UNIMODULAR CODE DESIGN FOR MIMO RADAR USING BHATTACHARYYA DISTANCE

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ABSTRACT

In this paper, we study the problem of unimodular code design to improve the detection performance of statistical multiple-input multiple-output (MIMO) radar systems. To this end, we consider a system transmitting arbitrary unimodular signals and a discrete-time formulation of the problem. Due to the complicated form of the performance metric of the optimal detector, we resort to the Bhattacharyya distance for code design. We devise a novel method based on the majorization of matrix functions to obtain solutions to the constrained design problem. Simulation results show the effectiveness of the proposed method.

Keywords: Bhattacharyya distance, code design, majorization-minimization, MIMO radar, unimodular signals.

1. INTRODUCTION

MIMO radar has recently been a topic of interest for many researchers. Angular diversity (in statistical MIMO[1]) and waveform diversity (in collocated MIMO) provide more degrees of freedom for MIMO systems as compared with conventional systems. The provided degrees of freedom lead to performance improvement in target detection, parameter estimation, target identifiability, clutter rejection, as well as spatial and temporal beampattern [1].

Waveform design for MIMO radars plays an important role in utilizing the available resources of the system. Transmit waveforms can be optimized to achieve performance improvement in detection, estimation, target classification, identification, and beampattern synthesis [1]. Waveform design for statistical MIMO radars is mainly concerned with detection performance improvement. In [2], a unified optimization framework has been developed for code design in MIMO radars with orthogonal transmission using information theoretic criteria. Regarding MIMO radars employing arbitrary signals, reference [3] considers space-time code design for the systems using mutual information criterion (see also [4]). Moreover, diversity-integration trade-off has been addressed in [3]. In [5], a similar problem has been taken into account via employing the Kullback-Leibler (KL) divergence as a design metric. The cited paper uses a weighted sum of the KL-divergences associated with probability density functions for observations in the detection problem. The authors of [6] consider both mutual information and KL-divergence as design metrics for a problem related to that of [3]. Note that in a majority of MIMO radar code design studies in the literature, no constraints (on codes) other than energy constraint are considered. Additionally, identical statistics for the target and interference have been assumed at various receivers. However, designing unimodular codes is of practical interest due to avoiding non-linear effects and employing the available power at the transmitter [7]. Furthermore, target and interference possess different statistics at various receivers.

In this paper, we study the problem of code design for MIMO radars with arbitrary unimodular signals. Under the Gaussian assumption, we present the optimal detector for arbitrary covariance matrices of the target and interferences at various receivers. The performance metric of the detector is too complicated to be used for code design; hence we resort to the Bhattacharyya distance as the design metric. In Section 2, we state the code design problem with unimodularity constraint. Next we devise a novel method based on the majorization-minimization (MaMi) approach to obtain a solution to the non-convex design problem (see Section 3). We show that each iteration of the proposed method can be handled via solving a unimodular quadratic program (UQP). This makes the proposed method quite fast as there exists an efficient method to tackle the UQP. Finally, Section 4 contains numerical examples and discussion.

2. DESIGN PROBLEM

We consider a MIMO system with N_t transmit antennas (transmitting arbitrary waveforms) and N_r receive antennas. Let $\mathbf{a}_m = [a_{m1}\ a_{m2}\ ...\ a_{mN}]^T$ denote the transmit code of the m^{th} transmitter. The problem of detecting a target (in the cell under test) at the k^{th} receive antenna can be expressed

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[1]Also known as MIMO Radars with widely separated antennas.

as [3] [8]:

$$\begin{cases} \mathrm{H}_0: & \mathbf{r}_k = \mathbf{n}_k \\ \mathrm{H}_1: & \mathbf{r}_k = \mathbf{A}\boldsymbol{\alpha}_k + \mathbf{n}_k \end{cases}, k = 1, 2, ..., N_r \tag{1}$$

where $\mathbf{r}_k$ denotes the discrete-time received signal at the k^{th} receive antenna, $\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2\ ...\ \mathbf{a}_{N_t}] \in \mathbb{C}^{N \times N_t}$ is the code matrix, $\boldsymbol{\alpha}_k \in \mathbb{C}^{N_t}$ describes target scattering effects associated with various transmitters at the k^{th} receive antenna, and $\mathbf{n}_k$ denotes the interference at the k^{th} receive antenna.

