DISTRIBUTED BLIND SYSTEM IDENTIFICATION IN SENSOR NETWORKS

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ABSTRACT

This paper studies the blind identification of multi-channel FIR systems in the context of sensor networks. Distributed identification algorithms are developed for both noise-free and noise-contaminated networked systems. The proposed algorithms distribute the data storage and computational load among multiple agents connected by a specified topology, and are fulfilled via information exchanges among neighboring agents without the need of fusion centers. In the presence of measurement noises, a stabilized distributed algorithm is provided which can avoid trivial estimations of the multiple channels. In addition, convergence properties of the proposed algorithms are provided, and simulation examples are given to show the performances of the proposed algorithms.

Index Terms— Blind identification, multi-agent system, consensus based gradient method.

1. INTRODUCTION

Motivated by the emergence of large-scale and inexpensive sensor networks, intensive researches have been done on distributed cooperative control and optimization [1]. In this paper, the blind identification of multiple channels in a networked system is investigated. A common source signal, emitted from a moving target on the ground or an unmanned aerial vehicle (UAV) in the air, is acquired by multiple sensors in a network, which are connected following a communication topology (see Fig. 1). Due to the inhomogeneous transmission medium, the acquired signals of the sensors deployed at different places are distinct. Here, the signal acquisition process is described by a single-input multi-output (SIMO) convolutional system model. Since only the observed signals are available, the estimation of the multiple channels needs to be carried out blindly.

For the centralized blind identification of SIMO systems, a great number of algorithms are included in [2] and the references therein. Generally, there are two kinds of deterministic algorithms for blind system identification: maximum likelihood method [3, 4, 5] and subspace-based method [6, 7]. The maximum likelihood method aims to maximize a nonlinear likelihood function using iterative optimization methods, so the final solution is sensitive to the selected initial condition. Although the adaptive algorithm presented in [5] can obtain a global optimal solution, it is very computationally expensive so that its applications may be limited. The subspace-based method can obtain an optimal solution by eigenvalue decomposition; however, it requires huge storage space and heavy instantaneous computational burden. To mitigate such a problem, several adaptive algorithms have been proposed, such as adaptive least squares smoothing algorithm [8] and stochastic approximation based algorithm [9].

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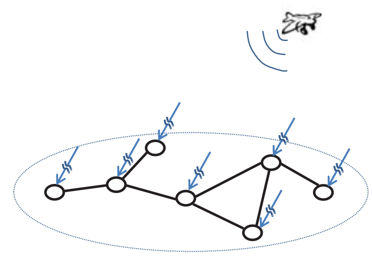

Fig. 1. Source signal acquisition by multiple agents in a network.

For the networked SIMO system as shown in Fig 1, we consider the distributed blind channel identification using only the acquired signals. It is noteworthy that a distributed blind adaptive algorithm was developed in [10] for constant modulus restoration; however, it is different from our work in the following aspects: (a) it does not deal with the explicit estimations of the multiple channels; (b) all sensors are linked by a Hamiltonian cycle rather than a general communication topology. The developed distributed blind estimation algorithms in the present paper adapt the blind system identification technique for the sensor network based applications. They are carried out by information exchanges among neighboring agents without the need of fusion centers, hence reducing the requirements of the storage space and the computational capability for a single agent. Although the centralized algorithms in [9, 11] can be realized

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in distributed environments; however, when there exist measurement noises, it will lead to a trivial solution. To cope with the above mentioned problem, we shall develop a stable distributed algorithm by combining the self-stabilized minor subspace rule [12] and the cross relation equalities [7].

2. PROBLEM FORMULATION

The signal acquisition process is described by an SIMO system as follows:

$$\begin{aligned} x_i(n) &= H_i(q)s(n) = \sum_{k=0}^{M} h_{i,k} s(n-k), \\ y_i(n) &= x_i(n) + w_i(n) \quad i = 1, 2 \cdots L, \end{aligned} \tag{1}$$

where $s(n)$ is the source signal, $w_i(n)$ is the measurement noise, $y_i(n)$ is the observation of the i-th sensor, L is the number of sensors, $H_i(q) = \sum_{k=0}^{M} h_{i,k} q^{-k}$ denotes the transfer function of the i-th channel, and q^{-1} denotes the backward shift operator in time domain.

