WEIGHTED SUM RATE OPTIMIZATION FOR MULTICELL MIMO SYSTEMS WITH HARDWARE-IMPAIRED TRANSCEIVERS

Rasmus Brandt[⋆] *Emil Björnson*[⋆,†] *Mats Bengtsson*[⋆]

[⋆]ACCESS Linnaeus Centre, Dept. of Signal Processing, KTH Royal Institute of Technology, Sweden
[†]Alcatel-Lucent Chair on Flexible Radio, SUPÉLEC, France

ABSTRACT

Physical transceivers exhibit distortions from hardware impairments, of which traces remain even after compensation and calibration. Multicell MIMO coordinated beamforming methods that ignore these residual impairments may suffer from severely degraded performance. In this work, we consider a general model for the aggregate effect of the residual hardware impairments, and propose an iterative algorithm for finding locally optimal points to a weighted sum rate optimization problem. The importance of accounting for the residual hardware impairments is verified by numerical simulation, and a substantial gain over traditional time-division multiple access with impairments-aware resource allocation is observed.

Index Terms— Weighted sum rate optimization, transceiver hardware impairments, interference alignment.

1. INTRODUCTION

For wireless networks with multiple antennas at the transmitters and receivers, spatial selectivity can be exploited to serve several users simultaneously. Spectral efficiency can then be improved over traditional orthogonal multiple access schemes, such as time-division multiple access (TDMA) [1]. In particular, for multicell MIMO networks, the concept of interference alignment (IA) [2] has lately gained traction. IA is able to completely remove the inter-user interference, by restricting the interference to a lower-dimensional subspace at all receivers simultaneously, and then applying zero-forcing filters at the receivers. In terms of sum rate, it is suitable to apply IA at high SNRs, when inter-user interference is the main performance-limiting factor [3,4]. For practical networks, there are typically other important performance-limiting factors as well, such as low to intermediate SNR [5], uncoordinated interferers [6], imperfect channel state information [7], and imperfect hardware [8]. In this work, we focus on the latter and perform resource allocation and coordinated beamforming for any SNR by finding a local optimum to a weighted sum rate optimization problem.

Any physical wireless transceiver will have hardware impairments, such as phase noise, I/Q imbalance, power amplifier non-linearities, and sampling-rate and carrier frequency offsets [8]. For each of these impairments, compensation schemes are typically applied to limit their negative effect. However, in practice the compensation and calibration will not be perfect, and distortion noises from *residual hardware impairments* will still remain [9]. These distortions may have a large impact on end-to-end performance [10, 11], and should therefore be taken into account when performing coordinated beamforming and resource allocation in practice [11].

1.1. Previous Work and Contributions

A large amount of previous work has focused on individual hardware impairments and their respective compensation schemes; see [8] and references therein. In [9], the *aggregate effect* of residual hardware impairments after such compensation was studied, and a model for the residual impairments was proposed and verified through measurements. In [10], it was shown that the point-to-point MIMO capacity is *fundamentally limited* in the high-SNR regime, due to the residual impairments. However, [10] further showed that the relative gain over SISO systems with residual impairments could still be large. The impact of path loss and power budget for systems with residual impairments was studied through simulations in [12]. Generalizing the model from [9], reference [11, Ch. 4.3] also described the optimal beamforming solution for the multicell MISO downlink.

For the multicell MIMO downlink without hardware-impaired transceivers, several iterative and distributed methods for coordinated beamforming exist [3, 13, 14]. In particular, [13] stands out as a constructive way of finding a local optimum to the non-convex weighted sum rate optimization problem, by reformulating the problem as a weighted minimum mean squared error (MMSE) problem.

In this work, we devise an iterative method for finding a local optimum to the weighted sum rate problem for the multicell MIMO downlink with residual hardware impairments. This is done by applying the general residual impairments model of [11, Ch. 4.3] to the MIMO case, and extending the weighted MMSE approach of [13] to the problem with hardware-impaired transceivers. System performance is evaluated through numerical simulations, and the advantage of accounting for the residual hardware impairments is verified. Further, we note a large relative gain for the proposed method, over TDMA with hardware-impaired transceivers.