Let $\mathbf{M}_k = \mathrm{E}\{\mathbf{n}_k\mathbf{n}_k^H\}$ and $\mathbf{R}_k = \mathrm{E}\{\boldsymbol{\alpha}_k\boldsymbol{\alpha}_k^H\}$. The optimal detector corresponding to the above problem (assuming Gaussian target and interference) is obtained by applying the estimator-correlator theorem [9]:

$$T_c = \sum_{k=1}^{N_r}\sum_{n=1}^{\delta_k}\left(\frac{\lambda_{kn}}{1+\lambda_{kn}}\right)|y_{kn}|^2 \underset{\mathrm{H}_1}{\overset{\mathrm{H}_0}{\lessgtr}} \eta \tag{2}$$

with $\{\lambda_{kn}\}_{n=1}^{\delta_k}$ being the nonzero eigenvalues of the matrix $\mathbf{H}_k \triangleq \mathbf{M}_k^{-1/2}\mathbf{A}\mathbf{R}_k\mathbf{A}^H\mathbf{M}_k^{-1/2}$, η being the detection threshold, and $\mathbf{y}_k = \mathbf{V}_k^H\mathbf{M}_k^{-1/2}\mathbf{r}_k$. Herein $\mathbf{V}_k$ represents the matrix of eigenvectors associated with $\{\lambda_{kn}\}$.

We aim to find the optimal code matrix $\mathbf{A}$ for improvement of the detection performance. The performance metrics of the detector in (2) can be obtained using the results of [10]. However, the corresponding expressions appear to be too complicated for code design and we do not provide them herein (see [10]). Therefore, we consider Bhattacharyya distance as an information-theoretic criterion for code design. Note that the Bhattacharyya distance provides an upper bound on the probability of false alarm P_{fa}, and at the same time yields a lower bound on the detection probability P_d [11]. Therefore, maximization of the Bhattacharyya distance minimizes the upper bound on P_{fa} and, at the same time, it maximizes the lower bound on P_d.[2] The Bhattacharyya distance $\mathcal{B}$ for two multivariate Gaussian distributions, $\mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_1)$ and $\mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_2)$, can be expressed as [11]:

$$\mathcal{B} = \log\left(\frac{\det(0.5(\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2))}{\sqrt{\det(\boldsymbol{\Sigma}_1)\det(\boldsymbol{\Sigma}_2)}}\right). \tag{3}$$

By applying (3) to the problem in (1) we obtain

$$\mathcal{B} = \sum_{k=1}^{N_r}\Big(\log\det(\mathbf{I} + 0.5\mathbf{H}_k) - 0.5\log\det(\mathbf{I} + \mathbf{H}_k)\Big). \tag{4}$$

Consequently, the problem of unimodular code design via maximization of $\mathcal{B}$ can be cast as:

$$\begin{cases} \max\limits_{\mathbf{A},\{\mathbf{H}_k\}} \sum\limits_{k=1}^{N_r}\Big(\log\det(\mathbf{I} + 0.5\mathbf{H}_k) - 0.5\log\det(\mathbf{I} + \mathbf{H}_k)\Big) \\ \text{subject to} \quad \mathbf{H}_k = \mathbf{M}_k^{-1/2}\mathbf{A}\mathbf{R}_k\mathbf{A}^H\mathbf{M}_k^{-1/2},\ \forall k \\ \qquad\qquad |[\mathbf{A}]_{m,n}| = 1,\ \forall\,(m,n) \end{cases} \tag{5}$$

[2] This is due to the fact that $P_d \geq 1 - \gamma^{-1/2}e^{-\mathcal{B}}$ and $P_{fa} \leq \gamma^{1/2}e^{-\mathcal{B}}$ where γ is the likelihood threshold [12].

