MIMO RADAR CAPABILITY ON POWERFUL JAMMERS SUPPRESSION

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ABSTRACT

The problem of jammers suppression in colocated multipleinput multiple-output (MIMO) radar is considered. We resort to reduced dimension (RD) beamspace designs with robustness/adaptiveness to achieve the goal of efficient jammers suppression. Specifically, our RD beamspace techniques aim at designing optimal beamspace matrices based on reasonable tradeoffs between the desired in-sector source distortion and the powerful jammer (possibly in-sector) attenuation when conducting the jammers suppression. These designs are cast as convex optimization problems which are derived using secondorder cone programming. Meanwhile, we study the MUSICbased direction-of-arrival estimation performance of the proposed beamspace designs by comparing to the conventional algorithms. Moreover, we demonstrate that the capability of efficient powerful in-sector jammers suppression using these designs is unique in MIMO radar.

Index Terms— Beamspace design, colocated MIMO radar, convex optimization, jammers suppression, robustness.

1. INTRODUCTION

The recently emerging concept of multiple-input multipleoutput (MIMO) radar has become the focus of intensive research [1]–[4]. It has been shown that MIMO radar with colocated antennas has advantages over phased-array (PA) radar such as improved parameter identifiability and angular resolution, increased upper limit on the number of detectable targets, and extended array aperture by virtual sensors [3]. Beamforming techniques have been employed in colocated MIMO radar to achieve coherent processing gain or desirable beampatterns [5]–[9]. Space-time adaptive processing techniques have also been exploited to mitigate clutter [10], [11]. One issue that is of great importance for colocated MIMO radar is to suppress the jamming signals which are typical interfering sources that take the form of high-power transmission and hence result in impairing the receive system. Terrain-scattered jamming occurs when the high-power jammer transmits its energy to ground, and it reflects the energy in a dispersive manner. Thus the jamming appears at the receive array as distributed source. This scenario becomes quite complicated when the jamming impinges on the receive array within the same spatial domain as the desired source [12]. To the best of our knowledge, the capability of efficient suppression on powerful jammers for MIMO radar has not been studied in previous works.

In this paper, we utilize robust/adaptive techniques to implement the suppression of powerful jammers in the context of MIMO radar with colocated antennas. We show that in MIMO radar the echoes reflected from the targets and the intentionally radiated jamming signals have different spatial signatures even if they impinge on the receive array from the same spatial angle. Using this observation, we provide a category of beamspace processing methods which employ mathematical optimization techniques to design the beamspace matrices by making tradeoffs among the in-sector source distortion, the in-sector powerful jammers suppression, and the out-of-sector interference attenuation. These matrices are of reduced dimension (RD), which saves the computational burden. We propose to incorporate robustness/adaptiveness against both the unknown in-sector jammers and the out-of-sector interference, and further cast the designs as convex optimization problems. The MUSIC-based direction-of-arrival (DOA) estimation performance of these designs is investigated.

2. SIGNAL MODEL

Consider a MIMO radar system equipped with colocated arrays which contain M transmit antenna elements and N receive antenna elements. We assume that both the transmit and receive arrays are close enough to each other such that they share the same spatial angle of a far-field target. Let $\mathbf{\Phi}(t) = [\phi_1(t), \dots, \phi_M(t)]^T$ be the $M \times 1$ vector that contains the complex envelopes of the transmitted waveforms $\phi_i(t)$, $i = 1, \dots, M$ which are assumed to be orthogonal, i.e., $\int_{T_p} \phi_i(t) \phi_j^*(t) dt = \delta(i-j)$, $i, j = 1, \dots, M$ where T_p is the pulse duration and $\delta(\cdot)$ is the Kronecker delta func-

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tion. Here $(\cdot)^T$ and $(\cdot)^*$ stand for the transpose and complex conjugate operations, respectively.

Let us assume that L targets including the desired and interfering sources are present in the background of noise. The $N \times 1$ complex vector of the received observations can be expressed as

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L} \alpha_l(\tau) \left[\mathbf{a}^T(\theta_l) \mathbf{\Phi}(t) \right] \mathbf{b}(\theta_l) + \mathbf{z}(t,\tau) \quad (1)$$

where τ is the slow time index, i.e., the pulse number, $\alpha_l(\tau)$ is the reflection coefficient of the *l*th source with variance σ_{α}^2 , θ_l is the spatial angle associated with the *l*th source, $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ are the steering vectors of the transmit and receive arrays, respectively, and $\mathbf{z}(t,\tau)$ is the $N \times 1$ zero-mean white Gaussian noise term.