Throughout the paper, the following standard assumptions are made.
A1. All transfer functions $\{H_i(q)\}_{i=1}^{L}$ do not share common zeros and the maximum channel order M is known.
A2. The deterministic source signal $s(n)$ has linear complexity larger than or equal to 2M+1.
A3. The measurement noise $w_i(n)$ is independent of $s(n)$.

In Assumption A2, the linear complexity of $s(n)$ is defined by the least value of c for which there exist $\{\gamma_i\}_{i=1}^{c}$ such that $s(n) = \sum_{i=1}^{c} \gamma_i s(n-i)$ for all $n \geq c$.

We assume that the data transmissions are perfect without any distortion and information exchanges only take place among neighboring agents. An $L \times L$ non-negative weight matrix $\mathbf{C}$ is introduced to describe the bidirectional topology of a network. If there is a link between agents i and j, $c_{i,j} > 0$; otherwise, $c_{i,j} = 0$. The value of $c_{i,j}$ represents the weight that agent i gives to the estimate received from agent j. Besides Assumptions A1-A2, we also require the following assumption:
A4. $\mathbf{C}$ is a symmetric stochastic matrix and the communication topology is strongly connected.

3. DISTRIBUTED IDENTIFICATION UNDER NOISE-FREE MEASUREMENTS

In this section, we try to identify the transfer functions $\{H_i(q)\}_{i=1}^{L}$ using the observations $\{x_i(n)\}_{i=1}^{L}$ from the multiple agents which are connected by a specified topology.

Suppose that the observation samples of $\{x_i(n)\}_{i=1,n=0}^{L,N}$ are available with $N \geq 4M$. The matrix-vector multiplication form of (1) is written by

$$\mathbf{x} = \boldsymbol{\mathcal{H}}\mathbf{s} \tag{2}$$

where $\mathbf{x} = [\mathbf{x}_1 \cdots \mathbf{x}_L]^T$, $\mathbf{x}_i = [x_i(N) \cdots x_i(M)]^T$, $\mathbf{s} = [s(N) \cdots s(0)]^T$, $\boldsymbol{\mathcal{H}} = \left[\boldsymbol{\mathcal{H}}_1^T \cdots \boldsymbol{\mathcal{H}}_L^T\right]^T$,

$$\boldsymbol{\mathcal{H}}_i = \begin{bmatrix} h_{i,0} & \cdots & h_{i,M} & & \\ & \ddots & & \ddots & \\ & & h_{i,0} & \cdots & h_{i,M} \end{bmatrix}_{(N-M+1)\times(N+1)}.$$

Denote by $\mathbf{h}_i = [h_{i,0} \cdots h_{i,M}]^T$. The matrix form of the cross relation equation between sensors i and j is written by [7]:

$$[-\boldsymbol{\mathcal{X}}_j \quad \boldsymbol{\mathcal{X}}_i] \begin{bmatrix} \mathbf{h}_i \\ \mathbf{h}_j \end{bmatrix} = 0,$$

where $\boldsymbol{\mathcal{X}}_i = \begin{bmatrix} x_i(N) & \cdots & x_i(N-M) \\ \vdots & \iddots & \vdots \\ x_i(2M) & \cdots & x_i(M) \end{bmatrix}$. The cross relation equation does not contain the source signal and is linear with respect to $\mathbf{h}_i$; thus, it is often used for the blind identification of practical systems. In a networked system, since only neighboring agents can exchange their information, a cross relation equation can be constructed from the observations of two neighboring sensors. This is the key point to design a distributed estimation algorithm.

