

MIMO RADAR DEMYSTIFIED AND WHERE IT MAKES SENSE TO USE

Dr. Eli Brookner

Raytheon Co. (Retired), 282 Marrett Road, Lexington, MA 02421
Tel: 781-862-7014; Cell: 781-654-5550; E-mail: Eli.Brookner@gmail.com

Abstract

Contrary to claims made Multiple Input and Multiple Output (MIMO) radars do not provide an order of magnitude or better angle resolution, accuracy and identifiability (the ability to resolve and identify targets) over conventional radars. This claim is based on using a MIMO array radar system consisting of a full transmit array and thinned receive array (or vice versa; called here a full/thin array). This claim for MIMO results from making the wrong comparison to a full conventional array rather than to a conventional full/thin array. It is shown here that a conventional full/thin array radar can have the same angle accuracy, resolution and identifiability as a MIMO full/thin array. Where does the MIMO radar provide a better angle accuracy than a conventional radar? A monostatic MIMO array radar does provide a better angle accuracy than its conventional monostatic equivalent, but it is only about a factor of $1/2$ (29 percent) better and its resolution is the same.

Keywords: MIMO, MIMO Radar, Multiple Input and Multiple Output, radar, phased array, adaptive arrays.

1. WHAT IS MIMO RADAR?

Consider a linear array of N equally spaced elements. When this array is used in a conventional radar each element transmits the same waveform at the same frequency but with a different phase shift per element. This array will form what is called a focused beam. The angle at which the beam steers is determined by the phase shift between elements. The i th element would have a phase shift i to shift the beam to an angle. For a 100 element array having $\lambda/2$ spacing between elements the beamwidth would be about 1° at boresight. In contrast when this linear array is used for a MIMO radar typically each element transmits a different waveform with these waveforms being orthogonal to each other [1, 2]. Because of this the antenna does not form a focused beam as done with a conventional array. In particular each element here for a MIMO array radiates over a beamwidth determined by the beamwidth of each element. This beamwidth might typically be about 120° wide. Thus with the simultaneous transmission of the orthogonal waveforms from the N elements one is illuminating a 120° field-of-view (FOV). These N orthogonal signals leaving the N transmit elements will go to the target and be reflected back. On receive each element will receive the N reflected orthogonal echo waveforms from the target. To process these signals each element needs N matched filters (MFs) for the N orthogonal echo signals; see Fig. 1. Actually FN MFs may be needed per element, where $F > 1$. This is because orthogonal waveforms are often doppler intolerant and to pulse compress one orthogonal signal echo received by one element a bank of F Doppler filters is needed. For simplicity it was assumed that $F=1$ for Fig. 1. Fig. 1 shows that in the receiver the N outputs

from the N matched filters at the i th element are weighted and then summed. The weighting and summing in the i th receiver element for the N echoes from the N transmit elements is focusing the transmit signal. If the array consists of 100 elements with a spacing of $\lambda/2$ then the focused beamwidth of the transmit beam will be 1° . What is happening here is that the focusing of the transmit signal from the N elements is done in the receiver as indicated in Fig. 1. So although we are illuminating a 120° FOV in the transmitter, a typical FOV for an element, we are focusing the beam in the receiver. This is a very nice feature of the MIMO array.

The outputs of the focused beams are designated as E_{ik} in Fig. 1 for the i th element. E_{ik} represents the i th element receiver output for a transmit beam focused in the direction θ_k . The N outputs E_{ik} are combined with weightings W_{Rik} to form a focused receive beam in the direction θ_k using all the N receiver elements. Not shown N focused beams are formed at θ_k , $k = 1$ to N , covering the 120° FOV using the same MF outputs with other weights W_{Tijk} and W_{Rik} . These receiver weights can be adjusted non-adaptively or adaptively to put a nulls in the transmit antenna and receive sidelobes where the clutter is to achieve maximum signal-to-interference ratio. There are some costs though. The signal processing is much more costly than for a conventional array because we need FN matched filters per element for a total of FN^2 matched filters whereas for a conventional array only N matched filters are needed if a doppler tolerant chirp waveform is used as is often done. Thus the MIMO array requires FN more matched filters. Assume a MIMO radar consisting of an array of 100 elements and that F is 30 then one would need $FN=3000$ more MFs per receive element for a total of 300,000 MFs versus only 100 for conventional radar using this 100 element array. We are not making a fair comparison yet, however, because the MIMO array is