Notation: The pth row of a matrix $\mathbf{A}$ is $[\mathbf{A}]_{p,:}$, and the qth column is $[\mathbf{A}]_{:,q}$. The zero-mean circularly symmetric complex Gaussian distribution is denoted $\mathcal{CN}(\mathbf{0}, \mathbf{B})$ with covariance matrix $\mathbf{B}$. The Frobenius norm is $\|\cdot\|_{\mathrm{F}}$ and diag $(\cdot)$ creates a diagonal matrix.

2. MULTICELL MIMO WITH IMPAIRED TRANSCEIVERS

Our system model is a multicell MIMO downlink with hardware-impaired transceivers. There are K_t base stations (BSs), each serving K_c user equipments (UEs). We index the kth UE associated with the ith BS as i_k. Orthogonal frequency-division multiplexing (OFDM) is used to transform the wideband channel into a set of orthogonal narrowband channels, or subcarriers. We study the subcarriers independently, and do not include the subcarrier index for

This work was supported by the FP7 project HIATUS (FET-Open grant #265578). E. Björnson is funded by the Int'l Postdoc Grant 2012-228 from The Swedish Research Council and by ERC Starting Grant 305123 MORE.

We would like to extend our gratitude to Assoc. Prof. Johan Löfberg at Linköping Univ. for the extensive help with shortening our Yalmip run times.

$$\mathbf{Q}_{i_k} = \mathbb{E}\left(\mathbf{y}_{i_k}\mathbf{y}_{i_k}^{\mathsf{H}}\right) = \underbrace{\mathbf{H}_{i_k i}\mathbf{V}_{i_k}\mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}^{\mathsf{H}}}_{\text{useful signal}} + \underbrace{\sum_{(j,l)\neq(i,k)} \mathbf{H}_{i_k j}\mathbf{V}_{j_l}\mathbf{V}_{j_l}^{\mathsf{H}}\mathbf{H}_{i_k j}}_{\text{inter-cell and intra-cell interference}} + \underbrace{\sum_{j=1}^{K_t} \mathbf{H}_{i_k j}\mathbf{C}_j^{(t)}\mathbf{H}_{i_k j}^{\mathsf{H}}}_{\text{impact of transmitter distortions}} + \underbrace{\mathbf{C}_{i_k}^{(r)}}_{\substack{\text{receiver thermal noise}\\ \text{and distortions}}} \tag{3}$$

notational simplicity. At a given subcarrier, the received signal at UE i_k is then

$$\mathbf{y}_{i_k} = \mathbf{H}_{i_k i}\mathbf{V}_{i_k}\mathbf{x}_{i_k} + \sum_{(j,l)\neq(i,k)} \mathbf{H}_{i_k j}\mathbf{V}_{j_l}\mathbf{x}_{j_l} + \sum_{j=1}^{K_t} \mathbf{H}_{i_k j}\mathbf{z}_j^{(t)} + \mathbf{z}_{i_k}^{(r)} \tag{1}$$

where $\mathbf{H}_{i_k j} \in \mathbb{C}^{M_r \times M_t}$ is the flat fading MIMO channel[1] from BS j to UE i_k. We let the signal intended for UE i_k, $\mathbf{x}_{i_k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_d})$, be linearly precoded with a transmit filter $\mathbf{V}_{i_k} \in \mathbb{C}^{M_t \times N_d}$. The received signal in (1) contains the desired signal, inter- and intra-cell interference, and transceiver distortion noises. The terms $\mathbf{z}_j^{(t)}$ and $\mathbf{z}_{i_k}^{(r)}$ in (1) model the distortion noises from the residual hardware impairments after compensation and calibration at the transmitter and receiver, respectively. The receiver thermal noise is part of $\mathbf{z}_{i_k}^{(r)}$.

