

JOINT PRECODING OVER A MASTER-SLAVE COORDINATION LINK

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ABSTRACT

This paper considers the problem of transmitter (TX) cooperation with distributed channel state information (CSI), where two or more transmitters seek to jointly precode messages while communicating over a rate-limited coordination link. Specifically we address a so-called *master-slave* scenario where one master (M-) TX is endowed with perfect CSI while K slave (S-) TXs have zero prior CSI information. We are interested in possible strategies for how the M-TX may efficiently guide the S-TXs over the coordination links so as to maximize the network's figure of merit. Strategies related to communicating quantized CSI or quantized precoding decisions are described and compared. Optimal and sub-optimal low complexity approaches are shown, exhibiting gains over conventional methods.

Index Terms— distributed CSIT, master-slave, network MIMO, precoding

1. INTRODUCTION

Transmitter cooperation is considered a promising tool for dealing with interference in wireless networks with an aggressive reuse policy for spectral resources. Cooperation is meant here as the joint optimization of certain transmission parameters, where such optimization can be carried out over several independent domains such as power control, user selection in time/frequency, antenna selection or beam/precoder design [1]. Cooperative multi-antenna precoder design itself has been the focus of a large body of literature, highlighting its potential benefit in harnessing interference limitations in next generation cellular networks [2] as well as some of its shortcomings [3].

Although transmitter cooperation comes in many flavors, a recurrent assumption behind proposed methods lies in the need for cooperating devices to (i) acquire, and (ii) share information pertaining to the propagation channel toward the multiple receivers. This holds true for instance for coordinated beamforming methods [4] [5] [6] and, to an even greater extent, for network-MIMO (Joint Processing CoMP in the LTE terminology) [2] [7] [8]. As feedback and exchange of CSI come at a price in terms of signaling overhead, a substantial effort has recently been aimed at developing techniques

which can reap the benefits of cooperation while living with a coarser channel representation. Within such efforts, two well distinct research directions emerge. In the first, emphasis is placed on limiting the rate of the over-the-air feedback link between RXs back to TXs, with special relevance to FDD settings [9] [10]. These studies assume an identical (albeit imperfect) CSI is available across all TXs and are rooted in classical limited feedback MIMO design [11] [9]. In the second direction, attention is brought on the fact that TXs have limited opportunities to exchange CSI, hence must solve what is fundamentally a distributed precoding problem based on *local* CSI only [12] [13].

In this paper, we address the second setting. We formulate a distributed precoding problem whereby multiple transmitters seek to coordinate their precoding decisions by communicating over a signaling link with a specified limited rate R . Unlike previous work, we assume the coordination link can be used just once so as to place a limit on the signaling overhead, akin to a realistic constraint on so-called X2 interface links in cellular networks [14]. This model prevents the use of certain message-passing iterative approaches [15] [16]. It is also assumed that transmitters cannot communicate on other independent radio channels, thereby preventing the use of so called implicit coordination techniques in, e.g., [17] [18].

As we focus on the limited coordination rate aspects and *asymmetric* information structures, we assume an extreme case where perfect over-the-air CSIT feedback is available at at least one of the transmitters while others have no prior CSI on their own. Although this model is justified in practical settings where, e.g., a base station cooperates with surrounding relays to serve cell users, it has received little attention before. In such a master-slave scenario, unique questions arise as to (i) how can the master TX (M-TX) and slave TXs (S-TXs) coordinate precoding decisions? and importantly (ii) what is the nature of information which should be conveyed over the coordination link? CSI or precoder related?

We provide initial answer to these questions. More specifically our contributions are as follows: We formulate strategies for finite rate coordination over the master-slave MIMO channel with 1 master and K slaves. We start with $K = 1$ and extend the results later. Intuitively, coordination signals are of a quantization nature where the quantity subject to quan-

tizing can be CSI, precoders or a combination of these. We describe naive (matching current practice) and optimal strategies for both cases and establish a simple equivalence result. Finally we propose a sub-optimal yet low complexity algorithm which is shown to compare favorably with respect to optimal strategies and has large gains over conventional approaches.

