jamming. A design method which allows for controlling the peak autocorrelation sidelobe and cross-correlation levels while effectively mitigating the interference is introduced. Even if the mismatched filter introduces a SNR loss, it is still possible to suppress interference. With the optimal filter design, a good trade-off is achieved so that SNR loss remains tolerable while both unintentional and intentional interference is effectively suppressed. Indeed, it will be shown in the examples that the proposed filter can provide a gain of over 10 dB compared to the matched filter.

This paper is organized as follows: The proposed filterbank design method for interference reduction is described in Section 2. Numerical results will be provided in Section 3 and Section 4 gives the concluding remarks.

2. FILTERBANK DESIGN

The objective is to design a filter for a MIMO radar receiver in such a way that the noise plus interference power is minimized at the filter output while maintaining unit response for the signal of interest. We assume that the transmit waveforms are fixed and filtering is applied at the receiver. The autocorrelation function of the interference is assumed to be either known or it may be reliably estimated in practice. The optimization of the filter coefficients can be done separately at each receiver.

We consider designing the filter coefficients for receiving one of the transmitted waveforms. The process can then be repeated for all the employed MIMO radar waveforms to form a filterbank. The design method can be applied to both distributed and colocated MIMO radar configurations. The filter coefficients are denoted by a $L \times 1$ vector $\mathbf{w}$. The waveform that the filter is intended to receive is denoted by $\mathbf{c}_k$. Since the mismatched filter can be longer than the waveform, $\mathbf{c}_k$ is padded with zeros to match the number of filter coefficients in $\mathbf{w}$. Without loss of generality, $\mathbf{c}_k$ is normalized to have unit norm, i.e. $\|\mathbf{c}_k\|_2 = 1$. It will be seen that the norm of the filter $\mathbf{w}$ is related to the SNR for i.i.d Gaussian noise.

The filter design problem can be written as minimization of the output power[9]

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \text{ s.t. } \mathbf{w}^H \mathbf{c}_k = 1, \tag{1}$$

where $\mathbf{R}_{i+n}$ is the covariance matrix of the interference plus noise. The solution to this problem,

$$\mathbf{w} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{c}_k}{\mathbf{c}_k^H \mathbf{R}_{i+n}^{-1}\mathbf{c}_k}, \tag{2}$$

is the well-known MVDR beamformer[10]. However, this design has the disadvantage that the cross-correlation and autocorrelation sidelobes cannot be controlled so that tolerable levels could be guaranteed. Therefore, additional constraints are required.

The response to a waveform $s_k(n)$ of a receiver filter is given by

$$w * s_k(n) = \sum_{m=0}^{L-1} w(m) s_k(n-m). \tag{3}$$

Now, define $L \times 1$ vectors

$$\mathbf{w} = \begin{bmatrix} w(0) & w(1) & \dots & w(L-1) \end{bmatrix} \tag{4}$$

and

$$\mathbf{c}_k(n) = \begin{bmatrix} s_k^*(n-0) & \dots & s_k^*(n-L+1) \end{bmatrix}. \tag{5}$$

The value of $s_k^*(n-m)$ is considered zero when $n-m$ falls outside the actual signal. The filter output can be written now as

$$\mathbf{c}_k^H(n)\mathbf{w} = w * s_k(n). \tag{6}$$

We can then write the filter design problem for the kth transmitted waveform as a constrained optimization problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \text{ s.t.} \tag{7}$$

$$\mathbf{c}_k^H(0)\mathbf{w} = 1 \tag{8}$$

$$|\mathbf{c}_k^H(n)\mathbf{w}|^2 \le \alpha,\ n = -L+1 \dots -1, 1 \dots L-1 \tag{9}$$

$$|\mathbf{c}_m^H(n)\mathbf{w}|^2 \le \beta,\ m \ne k,\ n = -L+1 \dots L-1. \tag{10}$$

Eq.(9) controls the autocorrelation value for offset n and (10) the cross-correlation, so α and β are the maximum autocorrelation sidelobe and cross-correlation levels, respectively. Since $\mathbf{c}_m$ is deterministic and $\mathbf{c}_m(n)\mathbf{c}_m^H(n)$ is necessarily a rank-one positive-semidefinite matrix for any m and n, this problem is a convex, quadratically constrained quadratic problem (QCQP), which can be solved efficiently[11].

