DESIGN OF SPARSE-SIGNAL PROCESSING IN RADAR SYSTEMS

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ABSTRACT

Sparse-signal processing (SSP) is interpreted in this paper as a sparse model-based refinement of typical steps in radar processing. Matched filtering remains vital within SSP but joined with radar detection promoting the sparsity. Realistic measurements are also supported in SSP by using Monte-Carlo (MC) methods. MC-based SSP promotes the sparsity by detection-driven MC-sampling that also improves efficiency. This MC extension aims for the stochastic description of sparse solutions, and the flexibility to use any prior on signals or on data acquisition, as well as any distribution of noise or clutter. Numerical experiments demonstrate favorable performance of the proposed SSP.

Index Terms— compressive sensing, radar systems, sparse recovery, detection, non-Gaussian distribution

1. INTRODUCTION

Sparse-signal processing (SSP) is being studied nowadays as a major part of compressive sensing (CS) (e.g. [1]-[4], [7]-[9], [22]-[23]). CS can improve radar performance because it is optimized to information in received data rather than only to the whole sensing bandwidth (e.g. [7], [10] and [13]). The information importance is also being emphasized in the information geometry (e.g. [5]). Pursuing practical CS in radar, we prefer stochastic SSP when treating noise, prior knowledge on signals or their data acquisition, and when providing results ([19]). In a way, stochastic SSP is giving a fresh boost to radar initiated in the fifties ([24]).

In CS, raw radar measurements $\boldsymbol{y}$ are described as:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{z}\,, \qquad (1)$$

by a sensing matrix $\boldsymbol{A}$, a sparse radar profile $\boldsymbol{x}$, signals $\boldsymbol{Ax}$ and (complex Gaussian) receiver noise $\boldsymbol{z}$ with zero mean and equal variances γ, $p(\boldsymbol{z}|\gamma) \propto \exp(-|\boldsymbol{z}|^2/\gamma)$. Matched filtering (MF) produces the main statistics $\boldsymbol{x}_{\text{MF}}$, $\boldsymbol{x}_{\text{MF}} = \boldsymbol{A}^{\text{H}}\boldsymbol{y}$, in detection via a generalized likelihood ratio test (GLRT) (e.g. [6]).

When the sparsity of $\boldsymbol{x}$ is formalized by a multivariate Laplace prior $p(\boldsymbol{x}|\lambda)$, $p(\boldsymbol{x}|\lambda) \propto \exp(-\lambda|\boldsymbol{x}|_1)$, the maximum a posteriori (MAP) estimator of $\boldsymbol{x}$, written as:

$$\boldsymbol{x}_{\text{MAP}} = \arg\min_{\boldsymbol{x}} \{\, |\boldsymbol{y}\text{-}\boldsymbol{Ax}|^2 + h|\boldsymbol{x}|_1 \,\}, \qquad (2)$$

gives the usual SSP from CS with the l_1-norm $|\boldsymbol{x}|_1$ promoting the sparsity and the l_2-norm $|\boldsymbol{y}\text{-}\boldsymbol{Ax}|$ for minimizing the noise, together with a threshold h that balances between the two tasks (e.g. [1], [7] and [22]). An underdetermined system can be solved by SSP from (2), i.e. M measurements in $\boldsymbol{y}$ can be enough for N outputs in $\boldsymbol{x}$, $M < N$, because of the sparsity, i.e. only K nonzeros in $\boldsymbol{x}$, $K < M$, and incoherence of $\boldsymbol{A}$, i.e. a low inner product between its different columns (e.g. [7]).

In a realistic radar case, the straight physical nature of $\boldsymbol{A}$ suits CS and the incoherence well (e.g. [26]). However, SSP from (2) can hardly work optimally. Firstly, the distribution of noise or clutter can often be non-Gaussian and even unknown in a closed form (e.g. [11]). Secondly, although a Laplace prior creates the usual SSP in (2), realizations from a Laplace distribution are hardly compressible ([2]). Thirdly, the control parameter h is signal dependent and cannot be known in advance. In radar, h shall be treated as a threshold, and thus, related to detection metrics. Finally, SSP estimates can have unknown and arbitrary probability distributions. The stochastic description of the SSP solutions is lacking or simply assumed Gaussian for the convenience.