2. SYSTEM MODEL AND PROBLEM DESCRIPTION

We consider transmitter cooperation in the form of a two TXs network MIMO setup, where TX1 acts as the master and TX2 as the slave. Both M-TX and S-TX jointly serve L receivers. Each TX is equipped with M transmit antennas while each RX j , $j = 1, \dots, L$ is equipped with N receive antennas. Let $\mathbf{x}_i \in \mathbb{C}^{M \times 1}$, $i = 1, 2$ be the signal transmitted by M-TX and S-TX respectively. Let the received signal at RX j be denoted as $\mathbf{y}_j \in \mathbb{C}^{N \times 1}$. The propagation channel between RX j and TX i is denoted as $\mathbf{H}_{ji} \in \mathbb{C}^{N \times M}$. The system model is

$$\mathbf{y}_j = \sum_{i=1}^2 \mathbf{H}_{ji} \mathbf{x}_i + \mathbf{n}_j. \quad (1)$$

$\mathbf{h}_{ji} = \text{vec}(\mathbf{H}_{ji}) \in \mathcal{CN}(\mathbf{0}, \mathbf{C}_{\mathbf{h}_{ji}})$ and $\mathbf{n}_j \in \mathcal{CN}(\mathbf{0}, \mathbf{I})$ represents the additive noise vector. The covariance matrix $\mathbf{C}_{\mathbf{h}_{ji}} = \mathbb{E}\{\mathbf{h}_{ji} \mathbf{h}_{ji}^H\}$ is positive semi-definite. The channel matrix for RX j is defined as $\mathbf{H}_j = [\mathbf{H}_{j1}, \mathbf{H}_{j2}] \in \mathbb{C}^{N \times 2M}$. Throughout this paper, we assume that data symbols are known at M-TX and S-TX hence the coordination link is dedicated to only convey CSI dependent information. Let $\mathbf{s}_j \in \mathbb{C}^{N \times 1}$ denote the data symbols intended for RX j and $\mathbf{s} = [\mathbf{s}_1^T, \dots, \mathbf{s}_L^T]^T \in \mathbb{C}^{NL \times 1}$. The data symbol vector \mathbf{s} has i.i.d entry $s_k : \sigma_{s_k}^2 = 1$. Consider the precoding matrix applied on TX i as $\mathbf{W}_i \in \mathbb{C}^{M \times NL}$, the signal transmitted by TX i is

$$\mathbf{x}_i = \mathbf{W}_i \mathbf{s}. \quad (2)$$

TX i is subjected to an individual power constraint of P_i , that is, $\mathbb{E}\{\|\mathbf{x}_i\|^2\} = \|\mathbf{W}_i\|_F^2 \leq P_i$. $\mathbf{H} = [\mathbf{H}_1^T \dots \mathbf{H}_L^T]^T \in \mathbb{C}^{NL \times 2M}$ denotes channel matrix and $\mathbf{W} = [\mathbf{W}_1^T \dots \mathbf{W}_L^T]^T \in \mathbb{C}^{2M \times NL}$ denotes the concatenated precoding matrix. Each RX j is equipped with a linear MMSE receive filter \mathbf{T}_j^H . This receive filter is calculated independently at each RX based on perfect CSI. The MMSE receive filter is

$$\mathbf{T}_j^H = \delta_j^H \mathbf{W}^H \mathbf{H}_j^H (\mathbf{H}_j \mathbf{W} \mathbf{W}^H \mathbf{H}_j^H + \mathbf{I})^{-1} \quad (3)$$

The mean square error (MSE) matrix for RX j is

$$\mathbf{E}_j = \delta_j^H (\mathbf{W}^H \mathbf{H}_j^H \mathbf{H}_j \mathbf{W} + \mathbf{I})^{-1} \delta_j \quad (4)$$

where $\delta_j^H = [\mathbf{0}, \dots, \mathbf{I}_N, \dots, \mathbf{0}] \in \mathbb{C}^{N \times NL}$, is a selection matrix with \mathbf{I}_N at the j th block position and zero matrices elsewhere.