Given the weight matrix $\mathbf{C}$, we define an augmented cross relation matrix $\boldsymbol{\mathcal{X}}$ whose rows are given by

$$\underbrace{[0 \cdots 0}_{i-1 \text{ block entries}} \quad -\boldsymbol{\mathcal{X}}_j \quad \underbrace{0 \cdots 0}_{J-i-1 \text{ block entries}} \quad \boldsymbol{\mathcal{X}}_i \quad 0 \cdots 0]$$

where $c_{i,j} > 0$ and $1 \leq i < j \leq L$. Denote by $\mathbf{h} = [\mathbf{h}_1, \cdots, \mathbf{h}_L]^T$. Under Assumption A4, the coefficient vector $\mathbf{h}$ for all channels can be estimated as a non-trivial solution of the following equation [13]:

$$\boldsymbol{\mathcal{X}}\mathbf{h} = 0. \tag{3}$$

The above equation can be solved using the stochastic optimization method [9] in a centralized manner as follows:

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) - \alpha_k \boldsymbol{\mathcal{X}}^H \boldsymbol{\mathcal{X}} \hat{\mathbf{h}}(k), \tag{4}$$

where α_k is the step size which satisfies the following standard assumption:
A5. $\alpha_k > 0$, $\alpha_{k+1} \leq \alpha_k$, $\alpha_k \to 0$ as $k \to \infty$, and $\sum_{k=1}^{\infty} \alpha_k = \infty$.

In a large-scale sensor network, each agent carries out estimations based on only its own and neighboring information; thus, we need to decentralize the computation in (3). To this end, the estimate of $\mathbf{h}_i$ at the agent i is updated as follows:

$$\hat{\mathbf{h}}_i(k+1) = \hat{\mathbf{h}}_i(k) - \alpha_k \sum_{j \in \mathcal{N}(i)} (\boldsymbol{\mathcal{X}}_j^H \boldsymbol{\mathcal{X}}_j \hat{\mathbf{h}}_i(k) - \boldsymbol{\mathcal{X}}_j^H \boldsymbol{\mathcal{X}}_i \hat{\mathbf{h}}_j(k)), \tag{5}$$

where $\mathcal{N}(i)$ denotes the index set of the i-th agent's neighbors.

For the distributed blind channel estimation in (5), it has the following convergence property.

Theorem 1 *Assume that Assumptions A1-A5 hold. Denote by* $\mathbf{h}^*$ *a nontrivial solution of the equation (3). If the initial condition* $\hat{\mathbf{h}}(0)$ *is not orthogonal to* $\mathbf{h}^*$*, then the estimate* $\hat{\mathbf{h}}(k) = \left[\hat{\mathbf{h}}_1^T(k) \cdots \hat{\mathbf{h}}_L^T(k)\right]^T$ *generated by the distributed algorithm in (5) satisfies that* $\hat{\mathbf{h}}(k) \to \frac{\mathbf{h}^*(\mathbf{h}^*)^H \hat{\mathbf{h}}(0)}{\|\mathbf{h}^*\|^2}$ *as* $k \to \infty$.

Due to space limitations, the theorems in this paper are provided without detailed proofs.

4. DISTRIBUTED IDENTIFICATION WITH NOISY MEASUREMENTS

In this section, the distributed identification of $\{H_i(q)\}_{i=1}^L$ with noisy measurements $\{y_i(n)\}_{i=1}^L$ will be investigated. All the notations are the same with the previous section, and all $x_i(n)$ related notations will be replaced by those of $y_i(n)$.