Note that related unconstrained design problems for special cases in which $\mathbf{R}_k = \mathbf{I}$ or $\mathbf{M}_k = \mathbf{M}, \mathbf{R}_k = \mathbf{R}$ were considered in previous works using KL-divergence, J-divergence, and mutual information [3, 5, 6]. In what follows, we devise a novel method based on the majorization of matrix functions to tackle the above non-convex problem.

3. THE PROPOSED METHOD

We use the majorization-minimization (or minorization-maximization) technique to tackle the problem in (5).
Majorization-minimization (MaMi) is an iterative technique that can be used for obtaining a solution , i.e. a stationary point, of the general minimization problem

$$\min_{\mathbf{z}}\ \widetilde{f}(\mathbf{z})\ \ \text{subject to}\ \ c(\mathbf{z}) \leq 0 \tag{6}$$

where $\widetilde{f}(.)$ and $c(.)$ might be non-convex functions. Each iteration (say the l^{th} iteration) of MaMi consists of two steps (see Fig. 1) [13]:
• Majorization Step: Finding $p^{(l)}(\mathbf{z})$ such that its minimization is simpler than that of $\widetilde{f}(\mathbf{z})$, and $p^{(l)}(\mathbf{z})$ majorizes $\widetilde{f}(\mathbf{z})$:

$$p^{(l)}(\mathbf{z}) \geq \widetilde{f}(\mathbf{z}),\ \ \forall\mathbf{z}\ \text{ and }\ p^{(l)}(\mathbf{z}^{(l-1)}) = \widetilde{f}(\mathbf{z}^{(l-1)}) \tag{7}$$

with $\mathbf{z}^{(l-1)}$ being the value of $\mathbf{z}$ at the $(l-1)^{th}$ iteration.
• Minimization Step: Solving the optimization problem,

$$\min_{\mathbf{z}}\ p^{(l)}(\mathbf{z})\ \ \text{subject to}\ \ c(\mathbf{z}) \leq 0 \tag{8}$$

to obtain $\mathbf{z}^{(l)}$.

We begin by substituting $\mathbf{H}_k$ in the objective function of (5) and rewriting the first logarithmic term (for each k):

$$\begin{aligned} &\log\det\left(\mathbf{I} + 0.5\mathbf{M}_k^{-1/2}\mathbf{A}\mathbf{R}_k\mathbf{A}^H\mathbf{M}_k^{-1/2}\right) = \\ &\log\det\left(\mathbf{I} + 0.5\mathbf{R}_k^{1/2}\mathbf{A}^H\mathbf{M}_k^{-1}\mathbf{A}\mathbf{R}_k^{1/2}\right) \end{aligned} \tag{9}$$

where we have used a standard determinant property. Now we reformulate the above expression as a convex function of another variable. Let $\mathbf{X}_k = \mathbf{I} + 0.5\mathbf{R}_k^{1/2}\mathbf{A}^H\mathbf{M}_k^{-1}\mathbf{A}\mathbf{R}_k^{1/2}$. By using the matrix inversion lemma we have

$$\mathbf{X}_k^{-1} = \mathbf{I} - 0.5\mathbf{R}_k^{1/2}\mathbf{A}^H\left(0.5\mathbf{A}\mathbf{R}_k\mathbf{A}^H + \mathbf{M}_k\right)^{-1}\mathbf{A}\mathbf{R}_k^{1/2}. \tag{10}$$

Next observe that for $\mathbf{U} = [\mathbf{I}_{N_t}\ \mathbf{0}_{(N+N_t)\times N_t}]^T$ and

$$\mathbf{B}_k \triangleq \begin{bmatrix} \mathbf{I} & (1/\sqrt{2})\mathbf{R}_k^{1/2}\mathbf{A}^H \\ (1/\sqrt{2})\mathbf{A}\mathbf{R}_k^{1/2} & \mathbf{M}_k + 0.5\mathbf{A}\mathbf{R}_k\mathbf{A}^H \end{bmatrix} \tag{11}$$

one can write $\mathbf{U}^H\mathbf{B}_k^{-1}\mathbf{U} = \mathbf{X}_k$. Therefore, the expression in (9) can alternatively be written as

$$\log\det\left(\mathbf{U}^H\mathbf{B}_k^{-1}\mathbf{U}\right). \tag{12}$$

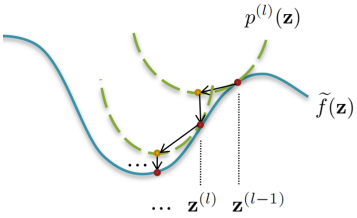

Fig. 1. An illustration of the MaMi technique.