2.1. Master-slave coordination

We assume that M-TX has perfect CSI while S-TX has no CSI whatsoever. Extensions to the case where slaves have prior information are left out due to space limitations and are addressed elsewhere. We consider the existence of a coordination link between M-TX and S-TX with a rate limited to R bits. Only a single use of the coordination link is allowed, by which M-TX will inform S-TX about the precoding strategy it should adopt. This setup is illustrated in Fig.1 with $L = 3$. Throughout this paper, we are interested in finding a linear

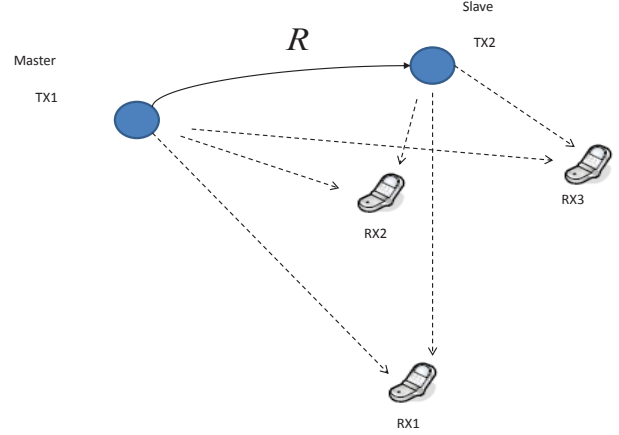


Fig. 1. Master-slave coordination with a M-TX and a S-TX jointly serving 3 RXs.

precoding matrix at each TX such that the system sum rate will be maximized to the extent of what the limited coordination rate allows. Note that the sum rate can be conveniently written based on MSE matrices by [19]:

$$f_{SR}(\mathbf{H}, \mathbf{W}) = f_{SR}(\mathbf{H}, \mathbf{W}_1, \mathbf{W}_2) = \sum_{j=1}^L \log \det(\mathbf{E}_j^{-1}) \quad (5)$$

3. COORDINATION STRATEGIES IN MASTER-SLAVE MODEL

Since the S-TX fully relies on M-TX for any channel dependent information, the strategies for coordination are limited. In this paper we consider the options where the coordination link is used to carry (i) quantized CSI, or (ii) quantized precoders.

3.1. M-TX sends precoder data

Let \mathcal{C}^{prec} denote a codebook for the precoder decision sent from M-TX to S-TX. Since the signaling rate is limited to R bits, the cardinality for \mathcal{C}^{prec} is $|\mathcal{C}^{prec}| = 2^R$. Note that the codebook design could in principle be optimized based on the sum rate maximizing precoder distribution.

3.1.1. Optimally quantized precoder

Under the finite coordination rate constraint, the sum rate optimal linear precoding strategy at each TX is obtained from the following hybrid continuous-discrete optimization problem

$$\begin{cases} \max_{\mathbf{W}_1, \mathbf{W}_2} & f_{SR}(\mathbf{H}, \mathbf{W}_1, \mathbf{W}_2) \\ \text{st :} & \mathbf{W}_1 \in \mathbb{C}^{M \times NL} \\ & \mathbf{W}_2 \in \mathcal{C}^{prec}, |\mathcal{C}^{prec}| = 2^R \\ & \|\mathbf{W}_i\|_F^2 \leq P_i, i = 1, 2 \end{cases} \quad (6)$$

where the M-TX proceeds with sending to S-TX the index of the optimal codeword for \mathbf{W}_2 in (6).

3.1.2. Naive quantized precoder

An intuitive yet naive algorithm (referred to later as naive quantized precoder) is to let M-TX compute in continuous domain the sum rate optimal precoders and send the quantized version of the obtained \mathbf{W}_2 to S-TX.