For the distortion noises, we use the model from [11, Ch. 4.3]. That is, $\mathbf{z}_i^{(t)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_i^{(t)})$ where $\mathbf{C}_i^{(t)} = \operatorname{diag}\left(c_{i,1}^{(t),2}, \ldots, c_{i,M_t}^{(t),2}\right)$. The transmitter distortion is modeled as Gaussian, since it is the sum of many residual impairments. Even if the antennas are served by different RF-chains, the distortions at the different antennas may be correlated due to the precoding [15]. Such correlations are, however, typically small [15], and we approximate them with zero for tractability. Due to its nature, being the residual of impairments after compensation and calibration for a given transmitted signal, we assume the transmit distortion noise to be independent of the transmitted signal. The *power* of the distortion noise at antenna m, $c_{i,m}^{(t),2}$, is however a function of the signal power allocated to that antenna [9]. Following [11, Ch. 4.3] we let $c_{i,m}^{(t)} = \eta\left(\sqrt{\sum_{k=1}^{K_c} \left\|[\mathbf{V}_{i_k}]_{m,:}\right\|_{\mathrm{F}}^2}\right)$, where the $\eta(\cdot)$ is a convex, nonnegative, and nondecreasing function describing how the magnitude of the signal maps to the magnitude of the transmitter distortions.

For the receiver distortion noise, the model in [11, Ch. 4.3] only considered single-antenna receivers. In order to support the MIMO case, we extend the model accordingly. Hence we assume that $\mathbf{z}_{i_k}^{(r)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{i_k}^{(r)})$ where $\mathbf{C}_{i_k}^{(r)} = \operatorname{diag}\left(c_{i_k,1}^{(r),2}, \ldots, c_{i_k,M_r}^{(r),2}\right)$. Similarly as above, we assume the distortions to be uncorrelated over antennas, and independent of the received signal. The distortion power is $c_{i_k,m}^{(r),2} = \sigma_r^2 + \nu^2\left(\sqrt{\sum_{(j,l)} \left\|[\mathbf{H}_{i_k j}\mathbf{V}_{j_l}]_{m,:}\right\|_{\mathrm{F}}^2}\right)$ where $\nu(\cdot)$ is a convex, nonnegative, and nondecreasing function describing how the magnitude of the received signal maps to the magnitude of the receiver distortions. σ_r^2 is the thermal noise power.

The impact of transceiver impairments is typically measured using the *error vector magnitude* (EVM). For our hardware impairments model, the EVM at transmitter antenna m is

$$\mathrm{EVM}_m^{(t)} \triangleq \sqrt{\frac{c_{i,m}^{(t),2}}{\sum_{k=1}^{K_c} \left\|[\mathbf{V}_{i_k}]_{m,:}\right\|_{\mathrm{F}}^2}} = \frac{\eta\left(\sqrt{\sum_{k=1}^{K_c} \left\|[\mathbf{V}_{i_k}]_{m,:}\right\|_{\mathrm{F}}^2}\right)}{\sqrt{\sum_{k=1}^{K_c} \left\|[\mathbf{V}_{i_k}]_{m,:}\right\|_{\mathrm{F}}^2}}.$$

[1] The system model can easily be extended to different number of antennas per transceiver, different number of data streams per user, etc.

The EVM at the receiver side antennas is similarly defined, with respect to the received power at that antenna. Depending on the required spectral efficiency, a typical maximum transmit-EVM range in the 3GPP LTE standard is $[0.08, 0.175]$ according to [16].

3. WEIGHTED SUM RATE OPTIMIZATION

Focusing on the effect of hardware impairments, we assume perfect channel state information at all nodes. Given the system model in (1), and assuming that the interference and distortions are treated as noise in the decoder, the achievable data rate for user i_k is

$$R_{i_k} = \log\det\left(\mathbf{I} + \mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}^{\mathsf{H}}\left(\mathbf{Q}_{i_k}^{\text{int+dist}}\right)^{-1}\mathbf{H}_{i_k i}\mathbf{V}_{i_k}\right), \tag{2}$$

where $\mathbf{Q}_{i_k}^{\text{int+dist}} = \mathbf{Q}_{i_k} - \mathbf{H}_{i_k i}\mathbf{V}_{i_k}\mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}^{\mathsf{H}}$ is the covariance matrix of interference and distortions for user i_k. The total received signal covariance matrix $\mathbf{Q}_{i_k}$ is given in (3), at the top of the page.