Alternatively, we can minimize the sidelobe and cross-correlation values while constraining the interference plus noise power, in which case the filter design problem can be formulated as

$$\min \alpha \text{ s.t.} \tag{11}$$

$$\mathbf{c}_k^H(0)\mathbf{w} = 1 \tag{12}$$

$$|\mathbf{c}_k^H(n)\mathbf{w}|^2 \le \alpha,\ n = -L+1 \dots -1, 1 \dots L-1 \tag{13}$$

$$|\mathbf{c}_m^H(n)\mathbf{w}|^2 \le t\alpha,\ m \ne k,\ n = -L+1 \dots L-1, \tag{14}$$

$$\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \le \gamma, \tag{15}$$

where t is the desired ratio of the maximum autocorrelation sidelobe and cross-correlation levels. Since the power of the desired signal is not changed by the filter due to unit-response constraint, the SINR gain of the filter is at least γ^{-1} in a linear scale. This optimization problem is also convex. Furthermore, we see that if the interference plus noise is white, $\mathbf{R}_{i+n}$ is proportional to an identity matrix, and the noise power changes by a factor of $\|\mathbf{w}\|^2$. In this case, γ is equal to the maximum SNR loss and the optimization problem becomes the same as the one presented in [6].

3. NUMERICAL EXAMPLES

Numerical examples of the proposed filterbank design are shown in this section. The starting point of the receiver filterbank design in this paper are polyphase Oppermann sequences proposed in [12]. Some of these sequences have autocorrelation and cross-correlation properties comparable to sequences obtained through optimization. For example, [3] used adaptive simulated annealing to find four polyphase codes of lenght 40 with PSL of 0.1820 and PCC of 0.2121. A genetic algorithm was used in [4] to obtain four polyphase codes of lenght 40 with PSL of 0.1581 and PCC of 0.2305. In [5], cross-entropy technique was used to obtain three polyphase codes of lenght 40 PSL of 0.1909 and PCC of 0.1405. Using the Oppermann sequences, one can get 40 polyphase codes of length 41 with PSL and PCC equal to 0.2362.

The transmitted waveforms in the examples were the first three polyphase sequences of the set of length 61 with parameters $m = 1$, $m = 3$, and $p = 1$ as defined in [12]. These waveforms have a normalized peak cross-correlation of 0.1765 and autocorrelation peak sidelobe of 0.1721.

In addition to the PCC and PSL of the receiver filter, a performance criterion used in [6] is the SNR loss defined as ratio of the SNR of matched filter to the SNR of a filter $\mathbf{w}$, given by

$$\mathcal{L}_{\mathrm{SNR}} = \frac{|\mathbf{c}_k^H \mathbf{c}_k|^2/\|\mathbf{c}_k\|^2}{|\mathbf{w}^H \mathbf{c}_k|^2/\|\mathbf{w}\|^2} = \frac{\|\mathbf{c}_k\|^2\|\mathbf{w}\|^2}{|\mathbf{w}^H \mathbf{c}_k|^2} = \|\mathbf{w}\|^2. \quad (16)$$

Similar to this, we define the SINR gain of a filter $\mathbf{w}$ as

$$\mathcal{G}_{\mathrm{SINR}} = \frac{|\mathbf{w}^H \mathbf{c}_k|^2/\mathbf{w}^H \mathbf{R}_{i+n}\mathbf{w}}{|\mathbf{c}_k^H \mathbf{c}_k|^2/\mathbf{c}_k^H \mathbf{R}_{i+n}\mathbf{c}_k} = \frac{\mathbf{c}_k^H \mathbf{R}_{i+n}\mathbf{c}_k}{\mathbf{w}^H \mathbf{R}_{i+n}\mathbf{w}}. \quad (17)$$

In these examples, the autocorrelation function of the interference was proportional to

$$r(k) = e^{-|k|/40 + j2\pi k/7}, \quad (18)$$

which corresponds to a narrow peak slightly off the center frequency of the used frequency band. The receiver noise was a i.i.d. white Gaussian noise. In the first examples, the interference to noise ratio (INR) was 20 dB.

The output of the matched filter and the conventional MVDR filter given by (2) are shown in Figure 1. The filters shown are designed for receiving the first sequence in the used set of polyphase sequences, and the output of the filter for the first sequence (autocorrelation) and the second sequence (cross-correlation) are plotted. The SINR gain of the MVDR filter is about 13.228 dB, so the interference has been suppressed significantly. However, the PSL and PCC values of 0.2179 and 0.2745 are much higher than those of the matched filter.

Figure 2 shows the response of the filter designed using the proposed method. The filter obtained with the method in [6], which minimizes the PSL and PCC while limiting the SNR loss to one dB, is shown for comparison. The minimum PSL and PCC design resulted in PSL and PCC of 0.1113. The SINR gain for the obtained filter was undesirably -0.7418 dB, so this filter obtained with the method of [6] in fact amplified the interference slightly. The proposed method was used constraining the PCC and PSL levels to the same value of 0.1113 while minimizing the interference power. The SNR loss, which was 2.8588 dB, was higher, but the SINR gain was 9.5087 dB, so the filter designed with the proposed method is able to suppress the interference significantly while having equal PSL and PCC levels.