Lemma. *Let* $\mathbf{U}$ *denote a full column-rank matrix. The function* $\log\det\left(\mathbf{U}^H\mathbf{Z}^{-1}\mathbf{U}\right)$ *is convex with respect to (w.r.t.)* $\mathbf{Z}\succ\mathbf{0}$.

Proof. see [14]. □

Note that $\mathbf{B}_k\succ\mathbf{0}$ which follows from the fact that $\mathbf{X}_k\succ\mathbf{0}$. Consequently, exploiting the convexity of the expression in (12) (via the above lemma), we obtain the following minorization for the matrix function in (12) at $\mathbf{B}_k=\widetilde{\mathbf{B}}_k$ using its supporting hyperplane:

$$\log\det\left(\mathbf{U}^H\mathbf{B}_k^{-1}\mathbf{U}\right)\geq\log\det\left(\mathbf{U}^H\widetilde{\mathbf{B}}_k^{-1}\mathbf{U}\right)+\mathrm{tr}\{\mathbf{T}_k(\mathbf{B}_k-\widetilde{\mathbf{B}}_k)\} \tag{13}$$

where $\mathbf{T}_k$ can be obtained, e.g., by employing the results of [15] as:

$$\mathbf{T}_k=-\widetilde{\mathbf{B}}_k^{-1}\mathbf{U}\left(\mathbf{U}^H\widetilde{\mathbf{B}}_k^{-1}\mathbf{U}\right)^{-1}\mathbf{U}^H\widetilde{\mathbf{B}}_k^{-1}. \tag{14}$$

As to the second term of the objective function in (5) (for each k), note that $-0.5\log\det\left(\mathbf{I}+\mathbf{H}_k\right)$ is convex w.r.t. $\mathbf{H}_k$. Consequently, a minorizer of this matrix function at $\widetilde{\mathbf{H}}_k$ can be obtained using its supporting hyperplane as follows

$$-\log\det\left(\mathbf{I}+\mathbf{H}_k\right)\geq-\log\det\left(\mathbf{I}+\widetilde{\mathbf{H}}_k\right)-\mathrm{tr}\{(\mathbf{I}+\widetilde{\mathbf{H}}_k)^{-1}(\mathbf{H}_k-\widetilde{\mathbf{H}}_k)\}. \tag{15}$$

In sum, using (13) and (15), the following optimization problem has to be solved for the $(l+1)^{th}$ iteration of the proposed algorithm:

$$\begin{cases}\max\limits_{\mathbf{A},\{\mathbf{H}_k\}} & \sum\limits_{k=1}^{N_r}\left(\mathrm{tr}\{\mathbf{T}_k^{(l)}\mathbf{B}_k\}-0.5\,\mathrm{tr}\{\left(\mathbf{I}+\mathbf{H}_k^{(l)}\right)^{-1}\mathbf{H}_k\}\right)\\ \text{subject to} & \mathbf{H}_k=\mathbf{M}_k^{-1/2}\mathbf{A}\mathbf{R}_k\mathbf{A}^H\mathbf{M}_k^{-1/2},\ \forall k\\ & |[\mathbf{A}]_{m,n}|=1,\ \forall\ (m,n)\end{cases} \tag{16}$$

Let

$$\mathbf{T}_k\triangleq\begin{bmatrix}\mathbf{T}_{k,11} & \mathbf{T}_{k,12}\\ \mathbf{T}_{k,12}^H & \mathbf{T}_{k,22}\end{bmatrix}. \tag{17}$$