3.2. M-TX sends CSI data

For this coordination strategy, let $\mathcal{C}^{CSI} = \{q_1, \dots, q_{2^R}\}$ denote the codebook for normalized channel matrix quantization with $|\mathcal{C}^{CSI}| = 2^R$.

3.2.1. Optimal quantized CSI

Under the finite coordination rate constraint, the optimal linear precoding will be the precoder pair $(\mathbf{W}_1, \mathbf{W}_2)$ where \mathbf{W}_2 is obtained with each possible quantized channel and \mathbf{W}_1 is the one that maximize the sum rate with aforementioned \mathbf{W}_2 :

$$\begin{cases} \max_{\mathbf{W}_1, \mathbf{W}_2} & f_{SR}(\mathbf{H}, \mathbf{W}_1, \mathbf{W}_2) \\ \text{st :} & \mathbf{W}_1 \in \mathbb{C}^{M \times NL} \\ & \mathbf{W}_2 \in \mathcal{D} = \{p_1, \dots, p_{2^R}\} \\ & \text{where } p_k = \mathbf{B}_2 \arg \max_{\mathbf{W}} f_{SR}(q_k, \mathbf{W}), k = 1, \dots, 2^R \\ & \|\mathbf{W}_i\|_F^2 \leq P_i, i = 1, 2 \end{cases} \quad (7)$$

where $\mathbf{B}_2 = [\mathbf{0}, \mathbf{I}_M] \in \mathbb{C}^{M \times 2M}$ is a selection matrix satisfies $\mathbf{B}_2 \mathbf{W} = \mathbf{W}_2$.

3.2.2. Naive quantized CSI

A common practice yet suboptimal algorithm (referred to later as naive quantized CSI) consists in quantizing \mathbf{H} at the M-TX and sending the codeword index to S-TX. The S-TX will calculate its precoder according to the quantized CSI ignoring the fact that M-TX will calculate its precoder with a more precise CSI. Just like the naive quantized precoder, the problem of this approach is that it ignores the asymmetry of CSI knowledge at the M-TX and S-TX.

3.3. An equivalence result

An interesting question is whether there is a fundamental advantage in signaling precoder-based vs. CSI-based data. The following provides an insight into the problem:

Proposition 1. *Given a codebook \mathcal{C}^{CSI} , There always exists a codebook \mathcal{C}^{prec} such that the optimization problem (7) and (6) attain the same optimum.*

Proof. This can be obtained by selecting a codebook \mathcal{C}^{prec} that satisfies $\mathcal{C}^{prec} = \mathcal{D}$, where \mathcal{D} is defined in problem (7). \square

Hence when the codebook is properly designed, the optimal coordination strategies involving a communication of quantized precoders or CSI have equivalent performance.

4. A GREEDY APPROACH FOR LOW COMPLEXITY COORDINATED PRECODING

According to proposition 1, without loss of generality, we will focus on a coordination strategy where M-TX sends the quantized precoder data, which is described by problem (6). This problem is difficult because it's a non-convex optimization problem over a non-convex set. Additionally, the complexity is prohibitive when R increases.

We propose an alternating maximization algorithm for problem (6). The optimization is decomposed into 2 phases. In phase 1, M-TX solves \mathbf{W}_2 based on a given \mathbf{W}_1^*

$$\begin{cases} \max_{\mathbf{W}_2} & f_{SR}(\mathbf{H}, \mathbf{W}_1^*, \mathbf{W}_2) \\ \text{st :} & \mathbf{W}_2 \in \mathcal{C}^{prec}, |\mathcal{C}^{prec}| = 2^R \\ & \|\mathbf{W}_2\|_F^2 \leq P_2 \end{cases} \quad (8)$$

