ROBUST DECISION FEEDBACK EQUALIZER SCHEME BY USING SPHERE-DECODING DETECTOR

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ABSTRACT

The decision feedback equalizer (DFE) is an efficient scheme to suppress intersymbol interference (ISI) in various communication and magnetic recording systems. However, most DFE implementations suffer from the phenomenon of error propagation, which degrades its bit error rate (BER) performance. In this paper, We use sphere detector (SD) to achieve maximum likelihood (ML) detection and significantly reduce the system symbol error rate (SER). Simulations show that the proposed scheme with sphere detector decision feedback equalizer (SD-DFE) algorithm can efficiently reduce the SER. At SNR=28, the SER can be improved from 2.0×10^{-5} (Ideal DFE) to 1.8×10^{-6} (six-stage SD-DFE).

Index Terms—decision feedback equalizer (DFE), sphere detector, error propagation.

1. INTRODUCTION

Data transmission through band-limited channel suffers from *inter-symbol interference* (ISI) and results in degradation of throughput rate [1]. The *decision feedback equalizer* (DFE) is widely utilized to mitigate the ISI effect. In DFE, a feed-forward filter (FFF) suppresses the precursor ISI, and a feed-backward filter (FBF) suppresses the postcursor ISI by using previous detected symbols.

The DFE needs to detect the received symbol and then feed back to FBF. When wrong detection occurs, the error will propagate throughout the feedback tap delay line in the FBF. Besides, it may affect the successive symbols for incorrect decisions. Many researches focus on analysis of the error propagation [2]-[4]. In [5], if the DFE output is smaller than a given threshold, 0 is fed back to FBF. In [6] and [7], a threshold technique is proposed to detect the error events. The key idea of [8] is to define an unreliable region. When the DFE output is in the unreliable region, the decision is not made instantly. Instead, the log-likelihood ratio (LLR) is fed to FBF, and the process is performed continuously until the DFE output becomes reliable. Then, the decisions of these symbols are made simultaneously, and the reliable symbols are helpful for detecting the former unreliable symbol. However, the joint detection of multiple symbols is an operation with high complexity, especially in higher order modulation or more joint detected symbols.

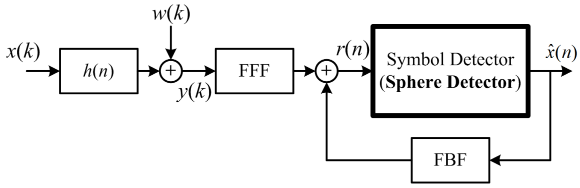

Fig. 1. Discrete-time channel model and the block diagram of the SD-DFE.

In this paper, we use three techniques to achieve better symbol error rate (SER) performance with reduced complexity in detection:

(1) *Matrix/vector model for signal detection in FBF*: we use the matrix/vector signal model to formulate the signal in the FBF. By jointly detecting all elements in the vector, the SER can be improved significantly.

(2) *The SD without QR decomposition*: After using the matrix/vector signal model, the signal model of the FBF is equivalent to the signals in the Multi-input-multi-output (MIMO) system. As shown in Fig. 1, we utilize the SD to detect the signal. Due to the upper triangular matrix in our proposed signal model, the SD does not need the QR decomposition.

(3) Auto-determined Threshold based SD/normal slicer switching mechanism: We extend the concept of [8], the threshold derived in [8] is utilized to detect the unreliable DFE output. As the DFE output falls in unreliable region, the symbol detection made by a normal slicer has high probability to be a wrong detection.

In our proposed SD-DFE, as the DFE output is in the reliable region, the slicer operates as a normal slicer. In other words, as the DFE output is unreliable, the SD is utilized as symbol detector. From our simulation results, the proposed SD-DFE has better SER performance than DFE with ideal feedback.

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The remainder of the paper is organized as: In section 2, we give some background knowledge about the SD and describe the signal model and the threshold value derived in [8]. Section 4 presents the proposed SD-DFE algorithm. Section 5 gives the simulation results and Section 6 concludes this paper.

2. REVIEW OF BACKGROUND KNOWLEDGE

2.1 Sphere Detector [9]

For a MIMO system with N_T transmitting antennas and N_R receive antennas, the received signal can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the received signal vector $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$, the transmit symbol vector $\mathbf{x} \in \psi^{N_T} \subset \mathbb{C}^{N_T \times 1}$, and the AWGN noise vector $\mathbf{n} \in \mathbb{C}^{N_R \times 1}$. The channel matrix $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ contains complex channel fading coefficients and is known before signal detection. Sphere detector (SD) can find the hard ML detection of $\mathbf{x}$ in the constellation set ψ^{N_T}. The SD can significantly reduce the number of candidate symbol vector in search space compared to exhaustive search algorithms.