By matched filtering the received data to the M transmitted orthogonal waveforms at the receiving end, the $MN \times 1$ virtual data vector can be obtained as

$$\mathbf{y}(\tau) = \operatorname{vec}\left(\int_{T_p} \mathbf{x}(t,\tau) \,\mathbf{\Phi}^H(t) \,dt\right)$$
$$= \sum_{l=1}^{L} \alpha_l(\tau) \left[\mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)\right] + \tilde{\mathbf{z}}(\tau)$$
(2)

where vec (\cdot) is the operator that stacks the columns of a matrix into one column vector, \otimes denotes the Kronecker product, $(\cdot)^H$ stands for the Hermitian transpose, and $\tilde{\mathbf{z}}(\tau) =$ vec $(\int_{T_p} \mathbf{z}(t,\tau) \mathbf{\Phi}^H(t) dt)$ is the $MN \times 1$ noise term whose covariance is given by $\sigma_z^2 \mathbf{I}_{MN}$ with \mathbf{I}_{MN} denoting the identity matrix of size $MN \times MN$.

In the presence of powerful jammers, the signal model (2) can be rewritten as

$$\mathbf{y}(\tau) = \sum_{l=1}^{L} \alpha_l(\tau) \mathbf{v}(\theta_l) + \sum_{j=1}^{J} \beta_j(\tau) \,\tilde{\mathbf{v}}(\theta_j) + \tilde{\mathbf{z}}(\tau) \quad (3)$$

where $\beta_j(\tau)$ is the signal of the *j*th jammer, θ_j is the presumed spatial angle associated with the *j*th jammer, *J* is the number of jammers, and $\mathbf{v}(\theta_l) \triangleq \mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)$ and $\tilde{\mathbf{v}}(\theta_j) \triangleq \mathbf{1}_M \otimes$ $\mathbf{b}(\theta_j)$ are the virtual steering vectors of the *l*th target and the *j*th jammer, respectively, with $\mathbf{1}_M$ denoting the $M \times 1$ vector of all ones. The reason that the virtual steering vector of the jammer contains $\mathbf{1}_M$ is because the terrain-scattered jammer does not originate from the MIMO radar transmit array and, therefore, does not depend on its transmit array steering vector.

3. SUPPRESSION OF POWERFUL JAMMERS

We assume that the desired targets are located within a known angular sector Θ [8] where powerful jamming sources are also present and can even have the same spatial angles as the targets. In what follows, we first introduce the beamspace signal model and present the MUSIC-based beamspace DOA estimator. Then, we propose three RD beamspace designs with robustness/adaptiveness against the in-sector jammers and the out-of-sector interfering sources whose performance is evaluated using the MUSIC-based DOA estimation.

3.1. MUSIC-Based Beamspace DOA Estimator

Let **B** be the $MN \times D$ ($D \ll MN$) **RD** beamspace matrix that transforms the original $MN \times 1$ received data vector $\mathbf{y}(\tau)$ to a new data snapshot $\tilde{\mathbf{y}}(\tau)$ of size $D \times 1$, i.e.,

$$\tilde{\mathbf{y}}\left(\tau\right) = \mathbf{B}^{H}\mathbf{y}\left(\tau\right). \tag{4}$$

Using (4), the covariance matrix of the reduced size vector $\tilde{\mathbf{y}}(\tau)$ can be expressed as

$$\mathbf{R}_{\tilde{\mathbf{y}}} \triangleq \mathbb{E}\left\{\tilde{\mathbf{y}}\left(\tau\right)\tilde{\mathbf{y}}^{H}\left(\tau\right)\right\} = \mathbf{B}^{H}\mathbf{R}_{\mathbf{y}}\mathbf{B}$$
(5)

where $\mathbf{R}_{\mathbf{y}} \triangleq \mathbb{E}\{\mathbf{y}(\tau)\mathbf{y}^{H}(\tau)\}$ denotes the covariance matrix of the original received data with $\mathbb{E}\{\cdot\}$ denoting the expectation operation. In practice, (5) is usually estimated using Pavailable sampling snapshots and thus it can be expressed as

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \frac{1}{P} \sum_{\tau=1}^{P} \tilde{\mathbf{y}}(\tau) \, \tilde{\mathbf{y}}^{H}(\tau) \,. \tag{6}$$