In phase 2, M-TX solves for \mathbf{W}_1 in continuous space with given \mathbf{W}_2^*

$$\begin{cases} \max_{\mathbf{W}_1} & f_{SR}(\mathbf{H}, \mathbf{W}_1, \mathbf{W}_2^*) \\ \text{st :} & \mathbf{W}_1 \in \mathbb{C}^{M \times NL} \\ & \|\mathbf{W}_1\|_F^2 \leq P_1 \end{cases} \quad (9)$$

According to [19], the optimal precoder \mathbf{W} maximizing the sum rate is the same as the precoder \mathbf{W} derived by a weighted sum MMSE minimization problem. Here we extend the result to the case of decentralized precoders:

Proposition 2. *The optimization problem (9) has the same KKT point as the optimization problem*

$$\begin{cases} \min_{\mathbf{W}_1} & \sum_{j=1}^L \text{tr}(\mathbf{M}_j \mathbf{E}_j) \\ \text{st :} & \|\mathbf{W}_1\|_F^2 \leq P_1 \end{cases} \quad (10)$$

while $\mathbf{W}_2 = \mathbf{W}_2^*$ and the weight matrix for TXj satisfies

$$\mathbf{M}_j = (\mathbf{E}_j)^{-1} \quad (11)$$

Proof. See Appendix. \square

4.1. Arbitrary number of Slave-TX

For master-slave scenario with arbitrary K S-TX, the proposed algorithm can be easily generalized. Details are omitted here. Problem (8) becomes a search over a large dimensional discrete space, where more efficient algorithms (e.g., branch-and-bound or genetic algorithm) will be considered.

5. NUMERICAL PERFORMANCE ANALYSIS

In this section the sum rate performance is evaluated for different settings using Mont-Carlo simulations. In all simulations $K = 2$ and $L = 3$, $M = N = 1$. R_2, R_3 are the rates for link between M-TX (TX1) and S-TXs (TX2, TX3). Each entry of \mathbf{H}_{ji} is generated independently by a complex Gaussian random variable $u \in \mathcal{CN}(0, 1)$ multiplying a path loss component $v_{ji} = \alpha d_{ji}^{-\varepsilon}$, where d_{ji} is the distance between RX j and TX i , α is the cell edge SNR, $\varepsilon = 2$. The TXs and RXs positions are generated uniformly in a circle cell with radius equal to 1km. The codebook are generated using random vector quantization (RVQ). The per-TX power constrain is $P_i = 1\text{Watt}$, $i = 1, \dots, 3$.

Fig.2 compares the sum rate for different strategies averaged over 100 realizations. Optimally quantized precoder is solved by complete discrete set search and then optimize in continuous space. The greedy quantized precoder outperforms the naive algorithms, as naive algorithms are unable to cope with the asymmetric nature of the CSIT. Our greedy algorithm perform close to the optimal quantized precoder, while having very clear advantage in complexity against it. Fig.3 confirms the improvement in performance as the signaling rate is increased.

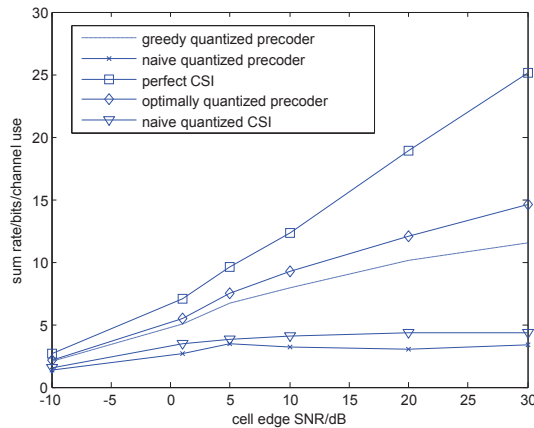


Fig. 2. Sum rate performance of 1 M-TX, 2 S-TX, and $R_2 = R_3 = 3\text{bits}$, the naive (conventional) techniques are not robust with respect to asymmetric CSI setting.