The QR decomposition of H needs to be done, and the considered candidates need to meet the condition:

$$\|\tilde{\mathbf{y}} - \mathbf{R}\mathbf{x}\|_2^2 \le C_0, \tag{2}$$

where C_0 is the squared radius of a N_R dimensional hyper-sphere, $\mathbf{R} \in \mathbb{C}^{N_R \times N_T}$ is an upper triangular matrix, $Q \in \mathbb{C}^{N_R \times N_R}$ is an orthogonal matrix, and $\tilde{\mathbf{y}} = \mathbf{Q}^H \mathbf{y}$. Depend on the search algorithm and the channel realization, SD searches a variable number of nodes in the constellation tree structure.

2.2 Signal Model of the DFE and threshold value [8]

In this paper, we are concerned with the system model shown in Fig. 1. The notation and the signal model are defined as the following:

- $x(k)$ is the transmitted data.
- $h(n)$ is the equivalent discrete time channel impulse response, which is linear and band-limited.
- $w(k)$ is Additive White Gaussian Noise (AWGN).
- $y(n)$ is the channel output.
- $r(n)$ is the output of the DFE and is expressed as:

$$r(n) = \sum_{m=0}^{N_b-1} b_m \times y(n-m) - \sum_{m=1}^{N_a} a_m \times \hat{x}(n-m) \tag{3}$$

where

- $\hat{x}_n$ is the detection of x_n.
- a_k is the k_{th} tap weight of the FBF.
- b_k is the $(k+1)_{th}$ tap weight of the FFF.
- N_a is the tap length of the FBF.
- N_b is the tap length of the FFF.

In most DFE design, the decision must be made for each symbol instantly. The STM-DFE [8] sets a threshold value to define unreliable region. [8] gives the closed-form analysis of the threshold value, and the threshold value can be approximated as:

$$L = a_1(1\text{-}a_1). \tag{4}$$

where L denotes the threshold value. For BPSK modulation, the transmitted data $x(k)$={-1, +1}, and the DFE output is unreliable as $|r(n)| < L$. This threshold value can be applied to higher order modulation. Without loss of generality, we give the example of 4 PAM in Fig. 2. If unreliable DFE output is detected, we need to use other received symbol to obtain a detection with much reliability.

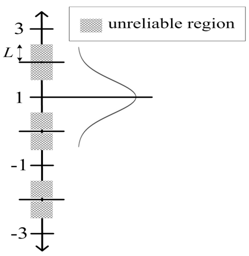

Fig. 2. Unreliable region of the 4 PAM modulation.

3. PROPOSED SPHERE DECODER BASED DFE

Before analyzing the DFE output error, several assumptions are made in this paper:

- With sufficient training symbols, the coefficients of the DFE are trained to be in the convergent state.
- $r(n) \approx x(n) + \varepsilon(n)$, where $\varepsilon(n)$ is AWGN with zeros mean and variance. This assumption is commonly used in many literatures.
- The detection for reliable DFE output is assumed to be correct.

To explain the SD-DFE algorithm easily, the signal model of three-stage SD-DFE is given first.

3.1 Signal Model of Three-Stage SD-DFE

As r(n) is an unreliable symbol, the three-stage SD-DFE will detect three successive DFE outputs jointly, r(n), r(n+1), and r(n+2). The error models of these three symbols are given in Eq.(5)~Eq. (7):

$$\begin{aligned} e(n) &= [\sum_{m=0}^{N_b-1} b_m \times y(n-m) - \sum_{m=1}^{N_a} a_m \times \hat{x}(n-m)] - \hat{x}(n) \\ &= f(n) - \hat{x}(n). \end{aligned} \tag{5}$$

$$\begin{aligned} e(n+1) &= [\sum_{m=0}^{N_b-1} b_m \times y(n+1-m) - \sum_{m=2}^{N_a} a_m \times \hat{x}(n-m)] \\ &\quad - a_1\hat{x}(n) - \hat{x}(n+1) \\ &= f(n+1) - a_1\hat{x}(n) - \hat{x}(n+1). \end{aligned} \tag{6}$$

$$e(n+2) = [\sum_{m=0}^{N_b-1} b_m \times y(n+2-m) - \sum_{m=3}^{N_a} a_m \times \hat{x}(n-m)]$$
$$- a_2\hat{x}(n) - a_1\hat{x}(n+1) - \hat{x}(n+2)$$
$$= f(n+2) - a_2\hat{x}(n) - a_1\hat{x}(n+1) - \hat{x}(n+2). \quad (7)$$