Due to the noise effect for the system model in (1), the matrix $\boldsymbol{\mathcal{Y}}^H\boldsymbol{\mathcal{Y}}$ may have full rank, namely it may not have any zero eigenvalue. According to the convergence analysis in the previous section, if we still use the distributed algorithm in the previous section, the estimate of $\mathbf{h}$ will converge to a trivial solution. To overcome such a problem, the following centralized iterative computation is adopted [12] for the channel identification:

$$\begin{aligned} \hat{\mathbf{h}}(k+1) &= \hat{\mathbf{h}}(k) - \alpha_k \underline{\|\hat{\mathbf{h}}(k)\|^4} \boldsymbol{\mathcal{Y}}^H\boldsymbol{\mathcal{Y}}\hat{\mathbf{h}}(k) \\ &+ \alpha_k \hat{\mathbf{h}}(k) \underline{\hat{\mathbf{h}}^H(k)\boldsymbol{\mathcal{Y}}^H\boldsymbol{\mathcal{Y}}\hat{\mathbf{h}}(k)}. \end{aligned} \tag{6}$$

The underlined terms in the above equation are two global variables, which need to be estimated before decentralizing the computations in each iteration. The two global variables can be represented as follows:

$$\begin{aligned} \|\hat{\mathbf{h}}(k)\|^4 &= L^2\left(\frac{1}{L}\sum_{i=1}^{L}\|\hat{\mathbf{h}}_i(k)\|^2\right)^2, \\ \|\boldsymbol{\mathcal{Y}}\hat{\mathbf{h}}(k)\|^2 &= \frac{L}{2}\left(\frac{1}{L}\sum_{i=1}^{L}\sum_{j\in\mathcal{N}(i)}\|\boldsymbol{\mathcal{Y}}_i\hat{\mathbf{h}}_j(k) - \boldsymbol{\mathcal{Y}}_j\hat{\mathbf{h}}_i(k)\|^2\right). \end{aligned} \tag{7}$$

It is easy to see that the global variables can be estimated using the average consensus techniques.

Denote $\phi_i(k) = \|\hat{\mathbf{h}}_i(k)\|^2$ and $\boldsymbol{\phi}(k) = [\phi_1(k)\cdots\phi_L(k)]^T$. Let $\varphi_i = \sum_{j\in\mathcal{N}(i)}\|\boldsymbol{\mathcal{Y}}_j\hat{\mathbf{h}}_i(k) - \boldsymbol{\mathcal{Y}}_i\hat{\mathbf{h}}_j(k)\|^2$ and $\boldsymbol{\varphi}(k) = [\varphi_1(k)\cdots\varphi_L(k)]^T$. Provided the weight matrix $\mathbf{C}$ and running R iterations of the average consensus operation, we can obtain that

$$\begin{aligned} \bar{\boldsymbol{\phi}}(k) &= \mathbf{C}^R\boldsymbol{\phi}(k), \\ \bar{\boldsymbol{\varphi}}(k) &= \mathbf{C}^R\boldsymbol{\varphi}(k), \end{aligned} \tag{8}$$

where $\bar{\boldsymbol{\phi}}(k) = \left[\bar{\phi}_1(k)\cdots\bar{\phi}_L(k)\right]^T$ and $\bar{\boldsymbol{\varphi}}(k) = [\bar{\varphi}_1(k)\cdots \bar{\varphi}_L(k)]^T$. Then, the estimates of $\|\hat{\mathbf{h}}(k)\|^4$ and $\|\boldsymbol{\mathcal{Y}}\hat{\mathbf{h}}(k)\|^2$ obtained by agent i are $L^2\bar{\phi}_i^2(k)$ and $\frac{L}{2}\bar{\varphi}_i(k)$, respectively. Therefore, the corresponding estimation for the i-th channel using the noisy measurements can be carried out as follows:

$$\begin{aligned} \hat{\mathbf{h}}_i(k+1) = \hat{\mathbf{h}}_i(k) - \alpha_k L^2\bar{\phi}_i^2(k)\sum_{j\in\mathcal{N}(i)}\Big(\boldsymbol{\mathcal{Y}}_j^H\boldsymbol{\mathcal{Y}}_j\hat{\mathbf{h}}_i(k) \\ -\boldsymbol{\mathcal{Y}}_j^H\boldsymbol{\mathcal{Y}}_i\hat{\mathbf{h}}_j(k)\Big) + \frac{\alpha_k L}{2}\bar{\varphi}_i(k)\hat{\mathbf{h}}_i(k). \end{aligned} \tag{9}$$

The proposed distributed identification algorithm is summarized in Algorithm 1.