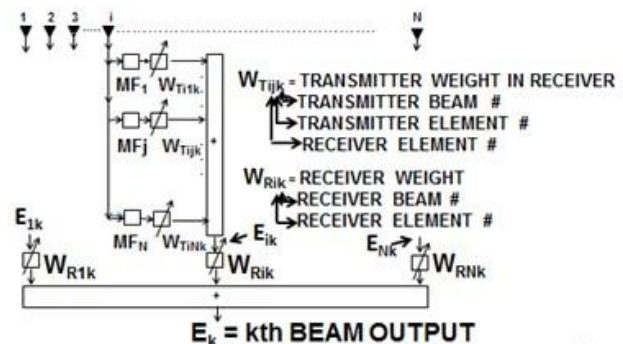


Figure 1: MIMO Monostatic Linear Array Used for Transmit & Receive Showing Receive Beamformer

illuminating essentially all of space 120° so that one searches out the whole of space on one transmission with the MIMO array. This has some advantages and also has some disadvantages as we shall point out shortly.

2. PROOF MIMO ARRAYS NOT ORDER OF MAGNITUDE BETTER THAN CONVENTIONAL ARRAYS

Consider a MIMO full/thin radar consisting of collocated, parallel, linear linear transmit and receive arrays each of $N=10$ elements with spacing $\lambda/2$ for the transmit elements and 2.5λ for the receive elements. Assume uniform weighting for receive and transmit. It has been shown [3] that for such a MIMO full/thin array for which orthogonal waveform are transmitted from the N elements is equivalent to what is called a virtual array consisting of N^2 elements having $\lambda/2$ spacing. Here the virtual array consists of 100 elements with $\lambda/2$ spacing, that is, a full array of 100 elements. What is done in the literature is to compare this to what we get with a conventional array having 10 elements with $\lambda/2$ spacing which is used for transmit and receive. The MIMO array in this case would have a resolution and accuracy 10 times better than the conventional array and would achieve this with 20 elements instead of requiring 100 elements as required for a conventional array. If N was a 100 the MIMO virtual array would have a resolution and accuracy 100 times better than its conventional full array of $N=100$ elements having a $\lambda/2$ equivalent spacing between elements. For $N=1000$, 1000 times better. The problem is that the wrong comparison is being made. As shall be shown we should be comparing to a full/thin conventional array.

For comparison with the above MIMO $N=10$ full/thin array we will start by using the same full/thin arrays for the conventional array, i.e., linear transmit and receive arrays each of $N=10$ elements with spacing $\lambda/2$ for the transmit elements and 2.5λ for the receive elements. For this case Fig. 2a shows the transmit beam pattern (labeled as unmodified) and receive antenna pattern obtained for this conventional array when both are pointing at boresight. Fig. 2b also shows the resulting 2-way beam pattern. What is apparent from this is that the resultant 2-way beam pattern for the conventional array has the same beamwidth as a full array consisting of 100 elements having a spacing of $\lambda/2$. And it has the same beamwidth and resolution as our $N=10$ MIMO full/thin array. Furthermore the conventional array does not have any grating lobes because grating lobes fall at the nulls of the transmit antenna pattern; see Fig. 2b. However, this comparison is not completely fair. The beamwidth for the transmit conventional beam in sine space (i.e., u space where $u = \sin\theta$) is $u = 1/5 = 0.2$ or 11.5° . The receive mainlobe beamwidth in contrast is 1.15° so we are wasting 10 dB of energy. To fix this problem we start by making the transmit antenna pattern have an ideal rectangular shape $1/5$ wide in sine space or 11.5° on boresight; see modified transmit antenna pattern in Fig. 2a. Next we generate ten simultaneous 1.15° wide focused receive beams to cover the 11.5° beamwidth. We scan our ideal beam to other 11.5° angles (actually $=1/5 = 0.2$ angles) until we cover the 120° FOV ($2\sin 60^\circ = 1.73$ in sine space). Nine such scans are needed ($1.73/0.2 = 8.7 \approx 9$). The ideal rectangular transmit pattern shown in Figs. 2a is generated by increasing the length of the transmit antenna a factor of 2 or 3 to 20 or 30 elements with the weighting across these elements being an approximation of that of a $\sin(x/x)$ pattern in order to get approximately a rectangular transmit pattern. This is done when using overlapped receive subarrays in order to generate multiple

simultaneous receive beams for efficient search or efficient limited scan; see Chap. 9 in [5]. It is what is done for the MIT Lincoln Laboratory (LL) MPAR array [6-8] where 2 8×8 subarrays are combined to form overlapped subarrays having twice the size of 16×8 .