In this paper, motivated by the practical CS in radar, we propose flexible SSP needed in realistic radar processing. Firstly, we recognize the natural role of radar detection in promoting the signal sparsity. In addition, we propose SSP based on Monte-Carlo (MC) methods that can accommodate realistic radar cases without restrictions from (2). This MC-based SSP (MC-SSP) provides the stochastic behavior of sparse solutions, and accommodates any prior knowledge on signals or their data acquisition, as well as any distribution of noise or clutter. Finally, we design MC-SSP that promotes the sparsity not only by an explicit prior but rather by an MC-sampling scheme and stopping criteria coming from an optimal detection strategy. Besides enabling the practical sparsity promotion, this approach also makes MC-SSP converge better and thus, be computationally efficient.

1.1. Related Work

MC-SSP is designed to perform tasks of MF and detection of radar processing. It promotes the sparsity by applying detection, and supports realistic cases by using MC methods.

SSP has already been seen from a detection point of view but only in basic SSP from (2) with an orthogonal $\boldsymbol{A}$ when h is related to detection via GLRT ([12]). The threshold h has

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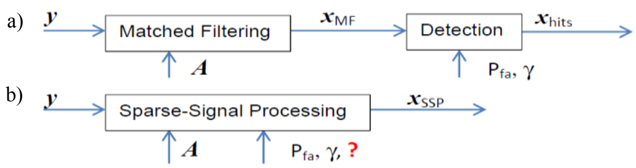

Fig.1. SSP in b) compared to a) traditional radar processing.

also been approximated for arbitrary distributions but only by assuming that the data dependent parameters are known ([20]). The control parameter h has also been studied in theory for specific CS goals (e.g. [8] and [25]).

Another class of stochastic SSP applies to random sensing matrices, such as e.g. belief propagation and approximate message passing (e.g. [22] and [8]). This SSP is deeply based on (asymptotic) Gaussian assumptions as needed for its closed form. Moreover, sensing matrices in radar are intrinsically deterministic what cannot be ignored as it is valuable in real-time implementation ([19] and [26]).

The MC approach in CS has also been studied in CS theory, e.g. for a universal goal in [3], for sparsity in [4] and [9], and for a combinatorial recovery in [15]. MC-SSP is more practical as aimed for the flexibility in radar systems.

1.2. Outline and Main Contributions

In Section 2, SSP is proposed with the main contributions of:

- promoting the signal sparsity by radar detection,
- using MC methods for non-Gaussian cases, and
- combining the radar detection and the MC methods in a greedy manner for a practical design of SSP.

In Section 3, the proposed-SSP performance is evaluated on simulated data in a range-only case. In Section 4, conclusions are drawn, and future work is indicated.

2. SPARSE-SIGNAL PROCESSING IN RADAR

In SSP for radar, estimated sparsity of a radar profile $\boldsymbol{x}$ is to be related to detection at a fixed probability of false alarms P_{fa} as shown in Fig.1. Other parameters will also be needed (shown by **?**) but let us start with P_{fa} and γ only as also used in typical radar detection based on GLRT. The variance γ is known (or estimated from data) while P_{fa} is fixed (and low!).

Typical radar detection starts with two hypotheses:

$$\mathcal{H}_1 : \boldsymbol{y} = x_i \boldsymbol{a}_i + \boldsymbol{z}; \text{ and } \mathcal{H}_0 : \boldsymbol{y} = \boldsymbol{z}; \qquad (3)$$

about $\boldsymbol{y}$ from (1) containing a target ($\mathcal{H}_1$) or not ($\mathcal{H}_0$) where $\boldsymbol{a}_i$ is an ith column of $\boldsymbol{A}$, and x_i is the ith element of $\boldsymbol{x}$. It results in the GLRT with $x_{MF,i}$, $x_{MF,i} = \boldsymbol{a}_i^H \boldsymbol{y}$, at a fixed P_{fa} where P_{fa} is probability defined by $P\{|x_{MF,i}|>\eta|\mathcal{H}_0\}$ where $\eta = \sqrt{-\gamma \ln P_{fa}}$. The probability of detection P_d is defined by $P\{|x_{MF,i}|>\eta|\mathcal{H}_1\}$.

In the MAP form of SSP from (2), h contains the noise variance γ and the sparsity parameter λ as: $h = \gamma\lambda$. If related to GLRT, h equals η ([12]). Such a link makes h (and λ) become known in SSP from (2), and related to P_{fa}. Note that other detection strategies may also apply (e.g. [18]).