If the PSL and PCC were constrained to same level as with the matched filter, the proposed method achieved a SNR gain of 12.675 dB, which is close the SINR gain of the MVDR. Thus, the proposed method provides excellent interference suppression performance without increasing PSL or PCC values in this example. The results are summarized in Table 1. It is clear that minimizing the interference power is significantly better design strategy for the receiver filterbank design compared to the SNR loss criterion when non-white interference is present. In the case of white noise only, the SINR and the SNR design criteria are the same. It is apparent that the proposed filter design method achieves a very good trade-off between the different design goals.

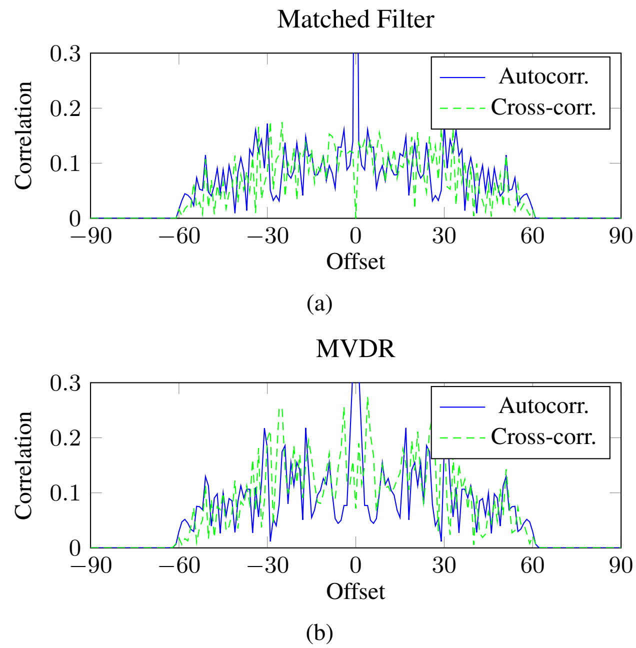

Fig. 1. Output of (a) the matched filter and (b) the MVDR filter of (2). The MVDR results in high waveform cross-correlation and autocorrelation sidelobes due to maximal interference suppression.

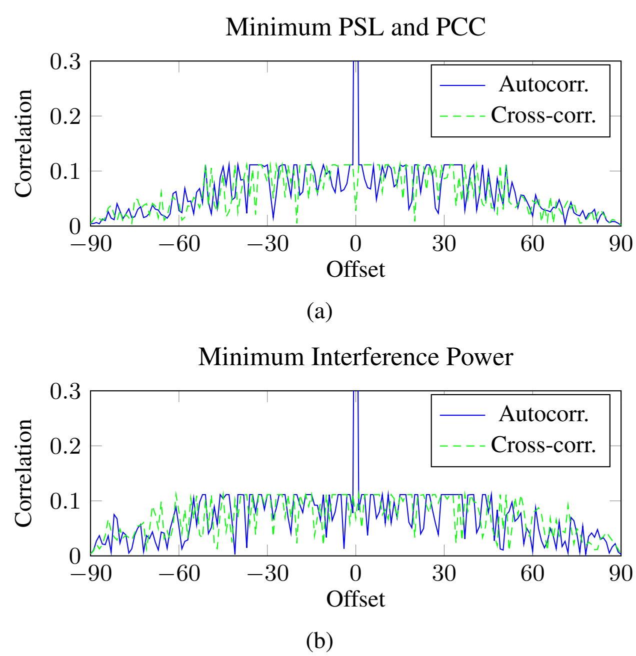

Fig. 2. Responses of (a) the minimum PSL and PCC with SNR loss of 1 dB as in [6] and (b) the proposed minimum interference design with equal PSL and PCC. Both methods have capped the autocorrelation sidelobes and cross-correlation peaks at the same level.

Table 1. The peak sidelobe and peak cross-correlation as well as SINR gain and SNR loss relative to the matched filter in dB. The SNR loss of the minimum PSL and PCC method of [6] was constrained to 1 dB. Two filters were designed with the proposed method, one to have the same PSL and PCC as the minimum PSL and PCC method and the other the same as the matched filter. The resulting filters have significantly better SINR gains than their design counterparts in both cases.