Now the conventional full/thin array of Fig. 2a has the same resolution and accuracy as the virtual full/thin MIMO array and at the same time provides coverage for the 120° wide horizon fence. Moreover, our conventional full/thin array can provide coverage of the 120° horizon fence potentially more efficiently with respect to energy utilization than the equivalent full/thin MIMO array. With the conventional full/thin array we dwell at the different $u=1/5$ sectors in u space according to the amount of energy needed based on the element pattern. For our full/thin MIMO array we cannot do this. Assume the MIMO full/thin array has an element pattern given by $\cos^{2n}(\theta)$ 2-way, where n is the element ideality factor which typically ranges from 1 to 1.5. If $n=1$, the MIMO full/thin array in order to provide coverage of the $\pm 60^\circ$ fence has to put out enough energy to provide that coverage at 60° . When doing this the energy it is putting out at boresight is $4 \times$ bigger or 6 dB higher than needed because $\cos 60^\circ = 0.5$. For $n=1.5$ we would be putting out $8 \times$ as much energy on boresight or 9 dB more than needed. To search of the whole 120° fence for $n=1$ we need 3.7 dB more energy for the MIMO full/thin than for the conventional full/thin array, for $n=1.5$, 5.2 dB more energy. One way to fix this for the MIMO array is to have array elements that have an element gain given by $1/\cos^{2n}(\theta)$ two-way. One possible way is to use a dome antenna [9], but is not easy. In the above we just increased the size of the conventional transmit array in order to increase the search efficiency. We could keep the transmit array at $N = 10$ at a loss in search efficiency. Instead of 10 simultaneous receive beams one could have 3, 4 or 5 receive beams. One would then have grating lobes. For 4 beams the maximum grating lobe is 15 dB down. One can determine whether the target is in the grating lobe by shifting where the grating lobes fall on successive search scans. By not increasing transmitter length and using 4 receive beams we reduce the search efficiency by 4 dB which is lower than the inefficiency loss of 5.2 dB for $n=1.5$ for the above equivalent MIMO full/thin array. There is a tradeoff. Thus we have shown it is possible with a conventional full/thin array to achieve the same resolution and accuracy as with its equivalent MIMO full/thin array and be more energy efficient possibly.

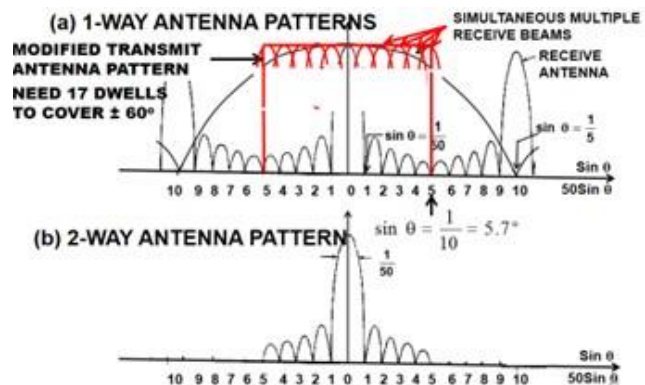


Figure 2: Full/Thin Array Used as Conventional Array .

In the literature [2; see Fig. 1.3, Chap. 1] it is shown that a MIMO full/thin array can get 5x better angle accuracy and identifiability as a conventional array. By identifiability we mean the ability to resolve and identify targets. The problem here again is that the wrong comparison is being made. For the MIMO array they use a full/thin array consisting of collocated transmit and receive linear arrays of 5 elements each with element spacings of respectively 2.5λ and $\lambda/2$. For the conventional array they use a 5 element receive linear array of $\lambda/2$ spacing and a one element transmit array. They should have used for the conventional array a full/thin array.

A full monostatic MIMO linear array of N elements having $\lambda/2$ spacing will provide better angle accuracy than its equivalent full monostatic conventional array having the same number of elements and spacing between elements, a $\sqrt{2}$ better angle accuracy [1,10,11], but it does not have better resolution and identifiability, instead the same.

3. COMPUTATION COMPLEXITY OF MIMO RADAR

As indicated a monostatic MIMO radar consisting of a linear array of N elements requires FN^2 receiver MFs; see Fig.1. Thus for $N=100$, 10,000F MFs are required, where F could be 30 or more. In contrast when the array of Fig. 1 is used for a conventional array radar only $N=100$ MFs are needed if a chirp doppler tolerant waveform is used, 3,000 fewer MFs. Thus the MIMO radar MF computation load for the MIMO radar can be orders of magnitude more than needed for a conventional radar.