Algorithm 1 Distributed blind system identification

1) Given the initial conditions $\hat{\mathbf{h}}_i(0)$ for $i = 1, \cdots, L$.
2) Run the average consensus: $\bar{\boldsymbol{\phi}}(k) = \mathbf{C}^R\boldsymbol{\phi}(k)$.
3) Run the average consensus: $\bar{\boldsymbol{\varphi}}(k) = \mathbf{C}^R\boldsymbol{\varphi}(k)$.
3) **for** $i = 1 : L$ **(in parallel)**
4)   Update $\hat{\mathbf{h}}_i(k)$ according to (9).
5) **end for**
6) $k \leftarrow k + 1$ and go to 2).

The proposed distributed identification algorithm using noisy measurements has the following convergence properties.

Theorem 2 *Assume that Assumptions A1-A5 hold. Let* η *be the second largest eigenvalue of* $\mathbf{C}$ *in modulus. If the number of consensus iterations* R *is large enough such that* $\eta^R \ll 1$*, then the estimate* $\hat{\mathbf{h}}(k)$ *generated from Algorithm 1 satisfies the following inequality when* $k \to \infty$*:*

$$\frac{1 - L\eta^R}{1 + 2L\eta^R} + \mathcal{O}(\eta^{2R}) \le \|\hat{\mathbf{h}}(k)\|^2 \le \frac{1 + L\eta^R}{1 - 2L\eta^R} + \mathcal{O}(\eta^{2R}).$$

Remark 1 *In the distributed algorithm, the estimation errors of the two global variables are reflected by* η^R*, i.e. the larger the number of average consensus iterations, the smaller the corresponding estimation errors. According to the result in the above theorem, the norm of* $\hat{\mathbf{h}}(k)$ *approaches to one as* $R \to \infty$*. When* R *is fixed, then the estimate of the channel coefficients will be bounded in a region around the unit circle.*

Theorem 3 *Assume that Assumptions A1-A5 hold. Assume also that* $\boldsymbol{\mathcal{Y}}^H\boldsymbol{\mathcal{Y}}$ *has a smallest eigenvalue with multiplicity one and* $\mathbf{h}^\star$ *is the corresponding normalized eigenvector. The estimate* $\hat{\mathbf{h}}(k)$ *generated from Algorithm 1 converges to a stationary point* $\mathbf{h}^0$ *which satisfies that*

$$\min_{|\kappa|=1}\|\mathbf{h}^0 - \kappa\mathbf{h}^\star\| \le c_0 L\eta^R + \mathcal{O}(\eta^{2R}),$$

where κ *is to eliminate the scalar ambiguity,*

$$c_0 = \begin{cases} \frac{3\rho}{\min\{\lambda_{min}, \lambda_{sse} - \lambda_{min}\}} & \lambda_{min} > 0 \\ \frac{3}{2} & \lambda_{min} = 0, \end{cases}$$

where ρ*,* λ_{min} *and* λ_{sse} *denote the spectrum, the smallest eigenvalue and the second smallest eigenvalue of* $\boldsymbol{\mathcal{Y}}^H\boldsymbol{\mathcal{Y}}$*.*

Remark 2 *In the above theorem, we assume that* $\mathcal{Y}^H\mathcal{Y}$ *has only one smallest eigenvalue. Otherwise, if the smallest eigenvalue of* $\mathcal{Y}^H\mathcal{Y}$ *has multiplicity two, then even the centralized blind identification algorithm may fail to obtain an accurate channel estimation. For the blind system identification, even if the result is normalized, there exists a unit norm scalar ambiguity. To this end,* κ *is introduced to eliminate the ambiguity in the above theorem. In addition, we can observe from the above results that the channel estimation performance may be seriously degraded when the least non-zero eigenvalue of* $\mathcal{Y}^H\mathcal{Y}$ *is small or the two smallest eigenvalues of* $\mathcal{Y}^H\mathcal{Y}$ *are very close.*