Table 1. Proposed MC-SSP scheme

Inputs: $\boldsymbol{y}$, $\boldsymbol{A}$, γ, P_{fa} (and a maximum number of iterations K_{max})

Outputs: sparse estimate $\boldsymbol{x}_{SSP}$ and its posterior $p(\boldsymbol{x}_{SSP}| \boldsymbol{y}, \gamma, P_{fa})$

Initialize: $\boldsymbol{y}_{res,1} = \boldsymbol{y}$, $\boldsymbol{x}_{res,1} = \boldsymbol{A}^H\boldsymbol{y}$, $\boldsymbol{x}_{SSP} = \boldsymbol{0}$, h (from γ and P_{fa})

Repeat for each iteration k till stopping (or *up to* K_{max}):

1. Draw L elements $\{n_{k,l}\}$ and estimate their weights $\{w_{k,l}\}$, $l = 1, .., L$, from *the importance density* $p(n \mid \boldsymbol{y}_{res,k}, \gamma, h)$
2. Select an element $n(k)$ from all L elements in $\{n_{k,l}\}$ with best weight $w_{n(k)}$ from all L weights $\{w_{k,l}\}$, $l = 1, .., L$
3. Estimate amplitude $x_{SSP,\,n(k)}$ and *posterior* of best $n(k)$
4. Update residuals $\boldsymbol{y}_{res,\,k+1} = \boldsymbol{y}_{res,\,k} - x_{SSP,\,n(k)}\, \boldsymbol{a}_{n(k)}$
5. Update remains $\boldsymbol{x}_{res,\,k+1} = \boldsymbol{A}^H \boldsymbol{y}_{res,\,k+1}$

Continue if any $|x_{res,\,k+1}| > h$ or $E[|\boldsymbol{y}_{res,\,k+1}|^2] > \gamma$, otherwise stop

In this paper, a detection-driven MC-SSP algorithm is presented in a basic case from (2) for the sake of clarity and fair comparison with the existing algorithm Complex Fast Laplace (cFL, [1] and [19] for complex signals). In cFL, the prior $p(\boldsymbol{x}|\lambda)$ is built from a complex Gaussian prior for $\boldsymbol{x}$ and a Γ hyper-prior for the variance of $\boldsymbol{x}$. cFL refines actually $\boldsymbol{x}_{MF}$ in a number of iterations by selecting significant elements based on increase in the assessed posterior in each element of $\boldsymbol{x}$. cFL is also fast because of a greedy implementation based on optimization separable for each element n, $n = 1,..,N$. Other algorithms can be fast but not stochastic (e.g. [23]).

MAP estimates can also be numerically approximated by using many MC realizations from an assessed posterior (e.g. [21]). Advances in MC techniques together with increasing computational power encouraged the development of feasible MC solutions (e.g. [17]). When translating the MAP estimation of cFL into an MC version, the SSP goal remains the same: to identify significant nonzeros $\{x_{n(k)}\}$ in an Nx1 vector $\boldsymbol{x}$ satisfying (2) where $n(k)$ indicates a column $\boldsymbol{a}_{n(k)}$ of an MxN matrix $\boldsymbol{A}$, and $\{n(k)\}$ is a resulting support set, k =1, .., K, $K < M < N$. A nonzero can be sought randomly, but it converges faster when the search is sensibly tailored.

An approximation $p(n\,|\boldsymbol{y},\gamma, h)$ of the individual posterior $p(x_n \mid \boldsymbol{y}, \boldsymbol{x}_{-n})$ (where the rest $\boldsymbol{x}_{-n}$ is known or zero) is created to perform MC (importance) sampling that encourages selecting a good candidate n based on the detection goal: optimal P_d at a fixed P_{fa}. In this case, it is built at $\boldsymbol{x}_{MF}$ and h with an LRT: $p(\boldsymbol{y}|x_n, \boldsymbol{x}_{-n})/p(\boldsymbol{y}|0,\boldsymbol{x}_{-n})$. An individual prior can be any prior but it serves the sparsity of $\boldsymbol{x}$ here via the detection threshold h.

The MC-SSP scheme is outlined in Table 1. In each iteration k, each MC-realization l is used to draw an element $n_{k,l}$ with the weight $w_{k,l}$, $w_{k,l} \propto p(n_{k,l} \mid \boldsymbol{y}_{res,k}, \gamma, h)$, l = 1,..,L. A single nonzero is sought from the greedy residuals $\boldsymbol{y}_{res,k}$, initially $\boldsymbol{y}$, or the greedy remains $\boldsymbol{x}_{res,\,k}$, initially $\boldsymbol{x}_{MF}$, i.e. $\boldsymbol{x}_{res,1} = \boldsymbol{A}^H\boldsymbol{y}$. The best candidate $n(k)$ with the highest weight assessed from $\{w_{k,l}\}$ is selected. An estimate $x_{SSP,n(k)}$ of its amplitude $x_{n(k)}$ is computed from $\boldsymbol{x}_{res,k}$. The MC generation repeats from $p(n \mid \boldsymbol{y}_{res,k+1}, \gamma, h)$ updated with the kth nonzero model-based contribution: $\boldsymbol{x}_{res,k+1} = \boldsymbol{x}_{res,k} - x_{SSP,n(k)}\, \boldsymbol{A}^H\boldsymbol{a}_{n(k)}$ and

$\boldsymbol{y}_{\mathrm{res},k+1} = \boldsymbol{y}_{\mathrm{res},k} - x_{\mathrm{SSP},n(k)}\, \boldsymbol{a}_{n(k)}$. This selection continues as long as the detection threshold h and convergence criteria hold.