The beam forming load is also larger. For the monostatic MIMO radar of Fig. 1, in the receiver each element forms N focused transmit beams covering the FOV to be searched. Because there are N elements N^2 such focused beams are formed to cover the FOV, 120° in the example of Sect. 1. In addition N receive focused beams are formed for a total of N^2+N . In comparison for a conventional monostatic array of N elements only N receive beams are needed for a total of only $N+1$ if a spoiled beam is used for transmit.

To improve the search energy efficiency and computational load for a monostatic MIMO array what can be done is to break up the array into subarrays. Consider a linear monostatic MIMO array consisting of 100 elements. Let us break the array into 10 subarrays of 10 elements each both on transmit and receive. All the subarrays are used as conventional arrays of 10 elements each. Specifically the 10 elements of a subarray use the same orthogonal waveform but the orthogonal waveforms are different for each subarray. Furthermore all the elements of all subarrays have phase shifts that steer the beam to a specific angle θ both on transmit and receive on one dwell time. Each subarray receives all the orthogonal waveforms simultaneously. We can now think of the 10 subarrays as being the elements of a 10 element MIMO radar, the subarrays being the elements. We will call such a MIMO array a subarray-MIMO array or SA-MIMO. This is in contrast to the MIMO array where we have a different orthogonal waveform for each of the elements, which we call an element-MIMO (E-MIMO) array. A SA-MIMO array overcomes the search energy inefficiency when searching a horizon fence as occurred in Sect. 2 when using E-MIMO array. This is because with the E-MIMO array we are illuminating the whole 120° horizon fence FOV with one illumination whereas with the SA-

MIMO array we will be only illuminating one small sector at a time just as done for the conventional full/thin array given in Sect. 2. We can not improve a lot the search efficiency of the full/thin MIMO array of Fig. 2a by using subarrays because with only 10 elements in the full transmit array, we can not form many subarrays.

Breaking up the E-MIMO array into subarrays to form the SA-MIMO array reduces the computational load. The SA-MIMO array is a MIMO array consisting of N_s elements instead of N elements where N_s is the number of subarrays. As a result instead of FN^2 matched filters being required, FN_s^2 are required. This results in reduction of number of match filters required by $(N_s/N)^2$. However, the computation has to be done N/N_s times as often in order to search out the whole volume of the search fence for the SA-MIMO array. As a result the throughput is reduced by the factor N_s/N instead of $(N_s/N)^2$. For the monostatic $N=100$ element MIMO radar using the subarraying of $N_s=10$ we get a throughput reduction of 10 or an order of magnitude. Not insignificant.

The 100 element SA-MIMO linear array example described above did not use overlapped subarrays. Using overlapped subarrays on receive offers several advantages. It allows the subarray receive patterns to be rectangular by having the weighting across the elements of each receive subarray be approximately that of a $\sin(x/x)$ pattern in order to get approximately a rectangular receive subarray pattern for each subarray. This is exactly what is done for the MPAR array [6-8] as described above in Sect. 2 to get a rectangular transmit pattern. A rectangular receive subarray pattern lowers the grating lobes as done in Fig. 2. It also leads to more efficient search as done for Fig. 2. Overlapped subarraying can be done on transmit also if linear power amplifiers (PA) are used for each array element on transmit in an active array. Linear PA however are not as power efficient as hard limited class C PAs. Ref. 21 considers the use of transmit subarrays for a MIMO array, called TS-MIMO, to reduce the computation load for putting nulls in the direction of interference. Consideration to overlapped subarraying is also given in [24]; see also [25].