Under the condition that all the jamming and interfering sources are well suppressed by the beamspace processing, the eigendecomposition of (6) can be denoted as

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \mathbf{E}_{\mathrm{s}} \mathbf{\Lambda}_{\mathrm{s}} \mathbf{E}_{\mathrm{s}}^{H} + \mathbf{E}_{\mathrm{n}} \mathbf{\Lambda}_{\mathrm{n}} \mathbf{E}_{\mathrm{n}}^{H}$$
(7)

where the $L_d \times L_d$ diagonal matrix Λ_s contains the largest (signal subspace) eigenvalues and the columns of the $D \times L_d$ matrix \mathbf{E}_s are the corresponding eigenvectors with L_d being the number of the desired targets within Θ . Similarly, the $(D-L_d) \times (D-L_d)$ diagonal matrix Λ_n contains the smallest (noise subspace) eigenvalues while the $D \times (D - L_d)$ matrix \mathbf{E}_n is built from the corresponding eigenvectors.

Applying the principle of the elementspace MUSIC estimator [13], we can obtain the beamspace spectral-MUSIC DOA estimator as

$$f(\theta) = \frac{\mathbf{v}^{H}(\theta)\mathbf{B}\mathbf{B}^{H}\mathbf{v}(\theta)}{\mathbf{v}^{H}(\theta)\mathbf{B}\mathbf{Q}\mathbf{B}^{H}\mathbf{v}(\theta)}$$
(8)

where $\mathbf{Q} \triangleq \mathbf{E}_{n} \mathbf{E}_{n}^{H} = \mathbf{I}_{D} - \mathbf{E}_{s} \mathbf{E}_{s}$ is the projection matrix onto the noise subspace.

3.2. Beamspace Design With Robustness/Adaptiveness

We assume that the interfering sources are present outside Θ and consider the general case that both the in-sector jamming and the out-of-sector interfering sources are unknown.

To efficiently suppress the unknown jammers and interference, we resort to RD beamspace design techniques to achieve the goal and later use the beamspace designs for the MUSIC-based DOA estimation. Specifically, these designs are expected to preserve desired signal energy received within the sector-of-interest (SOI) Θ , and to attenuate in-sector jammers and out-of-sector interference simultaneously. It is worth noting that these techniques significantly reduce the computational burden, and performing DOA estimation in beamspace leads to performance improvements such as enhanced source resolution, reduced DOA estimation bias, and reduced sensitivity to array calibration errors.

Technically, we first exploit the spheroidal sequences based methods [8], [14] to achieve a quiescent response beamspace matrix which ensures the preservation of energy received within the desired sector Θ . Then, we propose to design robust/adaptive RD beamspace processing, aiming at preserving the desired signal components while suppressing the in-sector powerful jammers and/or filtering out the interfering components that come from outside Θ . In other words, tradeoffs between the in-sector source distortion and the out-of-sector source attenuation are made while imposing a novel constraint used to nullify the jammers. It naturally leads to the robustness/adaptiveness against the unknown jammers. For example, we consider the case of distributed jammers that are located within the desired sector Θ . Although these jamming signals overlap with the desired sector, they can still be cancelled out by imposing the additional constraint.

For our beamspace designs, the beamspace dimension D depends on the width of Θ , and it can be obtained based on the principle that D should not be smaller than the number of the largest eigenvalues of the matrix $\mathbf{A} \triangleq \int_{\Theta} \mathbf{v}^{H}(\theta) \mathbf{v}(\theta) d\theta$ while simultaneously requires their sum to exceed a certain percentage (e.g., 99%) of the total sum of all eigenvalues. When the MUSIC-based DOA estimation is applied, D also needs to be no smaller than the number of the desired targets.