We rewrite Eq.(5)~Eq.(7) in matrix form:

$$\begin{bmatrix} e(n+2) \\ e(n+1) \\ e(n) \end{bmatrix} = \begin{bmatrix} f(n+2) \\ f(n+1) \\ f(n) \end{bmatrix} - \begin{bmatrix} 1 & a_1 & a_2 \\ 0 & 1 & a_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}(n+2) \\ \hat{x}(n+1) \\ \hat{x}(n) \end{bmatrix}. \quad (8)$$

Eq. (8) can be regarded as the case of 3×3 MIMO signal.

$$\mathrm{e} = \mathrm{f} - \mathrm{Rx}. \quad (9)$$

The optimal condition of Eq. (9) is similar to the optimal condition of Eq. (2). Based on maximum-likelihood (ML) criterion, we can obtain the optimal detection:

$$\mathbf{x}_{\text{optimal}} = \min \left\| \mathbf{f} - \mathbf{Rx} \right\|^2. \quad (10)$$

Because R is an upper triangular matrix, R does not need to do the QR decomposition. The SD can be applied to Eq. (10) directly to find optimal. The SD can find the ML detection of x in Eq. (9) with reduced complexity.

3.2 Design Example: The Three-stage SD-DFE

In Fig. 3, the architecture of the three-stage SD-DFE is given. The three-stage SD-DFE consists of a normal slicer, 3x3 SD, and unreliable symbol detector. In the three-stage SD-DFE, [x(n) x(n-1) x(n-2)] may be detected by 3x3 SD. The first three symbol registers in the FBF connect with multiplexor to select where the detected symbols come from. As mentioned above, it is not necessary to turn on the SD for all DFE outputs. There are three operation modes in the SD-DFE:

(1) *Normal mode:* As a reliable DFE output is received, the SD-DFE runs as a normal DFE. In this mode, the SD is not active, and the output of a normal slicer is fed to the FBF. The SD-DFE transfers to waiting mode, as an unreliable DFE output is detected.

(2) *Waiting mode:* As an unreliable symbol occurs, the DFE needs the SD to detect three symbols jointly. Hence, the DFE has to wait additional two symbols. In the meanwhile, the symbol ‘0’ is fed to the FBF.

(3) *Sphere detection mode*: As the SD collects three symbols, a joint SD-based ML detection is made. Then, the ‘0’ symbols stored in the FBF are replaced by these estimated symbols, and the SD-DFE turns into the normal mode.

There are three possible types of input for the register of the FBF: 1) output of a normal slicer; 2) ‘0’ symbol; 3) output of the SD.

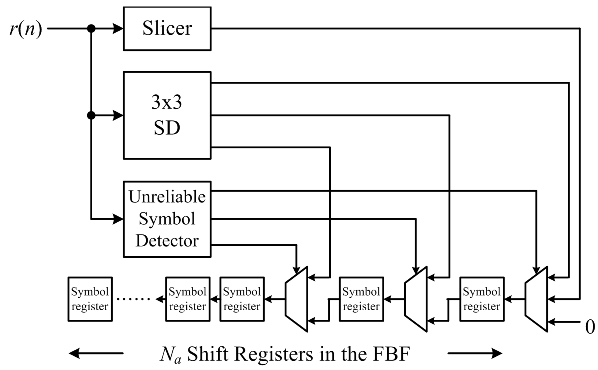

Fig. 3. The architecture of the three stage SD-DFE.

3.3 M-stage SD-DFE

In Section 4.1 and 4.2, we show that SD-DFE can detect the several symbols simultaneously, and an example of three-stage SD-DFE is given. It should be noted that the stage number is not fixed. The proposed SD-DFE can be extended to M-stage. For the extreme case, the signal model in Eq. (8) can be extended to contain the current DFE output and the Na previous DFE outputs. For a DFE that has FBF with Na taps, the signal model of a *(Na+1)*-stage SD-DFE is given as follows:

$$\begin{bmatrix} e(n+N_a) \\ \vdots \\ e(n+2) \\ e(n+1) \\ e(n) \end{bmatrix} = \begin{bmatrix} f(n+N_a) \\ \vdots \\ f(n+2) \\ f(n+1) \\ f(n) \end{bmatrix} - \begin{bmatrix} 1 & a_1 & \cdots & & a_{N_a} \\ & & \ddots & & \vdots \\ 0 & \cdots & 1 & a_1 & a_2 \\ 0 & \cdots & 0 & 1 & a_1 \\ 0 & \cdots & & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}(n+N_a) \\ \vdots \\ \hat{x}(n+2) \\ \hat{x}(n+1) \\ \hat{x}(n) \end{bmatrix}. \quad (11)$$

In Eq. (11), a *(Na+1)*× *(Na+1)* SD can give a ML estimation of $\hat{\mathbf{x}} = [\hat{x}(n+N_a)\ \ldots\ \hat{x}(n+1)\ \ \hat{x}(n)]^{\mathrm{T}}$.