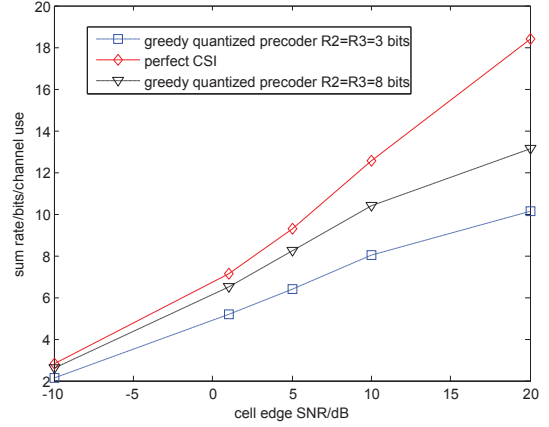


Fig. 3. Sum rate performance of 1 M-TX, 2 S-TX, various signaling rate.

6. CONCLUSION

We study coordinated precoding for the network MIMO channel with a master-slave CSI configuration. We compare signaling strategies based on exchange of CSI vs. precoder decisions. We exhibit optimal and suboptimal strategies with good performance complexity trade-off.

7. APPENDIX

Proof for Proposition 2: Let $[\mathbf{W}]_{nm}$ denote the n th row, m th column entry of the matrix. The lagrangian dual function for optimization problem (9) is $f(\mathbf{W}_1) = \sum_{j=1}^L -\log \det(\mathbf{E}_j) + \lambda(\text{tr}(\mathbf{W}_1 \mathbf{W}_1^H) - P_1)$.

Since $-\nabla_{[\mathbf{W}_1]_{nm}} \log \det(\mathbf{E}_j) = -\text{tr} \left(\mathbf{E}_j^{-1} \frac{\partial \mathbf{E}_j}{\partial [\mathbf{W}_1]_{nm}} \right)$

therefore, $[\nabla_{\mathbf{W}_1} f(\mathbf{W}_1)]_{nm} = \nabla_{[\mathbf{W}_1]_{nm}} f(\mathbf{W}_1)$

$= \sum_{j=1}^L \text{tr} \left(\mathbf{E}_j^{-1} \frac{\partial \mathbf{E}_j}{\partial [\mathbf{W}_1]_{nm}} \right) + \lambda \text{tr}(\mathbf{W}_1 \mathbf{J}_{nm})$, where \mathbf{J}_{nm} is a single entry matrix with 1 at (n, m) and 0 elsewhere.

The lagrangian dual function for optimization problem (10) with all slave precoders fixed is $g(\mathbf{W}_1) = \sum_{j=1}^L \text{tr}(\mathbf{M}_j \mathbf{E}_j) + \mu(\text{tr}(\mathbf{W}_1 \mathbf{W}_1^H) - P_1)$.

With a constant weight matrix \mathbf{M}_j for RX j , $j = 1, \dots, L$,

$[\nabla_{\mathbf{W}_1} \text{tr}(\mathbf{M}_j \mathbf{E}_j)]_{nm} = \text{tr} \left[\mathbf{M}_j \frac{\partial \mathbf{E}_j}{\partial [\mathbf{W}_1]_{nm}} \right]$.

Hence, $[\nabla_{\mathbf{W}_1} g(\mathbf{W}_1)]_{nm} = \nabla_{[\mathbf{W}_1]_{nm}} g(\mathbf{W}_1)$

$= \sum_{j=1}^L \text{tr} \left[\mathbf{M}_j \frac{\partial \mathbf{E}_j}{\partial [\mathbf{W}_1]_{nm}} \right] + \mu \text{tr}(\mathbf{W}_1 \mathbf{J}_{nm})$.

Comparing $[\nabla_{\mathbf{W}_1} f(\mathbf{W}_1)]_{nm}$ and $[\nabla_{\mathbf{W}_1} g(\mathbf{W}_1)]_{nm}$, it is clear that the two problem have the same KKT point if $\mathbf{M}_j = \mathbf{E}_j^{-1}$.

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