Our goal is then to maximize the *weighted sum rate* $R_{\mathrm{WSR}} = \sum_{(i,k)} \alpha_{i_k} R_{ik}$ of the system. The nonnegative data rate weights α_{i_k} determine the relative priorities of the users in the system level criterion, and are assumed to be given. Under a per-BS power constraint, the *impaired weighted sum rate* problem to be solved is

$$\begin{aligned} &\underset{\{\mathbf{V}_{i_k}\}}{\text{maximize}} && \sum_{(i,k)} \alpha_{i_k} R_{i_k} && \text{(ImpWSR)} \\ &\text{subject to} && \operatorname{Tr}\left(\mathbf{C}_i^{(t)}\right) + \sum_{k=1}^{K_c} \|\mathbf{V}_{i_k}\|_{\mathrm{F}}^2 \leq P_i,\ i = 1, \ldots, K_t. \end{aligned}$$

This is a non-convex problem, since (2) is non-convex in $\{\mathbf{V}_{i_k}\}$. Therefore, we only endeavour to find a *locally optimal point*. In order to do that, we first introduce the mean squared error (MSE) matrix for user i_k,

$$\begin{aligned} \mathbf{E}_{i_k} &= \mathbb{E}\left(\left(\mathbf{x}_{i_k} - \mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{y}_{i_k}\right)\left(\mathbf{x}_{i_k} - \mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{y}_{i_k}\right)^{\mathsf{H}}\right) \\ &= \mathbf{I} - \mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}\mathbf{V}_{i_k} - \mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}^{\mathsf{H}}\mathbf{U}_{i_k} + \mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{Q}_{i_k}\mathbf{U}_{i_k} \end{aligned}$$

where $\mathbf{U}_{i_k} \in \mathbb{C}^{M_r \times N_d}$ is a linear receive filter. Inspired by [13], we introduce the MSE weight matrices $\mathbf{W}_{i_k} \in \mathbb{C}^{N_d \times N_d}$ and formulate an *impaired weighted MMSE* problem:

$$\begin{aligned} &\underset{\substack{\{\mathbf{U}_{i_k}\}\\ \{\mathbf{W}_{i_k}\}\\ \{\mathbf{V}_{i_k}\}}}{\text{minimize}} && \sum_{(i,k)} \alpha_{i_k}\left(\operatorname{Tr}\left(\mathbf{W}_{i_k}\mathbf{E}_{i_k}\right) - \log\det\left(\mathbf{W}_{i_k}\right)\right) \\ &\text{subject to} && \operatorname{Tr}\left(\mathbf{C}_i^{(t)}\right) + \sum_{k=1}^{K_c} \|\mathbf{V}_{i_k}\|_{\mathrm{F}}^2 \leq P_i,\ i = 1, \ldots, K_t. \end{aligned} \quad \text{(ImpWMMSE)}$$

Proposition 1. *The optimization problems in* (ImpWSR) *and* (ImpWMMSE) *have the same global solutions* $\{\mathbf{V}_{i_k}^{\star}\}$.

Proof. Similarly as in the proof of Theorem 1 in [13], this follows by substituting the optimality conditions for $\{\mathbf{U}_{i_k}\}$ and $\{\mathbf{W}_{i_k}\}$ (derived in Sec. 3.1) into (ImpWMMSE). The remaining optimization problem (w.r.t. $\{\mathbf{V}_{i_k}\}$) can then be identified as (ImpWSR). $\square$

$$\begin{aligned} &\underset{\{\mathbf{V}_{i_k}\}}{\text{minimize}} && \sum_{i=1}^{K_t}\left[\operatorname{Tr}\left(\mathbf{T}_i\mathbf{C}_i^{(t)}\right)+\sum_{k=1}^{K_c}\left[\operatorname{Tr}\left(\mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{T}_i\mathbf{V}_{i_k}\right)-2\alpha_{i_k}\operatorname{Re}\left(\operatorname{Tr}\left(\mathbf{W}_{i_k}\mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}\mathbf{V}_{i_k}\right)\right)+\alpha_{i_k}\operatorname{Tr}\left(\mathbf{U}_{i_k}\mathbf{W}_{i_k}\mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{C}_{i_k}^{(r)}\right)\right]\right] \\ &\text{subject to} && \operatorname{Tr}\left(\mathbf{C}_i^{(t)}\right)+\sum_{k=1}^{K_c}\|\mathbf{V}_{i_k}\|_{\mathsf{F}}^2\leq P_i,\ i=1,\ldots,K_t. \end{aligned} \quad \text{(ImpWMMSE-BS)}$$

3.1. Alternating Minimization

The problem in (ImpWMMSE) is non-convex, but we can apply alternating minimization [17] to it over the three blocks of variables. First, by fixing $\{\mathbf{W}_{i_k},\mathbf{V}_{i_k}\}$ and optimizing over $\{\mathbf{U}_{i_k}\}$, the problem decouples over the users and we get $\mathbf{U}_{i_k}^{\star}=\mathbf{Q}_{i_k}^{-1}\mathbf{H}_{i_k i}\mathbf{V}_{i_k}$. This is the well-known MMSE receiver.