4. ADAPTIVE PROCESSING WITH MIMO RADAR

It was mentioned in Sect. 1 that an advantage of the MIMO array is that it can perform the adaptive nulling of the transmit beam in the receiver so that we can have low sidelobes in the direction of clutter for the transmit beam as well as the receive beam thus enabling a much lower 2-way sidelobe level for clutter rejection. This advantage though comes at a possible penalty. The penalty is in the increased computation. Assume no pulse Doppler processing is needed for the MIMO array, i.e., $F=1$. To establish the weights for the adaptive nulling for the transmit and receive beams of Fig. 1 for a monostatic MIMO array of N elements one needs to invert an $N^2 \times N^2$ matrix. For $N=100$ this becomes a 10,000 x 10,000 matrix to invert. Also the number of training samples needed for the MIMO array radar is N times that of a conventional array. For the case $N=100$ we would need 50,000 samples, or 50 ms for a 1 MHz bandwidth, to get a signal-to-interference ratio to within 1 dB of the ideal achieved if one knew the interference covariance matrix perfectly. The situation would be worse with pulse doppler processing. The size of the matrix that has to be inverted may be reduced using techniques used for Space-Time Adaptive Processing (STAP) [22; Sect. 4]. The block diagonal form of its interference

covariance matrix with each block having a Toeplitz form could help. Alternately for some applications it may be possible to separately optimize the transmit and receive weightings with $N \times N$ matrices having to be inverted [23].

In practice for conventional arrays we do not have to adaptively put nulls in the beam in the direction of the clutter. Instead we put open loop nulls in the direction of the clutter in the receive and transmit antenna patterns. For an active array using Class C power amplifiers (PAs) in the T/R modules this is done by controlling the phase weighting across the antenna. One can do this because typically one knows where the high clutter is. For example for a ground based radar the high clutter is on the horizon so one would put a null in the transmit and receive beams in the direction of the ground clutter. For an airborne radar in a look down mode one could have the clutter in the direction of the target so open loop nulling could not be used but adaptive nulling also would not work.

5. OTHER CLAIMS FOR MIMO ARRAYS

It has been claimed that MIMO can be put in a multistatic arrangement and provide target cross section angle diversity and better target location accuracy. However, the same is true for conventional radars put in a multistatic arrangement. Contrary to some claims MIMO radars can have same coherence dwell time as conventional radars when the proper comparison is made [1].

6. WHERE DOES IT MAKE SENSE TO USE MIMO AND VICE VERSA

A MIMO array is not efficient for track. If we have an E-MIMO array it has a FOV-of-view of a single element of the array which can be 120° . If we have the target in track we don't want to illuminate 120° , or even smaller angle if SA-MIMO is used. We want to use a high gain beam that is only illuminating the target. This is achieved by having the array be a conventional array using a focused narrow beam on transmit which if it has 100 elements with $\lambda/2$ spacing would have a beamwidth of about 1° . One could use an array in a MIMO mode for search and to use it as a conventional array with focused beams for track. This has to be traded off doing the search with a conventional array using a spoiled beam or machine gunning on transmit and a stacked beams on receive. The latter conventional approaches do not have a pulse chasing problem for a bistatic system just as is the case for a MIMO system. If one were doing track-while-scan (TWS) a MIMO array may make sense, especially a SA-MIMO array. Using MIMO for search does though in all cases have a processing penalty re its conventional equivalents. It is not clear that MIMO arrays would be less degraded by multipath.

Assume that one has in production a radar having a specific Power-Aperture-Gain (PAG) or Power-Aperture (PA) and that one would need to have a radar with a higher PAG and or PA. One would like to use the radar in production to achieve the higher PAG or PA. This can be done using MIMO, placing two of these radars near each other and cohering their transmitter illuminations on the target and also cohering their received echoes to achieve a radar system having a PAG that is 9 dB higher than that of a single radar and a PA that is 6 dB higher of a single radar [12-14]; see also [15]. Brookner et al [16] describe a patented incoherent MIMO method for combining two

radars to achieve an 8.7 dB PAG sensitivity improvement when the probability of detection required is 90%. It uses different carrier frequencies for the two radars with the frequencies separated far enough apart to provide frequency diversity for detection of a Swerling I target.

It was indicated before that for a monostatic MIMO array radar one can achieve an angle accuracy measurement on the target that is $1/\sqrt{2}$ better than that achieved with the conventional array. This would be critical for applications where the space is at a premium like for air, missile and space borne systems. It would have to be traded off against the option of increasing the power by a factor of 2 to achieve the $\sqrt{2}$ improvement. Also factored into the tradeoff is the increase in weight, size and power requirements to do the MIMO signal processing over that needed for a conventional array radar.

The use of MIMO for an OTH radar is another attractive near term application [17]. The bandwidth is small and the number of elements is not large. Hence the signal processing load would not be large. Low frequency MIMO radars in general have the advantage of small Doppler shifts and as a result may not have as severe a doppler variance problem.

This write-up did not address the possible usefulness of MIMO for airborne radars. No doubt MIMO will find applications in the future as the signal processing capabilities continues to increase. MIMO is successfully used at present for communication systems where it allows one to take advantage of multipath channels to increase channel data rate [18-20].