Using this notation, the objective in (16) can be explicitly expressed as a function of $\mathbf{A}$:

$$\textstyle\sum_{k=1}^{N_r}\mathrm{tr}\{0.5\mathbf{A}^H\mathbf{T}_{k,22}^{(l)}\mathbf{A}\mathbf{R}_k\}+\mathrm{tr}\{\frac{1}{\sqrt{2}}\mathbf{R}_k^{1/2}\mathbf{T}_{k,12}^{(l)}\mathbf{A}\}-0.5\,\mathrm{tr}\{\mathbf{A}^H\mathbf{M}_k^{-1/2}\left(\mathbf{I}+\mathbf{H}_k^{(l)}\right)^{-1}\mathbf{M}_k^{-1/2}\mathbf{A}\mathbf{R}_k\}. \tag{18}$$

Ultimately, the vectorized version of the optimization in (16) can be obtained via (18) as well as a standard property of the Kronecker product, viz. $\mathrm{tr}\{\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3\mathbf{X}_4\}=\mathrm{vec}^H(\mathbf{X}_1^H)\left(\mathbf{X}_4^H\otimes\mathbf{X}_2\right)\mathrm{vec}(\mathbf{X}_3)$. Consequently we obtain

$$\begin{cases}\max\limits_{\widetilde{\mathbf{a}}} & \widetilde{\mathbf{a}}^H\mathbf{Q}_{\mathcal{B}}^{(l)}\widetilde{\mathbf{a}}+2\Re\left((\mathbf{q}_{\mathcal{B}}^{(l)})^H\widetilde{\mathbf{a}}\right)\\ \text{subject to} & |\widetilde{a}_w|=1,\forall w\end{cases} \tag{19}$$

where $\widetilde{\mathbf{a}}=\mathrm{vec}(\mathbf{A})$, $\mathbf{q}_{\mathcal{B}}^{(l)}\triangleq\sum_{k=1}^{N_r}\frac{1}{\sqrt{2}}\mathrm{vec}(\mathbf{T}_{k,21}\mathbf{R}_k^{1/2})$, and

$$\mathbf{Q}_{\mathcal{B}}^{(l)}\triangleq\sum_{k=1}^{N_r}\left(0.5\mathbf{R}_k\otimes\left(\mathbf{T}_{k,22}^{(l)}-0.5\mathbf{Y}_k^{(l)}\right)\right) \tag{20}$$

with $\mathbf{Y}_k^{(l)}=\mathbf{M}_k^{-1/2}\left(\mathbf{I}+\mathbf{H}_k^{(l)}\right)^{-1}\mathbf{M}_k^{-1/2}$.

Remark: Note that $\mathbf{T}_k$ in (14) is negative definite for all k due to the fact that $\mathbf{B}_k\succ\mathbf{0},\forall k$. Therefore, $\mathbf{T}_{k,22}$ in (17) is negative definite as well. Furthermore, observe that $\mathbf{Y}_k^{(l)}\succ\mathbf{0}$. Now, considering properties of the Kronecker product, it follows that $\mathbf{Q}_{\mathcal{B}}^{(l)}$ is negative definite for all l. Therefore, in the case of unconstrained design, in which an energy constraint $\|\widetilde{\mathbf{a}}\|_2^2\leq e$ is considered in lieu of the unimodularity constraint in (19), the $(l+1)^{th}$ iteration of the proposed algorithm consists of solving a simple quadratic convex problem. ■

The constrained optimization problem in (19) is NP-hard in general [16]. However, a solution to this problem can be obtained using the iterative method discussed in [2, 16]. More concretely, the problem in (19) can be equivalently written as the following UQP:

$$\begin{cases}\max\limits_{\widetilde{\mathbf{a}}} & \widehat{\mathbf{a}}^H\mathbf{J}_{\mathcal{B}}^{(l)}\,\widehat{\mathbf{a}}\\ \text{subject to} & |\widetilde{a}_w|=1,\forall w\end{cases} \tag{21}$$