Note that SSP considers a set of nonzeros, i.e. multiple targets, and thus, facilitates actually the multi-target detection theory. E.g. probability of detecting the true support set of a sparse $\boldsymbol{x}$ depends on the signal-to-noise ratio (SNR), incoherence of $\boldsymbol{A}$, etc. ([25]). In MC-SSP, it depends also on a chosen MC-sampling strategy and its size L.

MC-SSP suits a realistic radar case whose likelihood and the priors are not restricted as in (2) to Gaussian likelihood or a Laplace prior only. MC-SSP enables any distribution including even empirical distributions being learned from measurements $\boldsymbol{y}$. Moreover, the sparsity is promoted not only by an explicit prior but also by MC sampling and stopping criteria based on an optimal detection strategy. This makes MC-SSP more greedy and thus, computationally efficient.

3. SIMULATION RESULTS

Simulated data are used to demonstrate performance of SSP and MC-SSP. To keep the tests simple and clear, measurements are range only in pulse radar. In this basic radar case, M measurements in $\boldsymbol{y}$ from (1) are taken over one pulse repetition time with pulse length 25 and M equal to 108. A sensing matrix $\boldsymbol{A}$ contains delayed replicas of a transmitted waveform that is a linear chirp with bandwidth equal to the unit sampling frequency. In order to have an underdetermined system (without compressive acquisition), M inputs remain, while the estimation grid is up-sampled to N outputs, N=250. The target locations are uniformly randomly chosen over all N possible range cells. The true amplitude of a target in $\boldsymbol{x}$ is set to one. A target SNR is given as an output SNR, SNR = $1/\gamma$.

Detection performance within SSP is tested at different values of a fixed P_{fa} and different values of SNR in a single-target test case. P_{d} and P_{fa} are measured as: $P_{\mathrm{dm}} = N_{\mathrm{d}} / N_{\mathrm{r}}$ and $P_{\mathrm{fam}} = N_{\mathrm{fa}} / N_{\mathrm{r}}(N\text{-}1)$, by counting N_{d} detections and N_{fa} false alarms in N_{r} noise runs of cFL at $\lambda^2 = -\ln P_{\mathrm{fa}}/\gamma$. When counting, a detection or a false alarm means an estimated nonzero being a true nonzero or a true zero, respectively.

At different SNR, measured P_{fam} remains fairly constant but differs from P_{fa} in the GLRT threshold, as shown in Fig.2. This indicates that this simple SSP threshold is a good start but needs further understanding and (fine) tuning.

Existing SSP (cFL, [19]) and its MC version (Table 1) are compared with the same simulated radar data. The normalized mean squared error (MSE) in the estimated $\boldsymbol{x}_{\mathrm{SSP}}$, MSE($\boldsymbol{x}_{\mathrm{SSP}}$) = $|\boldsymbol{x}_{\mathrm{SSP}}\text{-}\boldsymbol{x}|^2/|\boldsymbol{x}||\boldsymbol{x}_{\mathrm{SSP}}|$, is computed for nonzero (targets) and zero elements in the true profile $\boldsymbol{x}$, with the two SSP versions at different output SNR from 100 noise runs, all with 10 targets, and shown in Fig.3. The number L of the MC realizations in the MC-SSP tests is 100.

At lower SNRs the MSE performance is even better for nonzeros in $\boldsymbol{x}$, and slightly degraded for zeros as clear from Fig.3. At higher SNRs the performance is comparable.

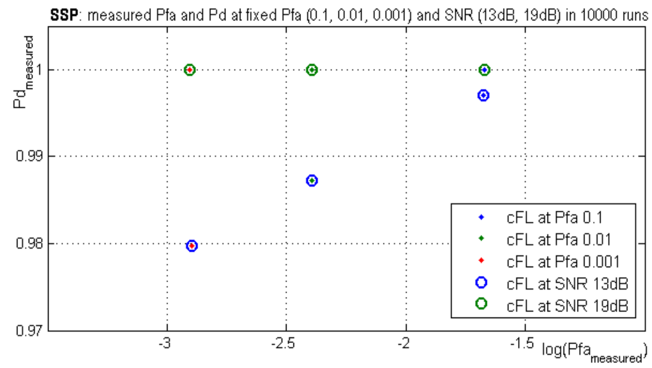

Fig.2. Measured detection performance from SSP at a fixed P_{fa}.