Similarly, fixing $\{\mathbf{U}_{i_k},\mathbf{V}_{i_k}\}$, the problem again decouples over the users and the optimal MSE weights are

$$\begin{aligned}\mathbf{W}_{i_k}^{\star}&=\underset{\mathbf{W}_{i_k}}{\arg\min}\left(\operatorname{Tr}\left(\mathbf{W}_{i_k}\mathbf{E}_{i_k}\right)-\log\det\left(\mathbf{W}_{i_k}\right)\right)=\mathbf{E}_{i_k}^{-1}\\ &=\mathbf{I}+\mathbf{V}_{i_k}^{\mathsf{H}}\mathbf{H}_{i_k i}^{\mathsf{H}}\left(\mathbf{Q}_{i_k}^{\text{int+dist}}\right)^{-1}\mathbf{H}_{i_k i}\mathbf{V}_{i_k}\end{aligned}$$

where the last equality comes from plugging in $\mathbf{U}_{i_k}^{\star}$ and applying the matrix inversion lemma. Notice that $\log\det\left(\mathbf{W}_{i_k}^{\star}\right)=R_{i_k}$, and hence the weight $\mathbf{W}_{i_k}^{\star}$ describes what data rate user i_k can achieve under the current interference and distortion conditions.

Finally, we fix $\{\mathbf{U}_{i_k},\mathbf{W}_{i_k}\}$ and optimize over $\{\mathbf{V}_{i_k}\}$. By dropping terms only containing $\mathbf{W}_{i_k}$ and rearranging the remaining terms using properties of the trace, the problem that should be solved is displayed in (ImpWMMSE-BS), at the top of the page. The matrix $\mathbf{T}_i=\sum_{(j,l)}\alpha_{j_l}\mathbf{H}_{j_l i}^{\mathsf{H}}\mathbf{U}_{j_l}\mathbf{W}_{j_l}\mathbf{U}_{j_l}^{\mathsf{H}}\mathbf{H}_{j_l i}$ is a virtual uplink signal and interference covariance matrix for BS i. Note that $\mathbf{C}_i^{(t)}$ and $\mathbf{C}_{i_k}^{(r)}$ are functions of $\{\mathbf{V}_{i_k}\}$, as described in Sec. 2. Since $\eta^2(\cdot)$ and $\nu^2(\cdot)$ are convex [18, Ch. 3.2], the problem is convex and can be solved by, for example, general-purpose interior-point methods [19]. When the optimal $\{\mathbf{V}_{i_k}^{\star}\}$ have been found, a new alternating minimization iteration is started by again optimizing over the $\{\mathbf{U}_{i_k}\}$, given the new $\{\mathbf{V}_{i_k}\}$. These iterations then continue until convergence.

Proposition 2. *The alternating minimization (AM) of* (ImpWMMSE) *monotonically converges and every limit point of the AM iterates is a stationary point of* (ImpWSR).

Proof. The AM objective values are monotonically nonincreasing, and the objective function can be lower-bounded. If any of the $c_{i,m}^{(t),2}$ or $c_{i_k,n}^{(r),2}$ are non-differentiable w.r.t. $\{\mathbf{V}_{i_k}\}$, introduce auxiliary optimization variables $d_{i,m}^{(t)}$ and $d_{i_k,n}^{(r)}$ to (ImpWMMSE). Replace $c_{i,m}^{(t),2}\to d_{i,m}^{(t)}$, $c_{i_k,n}^{(r),2}\to d_{i_k,n}^{(r)}$ and add inequality constraints

$$\begin{aligned} c_{i,m}^{(t),2} &= \eta^2\left(\sqrt{\sum_k\left\|\left[\mathbf{V}_{i_k}\right]_{m,:}\right\|_{\mathsf{F}}^2}\right)\leq d_{i,m}^{(t)}, \quad \forall\, i,m\\ c_{i_k,n}^{(r),2} &= \sigma_r^2+\nu^2\left(\sqrt{\sum_{(j,l)}\left\|\left[\mathbf{H}_{i_k j}\mathbf{V}_{j_l}\right]_{n,:}\right\|_{\mathsf{F}}^2}\right)\leq d_{i_k,n}^{(r)},\ \forall\, i_k,n\end{aligned}$$