Type	PSL	PCC	SINR Gain	SNR Loss
Matched	0.1721	0.1765	0.0000	0.0000
Proposed	0.1721	0.1765	12.6749	1.0242
MVDR	0.2179	0.2745	13.2277	1.6517
Min PSL&PCC	0.1113	0.1113	-0.7418	1.0000
Proposed	0.1113	0.1113	9.5087	2.8588

The interference mitigation performance of the minimum PSL and PCC method and the proposed method with equal PSL and PCC levels are compared in Figure 3. This figure shows the SINR gain of the filters as the function of the interference to noise ratio. As the minimum PSL and PCC method of [6] does not take the interference into account, the performance varies very little with the interference power. In this particular example, the obtained filter amplifies the interference regardless of the interference power, which is intolerable. The proposed minimum interference power filter design increases the SINR gain with increasing interference power demonstrating that the filter is actively suppressing the interference. As the interference power approaches zero, the SINR gains of the filters tend to the same value as the filters become essentially the same.

The reason why the minimum PSL and PCC design amplifies the interference can be seen in Fig. 4, which shows the frequency responses of the proposed and the minimum PSL and PCC design together with the periodogram of the interference. There is a deep notch near the interference peak in the frequency response of the filter designed with the proposed method. As a result, the interference is suppressed effectively. On the other hand, the minimum PSL and PCC filter does not incorporate interference cancellation in the design and as there is a peak in the frequency response near the strongest interference frequencies, the interference is amplified.

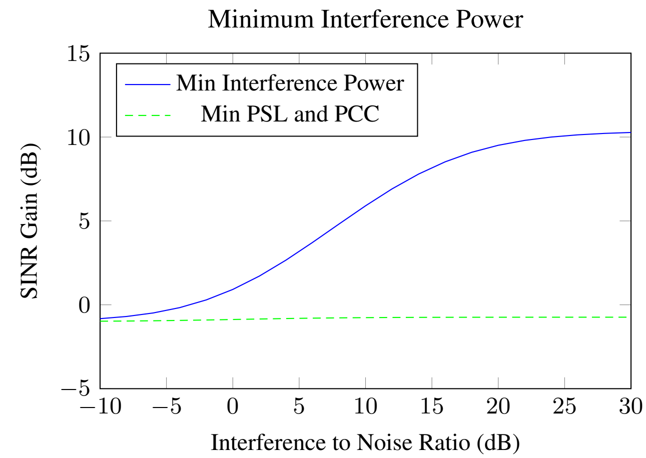

Fig. 3. Interference mitigation performance of the proposed minimum interference power approach and the minimum PSL and PCC design of [6]. The minimum interference power design actively suppresses the interference while maintaining equal PSL and PCC levels, whereas the the minimum PSL and PCC approach amplifies the interference in this example.

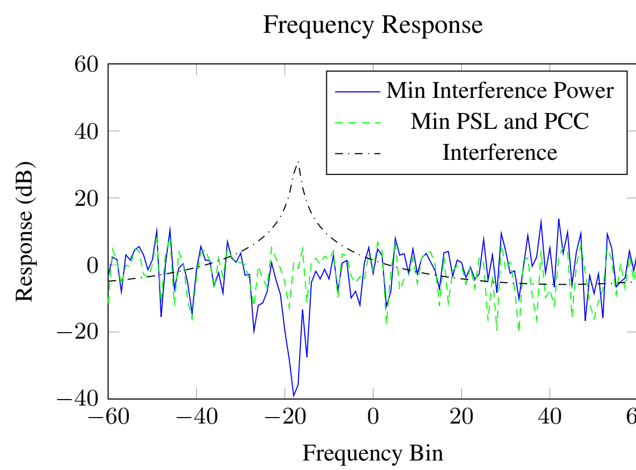

Fig. 4. Frequency response of the proposed filter design and that of the minimum PSL and PCC design together with the periodogram of the interference. There is a deep notch where the interference is strongest in the response of the proposed design, so the interference is effectively suppressed. In the minimum PSL and PCC design, however, there is a peak in the response near the interference peak leading to amplification of the interference.

4. CONCLUSIONS

In this paper, we proposed a method for designing a mismatched filterbank for MIMO radar system by minimizing interference power while constraining the peak autocorrelation sidelobe and peak cross-correlation to a desired level. The optimization can be formulated as convex, quadratically constrained quadratic problem and thus, a global optimum can be found efficiently. In the presence of a non-white interfering signal, using the interference power as optimization criterion produces much better performance in terms of interference and jammer cancellation while maintaining the peak sidelobe and cross-correlation levels compared to minimizing the SNR loss only. In a subsequent study, we will also include Doppler frequency in the mismatched filter design.

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