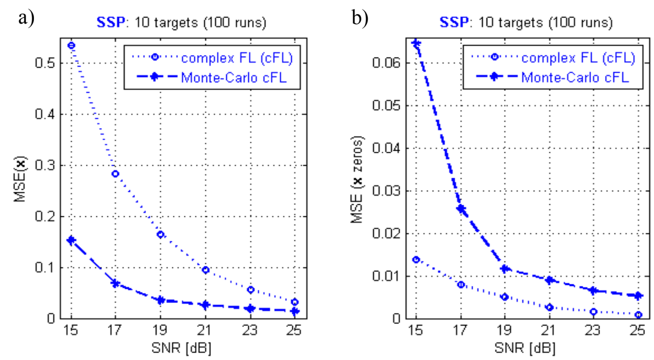

Fig.3. MSE of range profiles $\boldsymbol{x}$ estimated by existing SSP (cFL from [19]) and its MC version (Table 1) for: a) all elements and b) zeros only, at different SNR from 100 noise runs.

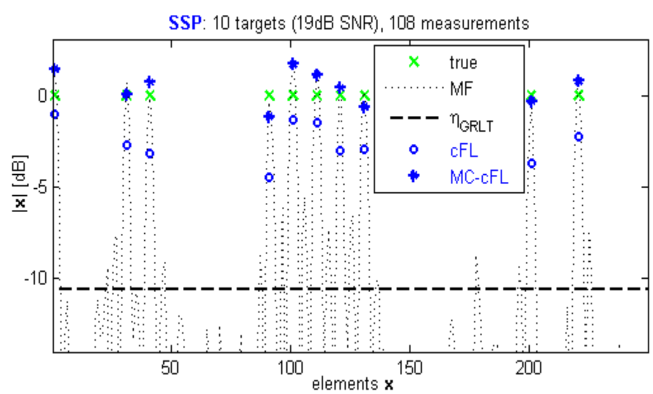

Fig.4. Range profile $\boldsymbol{x}$ estimated from a single run by MF, existing SSP (cFL from [19]) and its MC version (Table 1). The threshold η is computed at $P_{\mathrm{fa}} = 0.001$ in all runs.

For further clarity, MF and the GLRT threshold together with both SSP estimates are shown in a single run in Fig.4. Traditional radar processing (after radar detection) would give the whole MF response above the detection threshold. Since SSP is model based, it produces only points as it knows the point-spread functions (given by $\boldsymbol{A}$). Thus, SSP can replace not only MF followed by detection (Fig.1.) but even more, as its outcomes are comparable with the traditional outcomes from (sub)clustering, i.e. a radar-processing step even further than detection.

4. CONCLUSIONS

Practical SSP is designed to merge tasks of matched filtering and radar detection together as it refines their performance. This SSP promotes sparsity by applying radar detection, and also supports realistic radar cases by using MC methods. Practical MC-SSP promotes the sparsity via an MC-sampling strategy based on a detection test what also improves its convergence and computational efficiency.

The detection performance with a simple SSP threshold indicates a good start but needs fine tuning. The MSE performance of MC-SSP is comparable with existing SSP.

In ongoing work, the SSP is further being investigated and tuned, and accordingly, the detection performance metrics are being quantified. Furthermore, optimal detection strategies, priors on radar signals and their data acquisition, distributions of noise or clutter, grid matching and tracking are being embedded in this MC-SSP framework.

4.1 Future Work

Optimal detection strategies at a fixed P_{fa}, are being explored within the SSP framework so that the performance metrics can be quantified (e.g. [25]). Not only SNR and a fixed P_{fa} are relevant but possibly also other (known) SSP parameters.

Freedom in estimation (and observation) grids is being employed in CS radar (e.g. [14]). The estimation is preferred in a stochastic adaptive continuous grid. The grid design is also being studied from the Bayesian perspective ([16]), and within the scope of information geometry (e.g. [5]).

Subsequently, MC-SSP will be further compared with the closed-form SSP, especially, regarding the detection performance and moreover, the computational efficiency with the optimal sampling strategies. Finally, the MC-SSP performance will also be tested in non-Gaussian cases as MC-SSP shall support any noise or clutter and moreover, shall also employ not only sparsity but also other prior knowledge about radar signals and their data acquisition.

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