The first solution is to upper-bound the acceptable difference between the desired and quiescent response beamspace matrices while maximizing the worst-case in-sector jammers suppression. Additionally, the out-of-sector sidelobes can be kept below a certain level to insure interference attenuation. The corresponding optimization problem can be written as

$$\min_{\mathbf{B}} \max_{i} \|\mathbf{B}^{H} \tilde{\mathbf{v}}(\theta_{i})\|, \theta_{i} \in \mathbf{\Theta}, i = 1, \dots, Q$$
s.t.
$$\|\mathbf{B} - \mathbf{B}_{q}\|_{F} \leq \varepsilon \qquad (9)$$

$$\|\mathbf{B}^{H} \mathbf{v}(\bar{\theta}_{k})\| \leq \gamma, \ \bar{\theta}_{k} \in \bar{\mathbf{\Theta}}, \ k = 1, \dots, K$$

where \mathbf{B}_{q} is the quiescent response beamspace matrix, $\varepsilon > 0$ is the parameter that bounds the in-sector signal distortion caused by the beamspace matrix \mathbf{B} as compared to $\mathbf{B}_{q}, \gamma > 0$ is the parameter of the user choice that characterizes the worst acceptable out-of-sector attenuation, $\bar{\boldsymbol{\Theta}}$ combines a continuum of all out-of-sector directions, $\{\theta_i \in \boldsymbol{\Theta}, i = 1, \dots, Q\}$ and $\{\bar{\theta}_k \in \bar{\boldsymbol{\Theta}}, k = 1, \dots, K\}$ are grids of angles used to approximate the in-sector $\boldsymbol{\Theta}$ and the out-of-sector $\bar{\boldsymbol{\Theta}}$ by finite numbers Q and K of directions, respectively, $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm of a matrix, and $\|\cdot\|$ is the Euclidean norm.

An alternative robust approach is to minimize the difference between the desired and quiescent response beamspace matrices while keeping the in-sector jammers suppression higher than a certain desired level and, if needed, keeping the out-of-sector attenuation to an acceptable level. Hence, the corresponding optimization problem can be written as

$$\min_{\mathbf{B}} \|\mathbf{B} - \mathbf{B}_{\mathbf{q}}\|_{\mathbf{F}}$$
s.t.
$$\|\mathbf{B}^{H}\tilde{\mathbf{v}}\left(\theta_{i}\right)\| \leq \delta, \ \theta_{i} \in \Theta, \ i = 1, \dots, Q \qquad (10)$$

$$\|\mathbf{B}^{H}\mathbf{v}\left(\bar{\theta}_{k}\right)\| \leq \gamma, \ \bar{\theta}_{k} \in \bar{\Theta}, \ k = 1, \dots, K$$

where $\delta > 0$ is the parameter that characterizes the worst acceptable level of the jamming power radiation in the desired sector Θ . It is worth noting that the last set of constraints in (9) and (10) are needed only if there are interfering sources located in the out-of-sector area, and, therefore, they can be removed if only intentional jammer suppression is concerned.

As on-line computation becomes practical, it is meaningful to develop an approach that is data-adaptive for the beamspace design. This is particularly important when the jammers and/or the interfering sources are varying. To adaptively cancel out both types of sources, the data-adaptive formulation can be developed by minimizing the output power of the transformed vector $\tilde{\mathbf{y}}(\tau)$. This power can be denoted as

$$\mathbb{E}\left[\tilde{\mathbf{y}}^{H}\left(\tau\right)\tilde{\mathbf{y}}\left(\tau\right)\right] = \operatorname{tr}\left\{\mathbb{E}\left[\tilde{\mathbf{y}}\left(t\right)\tilde{\mathbf{y}}^{H}\left(t\right)\right]\right\} = \operatorname{tr}\left\{\mathbf{B}^{H}\mathbf{R}_{\mathbf{y}}\mathbf{B}\right\}$$
(11)

where $tr\{\cdot\}$ denotes the trace of a matrix. Finally, the corresponding data-adaptive beamspace design can be cast as

$$\min_{\mathbf{B}} \operatorname{tr} \left\{ \mathbf{B}^{H} \mathbf{R}_{\mathbf{y}} \mathbf{B} \right\}$$
s.t.
$$\|\mathbf{B} - \mathbf{B}_{q}\|_{F} \leq \varepsilon$$

$$\|\mathbf{B}^{H} \tilde{\mathbf{v}} \left(\theta_{i}\right)\| \leq \delta, \ \theta_{i} \in \Theta, \ i = 1, \dots, Q$$

$$\|\mathbf{B}^{H} \mathbf{v} \left(\bar{\theta}_{k}\right)\| \leq \gamma, \ \bar{\theta}_{k} \in \bar{\Theta}, \ k = 1, \dots, K.$$
(12)

The problems (9), (10), and (12) are convex and can be efficiently solved using second-order cone (SOC) programming. For given values of δ and γ , the feasibility of (12) is guaranteed if $\varepsilon \geq \varepsilon_{\min}$ is used where ε_{\min} is the minimum value of $\|\mathbf{B} - \mathbf{B}_{q}\|_{F}$ that can be calculated by solving (10).