5. NUMERICAL SIMULATIONS

In this section, two simulation examples are provided to show the performances of the proposed algorithms: one uses noise-free measurements and the other uses noisy measurements. The considered topology is shown in Fig. 2. The Metropolis-

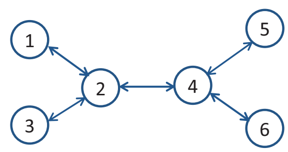

Fig. 2. Topology of a six-agent network.

based wight matrix is used [14], which is defined by

$$c_{i,j} = \begin{cases} \frac{1}{1+\max\{|\mathcal{N}(i)|,|\mathcal{N}(j)|\}} & \text{sensors } i \text{ and } j \text{ are linked,} \\ 1 - \sum_{j\neq i} c_{i,j} & i = j, \\ 0 & \text{otherwise.} \end{cases}$$

where $|\mathcal{N}(i)|$ denotes the number of neighbors for agent i. All the channel coefficients and the source signal are randomly generated such that Assumptions A1-A2 are satisfied with probability one. The step size is defined by $\alpha_k = \frac{1}{k}$ so that Assumption A5 is satisfied. To measure the identification performance, we define the normalized error as $\frac{\min_\gamma \|\gamma\hat{\mathbf{h}}(k)-\mathbf{h}^*\|}{\|\mathbf{h}^*\|}$, where $\hat{\mathbf{h}}(k)$ is the estimate of the coefficient vector at the k-th step and $\mathbf{h}^*$ denotes the true coefficient vector. The number of consensus iterations is set to $R = 50$.

Fig. 3 shows the identification performance of the algorithm in Section 3 with noise-free measurements. It can be found that the channel estimations corresponding to different agents converge after about 800 iterations. In addition, the associated normalized estimation errors converge to zero, namely the multiple channels can be accurately determined up to a scalar constant. Fig. 4 shows the identification performance of Algorithm 1 under noisy measurements, where the signal noise ratio is set to 20dB. We can find that the channel estimations converge after around 800 iterations. However, the normalized estimation errors of the multiple channels deviate from zero, which are caused by the measurement noises.

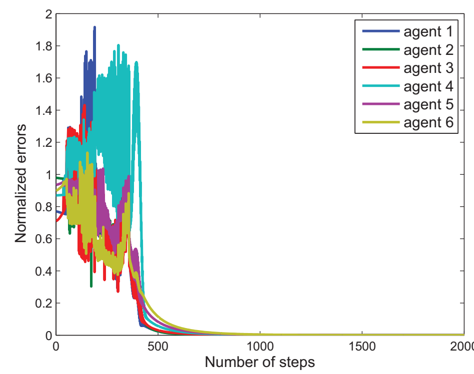

Fig. 3. Identification performances using noise-free measurements.

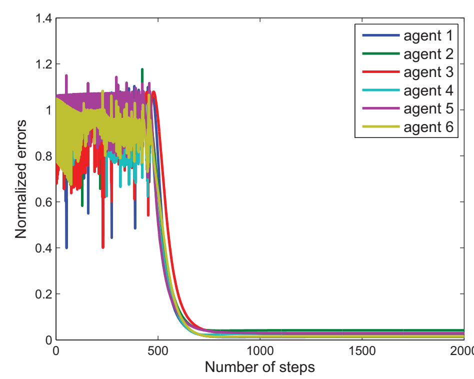

Fig. 4. Identification performances using noisy measurements.

6. CONCLUSION

In this paper, we have developed distributed blind system identification methods in the context of sensor networks. The key to the proposed algorithms lies in the adaption of the associated cross relation equations to the networked systems, namely one pair of neighboring agents can generate a cross relation equation by exchanging their information. The convergence properties of the proposed algorithms have been analyzed and simulation examples have been provided to validate the developed algorithms. In our future work, the proposed distributed identification algorithm will be extended for the recursive blind system identification, for which the estimation errors of the multiple channels can be reduced by adopting more observation samples.

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