where $\widehat{\mathbf{a}}=[\widetilde{\mathbf{a}}\ \ 1]^T$, $\mathbf{J}_{\mathcal{B}}^{(l)}=\mu^{(l)}\mathbf{I}_{NN_t+1}+\mathbf{K}_{\mathcal{B}}^{(l)}$, and

$$\mathbf{K}_{\mathcal{B}}^{(l)}=\begin{bmatrix}\mathbf{Q}_{\mathcal{B}}^{(l)} & \mathbf{q}_{\mathcal{B}}^{(l)}\\ (\mathbf{q}_{\mathcal{B}}^{(l)})^H & 0\end{bmatrix}$$

with $\mu^{(l)}>|\lambda_{min}(\mathbf{K}_{\mathcal{B}}^{(l)})|$. The code vector $\widetilde{\mathbf{a}}$ at the $(l+1)^{th}$ iteration of the proposed algorithm can be obtained by running the following iteration until convergence [16]:

$$\widetilde{\mathbf{a}}^{(p+1)}=\exp\left(j\arg(\breve{\mathbf{a}}^{(p)})\right) \tag{22}$$

where $\breve{\mathbf{a}}^{(p)}$ represents the vector containing the first NN_t entries of $\mathbf{J}_{\mathcal{B}}^{(l)}\,\widehat{\mathbf{a}}^{(p)}$.

The steps of the proposed algorithm are summarized in Table 1. Note that the code matrix $\mathbf{A}$ can be obtained from the solution $\widetilde{\mathbf{a}}$ using $\mathbf{A}=\mathrm{vec}^{-1}(\widetilde{\mathbf{a}})$.

4. NUMERICAL EXAMPLES AND DISCUSSION

In this section, we present numerical examples to examine the performance of the proposed algorithm. In addition to

Table 1. The proposed algorithm for maximizing the Bhattacharyya distance under unimodularity constraint

Step 0: Initialize $\widetilde{\mathbf{a}}$ with a random vector in $\mathbb{C}^{NN_t}$ and set the iteration number l to 0.
Step 1: Solve the UQP in (21) via the use of the iterations in (22); set $l \leftarrow l+1$.
Step 2: Compute $\mathbf{Q}_{\mathcal{B}}^{(l)}$ and $\mathbf{q}_{\mathcal{B}}^{(l)}$.
Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $\|\widetilde{\mathbf{a}}^{(l+1)} - \widetilde{\mathbf{a}}^{(l)}\|_2 \leq \xi$ for some $\xi > 0$.

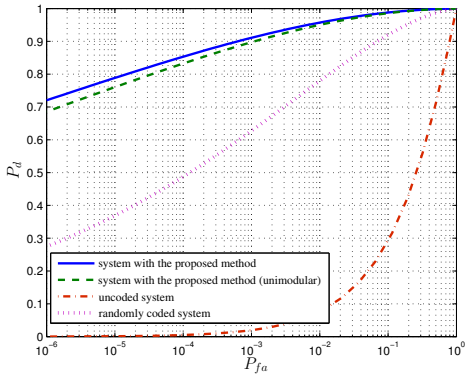

Fig. 2. ROC of the coded system, the uncoded system, and the system with random coding.

the comparison of the detection performance of the optimally coded system (the system that is coded using the proposed method and referred to as coded system in what follows), a numerical study of diversity-integration trade-off is provided.

We consider a MIMO system with $N_r = 2$ and the code length $N = 10$. For the interference at the k^{th} receiver, it is assumed that $[\mathbf{M}_k]_{l,l'} = p_{\text{int},k}\rho_k^{|l-l'|}$ with parameters $p_{\text{int},k}$ and ρ_k. For the target, we let $[\mathbf{R}_k]_{l,l'} = \hat{\rho}_k^{|l-l'|}$ with parameter $\hat{\rho}_k$. Moreover, in the case of unconstrained design, we set the total transmit energy e equal to NN_t.