It is worth noting that the capability of in-sector jammers suppression for MIMO radar is unique. PA radar is unable to achieve the goal of in-sector jammers suppression by utilizing the same way in spatial domain especially when the jammers and the desired targets are located at the same directions.

4. SIMULATION RESULTS

In our simulations, we use uniform linear arrays of M = 16 transmit and N = 8 receive antenna elements spaced half a wavelength apart. The presumed SOI area is $\Theta = [10^{\circ}, 25^{\circ}]$ and the out-of-sector area is $\bar{\Theta} = [-90^{\circ}, 0^{\circ}] \cup [35^{\circ}, 90^{\circ}]$. Two



Fig. 1. Beamspace attenuation versus angle.



Fig. 2. RMSEs of DOA estimation versus SNR.

desired targets with DOAs $\theta_t = 16.5^{\circ}$ and 18.5° are located in the SOI, and four interfering sources are assumed to be located at $\theta = -35^{\circ}$, -20° , -5° , and 50° , respectively. The signalto-noise ratio (SNR) and interference-to-noise ratio (INR) are set to be equal to 0 dB and 40 dB, respectively. The CVX toolbox [15] is used to solve the problems (9), (10), and (12).

In the first example, we assume that uniformly distributed jammers spaced 1° apart from each other are present in the SOI. The jammer-to-noise ratio (JNR) is assumed to be equal to 50 dB. Other parameters employed are as follows: D = 7, P = 500, $\gamma = 0.2$, $\delta = 0.1$, and $\varepsilon = 1.467$. Fig. 1 shows the beamspace attenuation $g(\theta) \triangleq ||\mathbf{B}^H \mathbf{u}(\theta)||^2/||\mathbf{u}(\theta)||^2$ $(\mathbf{u}(\theta) = \mathbf{v}(\theta)$ for targets and $\mathbf{u}(\theta) = \tilde{\mathbf{v}}(\theta)$ for jammers) for the spheroidal sequences based algorithm and the proposed adaptive beamspace design in (12). It can be clearly seen that the proposed data-adaptive beamspace design shows good capability of suppressing the out-of-sector interference and the in-sector jammers, even if they have the same directions as the targets. There is almost no target attenuation within the SOI.

In the second example, we evaluate the suppression performance of the beamspace designs by comparing the DOA estimation performance versus SNR with that of the conven-



Fig. 3. Probabilities of source resolution versus SNR.

tional elementspace MUSIC and spheroidal sequences based algorithms. The same scenario and parameters are selected as used in the first example except that only the 5 jammers located between 15.5° and 19.5° are present. The results are averaged over 200 independent simulation runs. Fig. 2 displays the root-mean-square errors (RMSEs) of the MUSIC-based DOA estimators, and Fig. 3 shows the corresponding probabilities of source resolution for different designs. The target sources are regarded as resolved in the *n*th run if $\sum_{i=1}^2 |\hat{\theta}_i(n) - \theta_i| < 2^\circ$ where $\hat{\theta}_i(n)$ is the estimated DOA of the *i*th target in the *n*th run. It can be seen that the performance of all the proposed beamspace designs outperform that of the conventional methods. In the presence of powerful in-sector jammers and outof-sector interfering sources, the conventional elementspace MUSIC and spheroidal sequences based algorithms can not accurately discriminate targets even if large SNR is employed. The proposed data-adaptive beamspace design gives the best RMSE and probabilities of source resolution only if the SNR is larger than 12 dB. The other two beamspace designs show approximately the same DOA estimation performance.

5. CONCLUSIONS

We have considered the jammers suppression problem for MIMO radar with colocated antennas, and have provided three RD beamspace designs to address the problem. Tradeoffs between the desired in-sector source distortion and the powerful jammer (possibly in-sector) attenuation are made when conducting the jammers suppression. We cast the designs as convex optimization problems using SOC programming, in which robustness/adaptiveness against the unknown in-sector jamming and out-of-sector interfering sources is incorporated. Moreover, we have investigated the MUSIC-based DOA estimation performance of the proposed designs. Simulation results show that the performance of the proposed designs outperforms that of the conventional methods. We have also shown that the capability of efficient in-sector jammers suppression using these designs is unique in MIMO radar.

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