7. ACKNOWLEDGMENT

Thanks due Mike Sarcione, Dr. Jama Mohamed, Dr. Dan Marshall, Joe Gwinn, Dr. Daniel Zwillinger, Dr. William P. Ballance, all at Raytheon Co., Prof. Jian Li (Un.Florida), Dr. Dan Rabideau (MIT Lincoln Lab.), Dr. Jian Wang (formerly Raytheon, now Rockwell Collins) Dr. Robert Francois (retired from Raytheon) and the reviewers for their inputs.

8. REFERENCES

- [1] E. Brookner, MIMO Radar: Demystified, Microwave J, 1, Jan. 2013.
- [2] J. Li and P. Stoica (editors), MIMO Radar Signal Processing, John Wiley & Sons Inc, 2009
- [3] K. W. Forsythe and D. W. Bliss, MIMO Radar: Concepts, Performance, Enhancements, and Applications, Chap. 2 in: J. Li and P. Stoica (editors), MIMO Radar Signal Processing, John Wiley & Sons Inc., Somerset, NJ, 2009.
- [4] E. Brookner, Radar Technology, Artech House, Norwood, MA, 1977.
- [5] E. Brookner, Practical Phased Array Antenna Systems, Artech, 1991.
- [6] Herd, J. et al, IEEE Radar 2010, Washington, DC
- [7] Conway, D., J. Herd and K. Hondl, "Digital Beam Phased Array Radar for Aircraft and Weather", EuRAD, 2012.
- [8] Brookner, E., "Recent Developments and Future Trends in Phased Arrays", IEEE International Symposium on Phased Array Systems and Technology, 2013, Boston, MA.
- [9] E. Brookner, Aspects of Modern Radar, Artech House, 1988.
- [10] D. Bliss, MIT Lincoln Laboratory, private communication.
- [11] Rabideau, D., MIMO Radar Optimization, IET, Radar Sonar Navig., 2011, v. 5, #2, pp.155-162.

- [12] G.D. Thome, R.P. Enzman and F. Steudel, "System and Method for Coherently Combining Plurality of Radars," US Patent Application Publication, dated October 5, 2006, Pub. No. US2006/0220951 A1.
- [13] G.D. Thome, R.P. Enzmann and F. Steudel, "System and Method For Coherently Combining Plurality of Radars," US Patent No. US 7,358,892 B2, dated April 15, 2008.
- [14] L. Green, "Cooperative Radar System," US Patent No. US 6,362, 774 B1, dated March 26, 2002.
- [15] Coutts and Coumo, "Radar Combining," IEEE SAM 2006, Boston, MA.F. Robey et al., "MIMO Radar Theory and Experimental Results," Proceedings of the 2004 Asilomar Conference, pp. 300-304.
- [16] E. Brookner, D.V. Manooogian and F. Steudel, "Multiple Radar Combining for Increased Range, Radar Sensitivity and Angle Accuracy," US Patent No. US 2005/0231420 A1, dated Oct. 20, 2005.
- [17] G.J. Frazer, Y.I. Abramovich, B.A. Johnson and F.C. Robey, "Recent Results in MIMO Over-the-Horizon Radar," IEEE Array-2010.
- [18] L. Bai and J. Choi, Low Complexity MIMO Detection, Springer, New York, NY, 2012.
- [19] D. Tse and P. Viswanath, Fundamentals of Wireless Communications, Cambridge University Press, New York, NY, 2005.
- [20] Simon, Haykin, Digital Communication Systems, Wiley, 2014 21. [21] H. Li, and B. Himed, "Transmit Subaperturing for MIMO Radars with Co-Located Antennas," IEEE Journal of Selected Topics in Signal Processing, February 2010, pp. 55-65.
- [22] Ward, J., "Space-Time Adaptive Processing for Airborne Radar", MIT Lincoln Lab. Tech. Rept. 1015, 12/13/94.
- [23] Marshall, Dan, Raytheon Co., private communication.
- [24] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," IEEE Trans. Signal Process., vol. 58, no. 6, pp. 3137–3151, Jun. 2010.
- [25] A. Hassanien and S. A. Vorobyov, "Transmit Energy Focusing for DOA Estimation in MIMO Radar With Colocated Antennas," IEEE Trans. Signal Process., vol. 59, no. 6, pp. 2669 - 2682, Jun. 2011.