We investigate the detection performance of the coded system, the uncoded system (employing a scaled version of the all-one matrix as the code matrix $\mathbf{A}$), and the system with random coding for the number of transmitters $N_t = 2$, target parameters $\hat{\rho}_k = (0.3, 0.2)$, and interference parameters $\rho_k = (0.8, 0.7)$ as well as $p_{\text{int},k} = (10, 10)$. The ROC is used to evaluate the detection performance of the system based on analytical expressions for the probability of detection and false alarm (see eqs. (32)-(34) in [10]). The results for the coded system (both unconstrained and constrained), the uncoded one as well as the system with random coding are plotted in Fig. 2. For the random coding system, the average result is depicted considering 100 randomly generated $\mathbf{A}$ with i.i.d Gaussian entries. It is observed that employing the proposed method for the system leads to a significant performance improvement as compared to the uncoded and randomly coded systems. Also, only a minor performance degradation is observed for unimodular code design as compared to the unconstrained design. Note that, in this example, the uncoded system behaves rather poorly. To get some insight into this observation, we remark on the fact that the code matrix for the uncoded system, i.e., $\mathbf{A}_{\text{uncoded}}$ is a scaled version of all-one matrix; hence the covariance of the target component at the first receiver, i.e., $\mathbf{A}_{\text{uncoded}}\mathbf{R}_1\mathbf{A}_{\text{uncoded}}^H$, is rank-one. Let $\mathbf{A}_{\text{uncoded}}\mathbf{R}_1\mathbf{A}_{\text{uncoded}}^H = \mathbf{u}_{\text{uncoded}}\mathbf{u}_{\text{uncoded}}^H$. The aforementioned observation can be explained via noting that the cross-correlations of $\mathbf{u}_{\text{uncoded}}$ with the principal eigenvectors of $\mathbf{M}_1$ and $\mathbf{M}_2$ are equal to 0.9922 and 0.9851, respectively. In other words, the energy of the target component (at the first receiver) for the uncoded system is concentrated in the direction in which the interferences have their maximum (negative) effect. A similar argument holds for the second receiver. On the other hand, it can be verified that for the coded system, the energy of the target components (at the receivers) is put in the direction of the minor eigenvectors of the interferences.

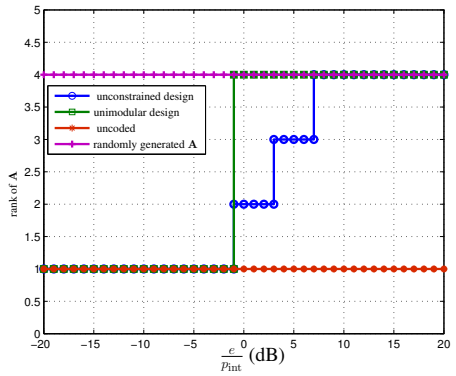

Fig. 3. The rank of $\mathbf{A}$ versus e/p_{int}.

Next we address the diversity-integration trade-off for a MIMO system with $N_t = 4$ and the other parameters (except for $p_{\text{int},k}$) identical to those of Fig. 2. Fig. 3 depicts the rank of the optimal code matrix $\mathbf{A}$ versus $\frac{e}{p_{\text{int}}}$. The figure also shows the case of $\mathbf{A}_{\text{uncoded}}$ and randomly generated $\mathbf{A}$. Here we keep the transmit energy e constant and plot the ranks via changing $p_{\text{int}} = p_{\text{int},k}$. Note that $\frac{e}{p_{\text{int}}}$ shows the variations of SNR at receivers. The rank of $\mathbf{A}$ is considered as the diversity order of the system and the integration refers to putting more energy on the only diverse path (see [3] for details). It is seen form Fig. 3 that by increasing $\frac{e}{p_{\text{int}}}$, the diversity order of the system increases. Indeed, in high SNR regimes, more diversity is preferable whereas in low SNR regimes, integration leads to a better performance. As expected, for the constrained design, the rank of $\mathbf{A}$ behaves slightly differently. The $\mathbf{A}_{\text{uncoded}}$ is always rank-one; whereas the randomly generated $\mathbf{A}$ has full rank (see [17]). We finally remark on the fact that not only the rank of $\mathbf{A}$ but also the allocated energy to diverse paths influences the detection performance.

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