In Eq. (11), *M* is *(Na+1)*, and we can decrease *M* for lower computation complexity. For FBF with Na taps, the possible value of *M* is from *2* to *(Na+1)*.

4. SIMULATION RESULTS

4.1 Channel Model and Simulation Setting

In this section, the channel model [10] we use is:

$$h = 0.5[1+\cos(2\pi/w)21+\cos(2\pi/w)], \quad (12)$$

where w = 3.3. The number of taps in FFF and FBF are 7 and 5, respectively. In general, the least mean square (LMS) algorithm is a well-known approach to obtain the filter coefficients. In our simulation, the coefficients of FFF and FBF are trained by LMS algorithm with sufficient training sequences.

Because we focus on high order modulation in this paper, we use 8 PAM modulation scheme in our simulations. Due to the flexibility of the SD, our proposed SD-DFE can be applied to other modulation scheme. We assume the data fed back into the FBF are always correct in the case of the ideal DFE (IDFE) in order to measure the SNR degradation of DFE due to the error propagation.

4.2 Comparison of BER

Fig. 4 gives the BER simulation results of the proposed SD-DFEs, normal DFE, and the IDFE. Due to joint detection for multiple symbols, the proposed SD-DFEs have better performance than normal DFE. Moreover, in high SNR condition, the SD-DFEs perform better than the IDFE. Even the IDFE does not suffer from error propagation in FBF, it cannot detect and correct the errors resulting from noise. The SD-DFEs make a joint detection for multiple symbols. In high SNR condition, a symbol with large noise occurs few times. As one symbol suffers from large noise interference, the SD can correct it by other symbols. Besides, as the SD-DFE makes a joint detection for more symbols, it has greater ability to correct a noisy symbol. Hence, the SD-DFEs have better SER than IDFE in high SNR condition.

Fig. 5 gives the percentage of the symbol detected by the SD. It is apparent that only few symbols are detected by the SD in high SNR condition. The computation of the SD can be significantly saved. Nevertheless, the SD-DFE can give much better performance than IDFE, as shown in Fig. 4.

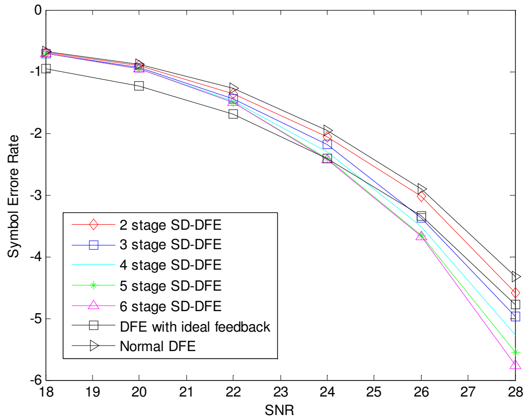

Fig. 4. SER comparison result.

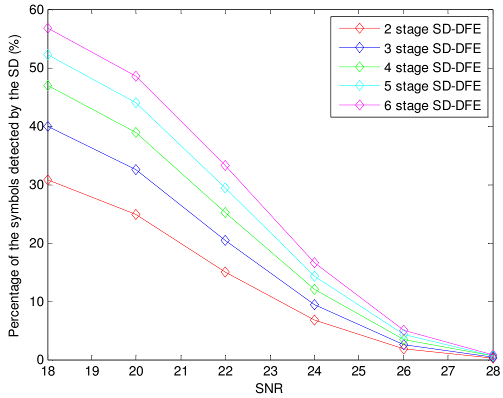

Fig. 5. Percentage of the symbol detected by the SD.

5. CONCLUSIONS

In this paper, we use matrix/vector model for the signal fed into FBF. Based the matrix/vector signal model that is similar to the MIMO detection problem, this work proposes a novel and effective SD-DFE algorithm to enhance the DFE performance. Besides, by using the threshold value derived in our prior work [8], the unreliable symbols can be found, and the SD can only be utilized to detect these unreliable symbols. From simulation result, the proposed SD-DFE algorithm can outperform the conventional DFE. In high SNR condition, the SD-DFEs have better SER performance than the IDFE.

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