to get the squared functions on epigraph form. For this equivalent problem, the objective function is continuously differentiable and the extended feasible set is convex. Then, since the subproblem for $\{\mathbf{U}_{i_k}\}$ is strictly convex, [17, Prop. 5] gives that every limit point of the AM iterates is a stationary point of (ImpWMMSE). That this is also a stationary point of (ImpWSR) follows directly from the proof of Theorem 3 in [13]. □

The alternating minimization is distributed over the UEs, but (ImpWMMSE-BS) only decouples over the BSs if the term $\sum_{(i,k)}\operatorname{Tr}\left(\mathbf{U}_{i_k}\mathbf{W}_{i_k}\mathbf{U}_{i_k}^{\mathsf{H}}\mathbf{C}_{i_k}^{(r)}\right)$ decomposes over the BSs. In the next section we investigate one such particular case.

3.2. Distributed Solution for Constant-EVM Impairments

We now exemplify how a distributed, semi-closed form, precoder solution can be achieved under a certain impairment model: constant-EVM impairments. In particular, we let $\eta(x)=\kappa_t x$ and $\nu(x)=\kappa_r x$; thus, we have that $\text{EVM}_m^{(t)}=\kappa_t$ and $\text{EVM}_n^{(r)}=\kappa_r$ for all users, all receive antennas n, and all transmit antennas m. Consequently, $\mathbf{C}_i^{(t)}=\kappa_t^2\sum_{k=1}^{K_c}\operatorname{diag}\left(\mathbf{V}_{i_k}\mathbf{V}_{i_k}^{\mathsf{H}}\right)$ and $\mathbf{C}_{i_k}^{(r)}=\sigma_r^2\mathbf{I}+\kappa_r^2\sum_{(j,l)}\operatorname{diag}\left(\mathbf{H}_{i_k j}\mathbf{V}_{j_l}\mathbf{V}_{j_l}^{\mathsf{H}}\mathbf{H}_{i_k j}^{\mathsf{H}}\right)$, where $\operatorname{diag}(\cdot)$ only retains the diagonal elements. Due to the affine structure of $\mathbf{C}_{i_k}^{(r)}$, the terms in (ImpWMMSE-BS) can be rearranged so the problem decomposes into one subproblem per BS. With $\widetilde{\mathbf{T}}_i=\sum_{(j,l)}\alpha_{i_k}\mathbf{H}_{j_l i}^{\mathsf{H}}\operatorname{diag}\left(\mathbf{U}_{j_l}\mathbf{W}_{j_l}\mathbf{U}_{j_l}^{\mathsf{H}}\right)\mathbf{H}_{j_l i}$, the solution for UE i_k can be locally calculated at BS i as

$$\mathbf{V}_{i_k}^{\star}=\alpha_{i_k}\left(\mathbf{T}_i+\kappa_t^2\operatorname{diag}\left(\mathbf{T}_i\right)+\kappa_r^2\widetilde{\mathbf{T}}_i+\mu_i^{\star}\mathbf{I}\right)^{-1}\mathbf{H}_{i_k i}^{\mathsf{H}}\mathbf{U}_{i_k}\mathbf{W}_{i_k}.$$

The term $\mu_i^{\star}\geq 0$ is a Lagrange multiplier, which can be found by bisection such that $\left(1+\kappa_t^2\right)\sum_{k=1}^{K_c}\left\|\mathbf{V}_{i_k}^{\star}\right\|_{\mathsf{F}}^2\leq P_i$ is satisfied.

4. PERFORMANCE EVALUATION

We investigate the performance of the proposed method by means of numerical simulation. For this purpose, we let the impairment functions be

$$\eta(x)=\kappa_t x\left(1+\left(\frac{x}{\kappa_t^{(\text{NL})}}\right)^2\right),\quad \nu(x)=\kappa_r x, \tag{4}$$

which means that the receivers have a constant EVM of κ_r and the transmitters have a third order non-linearity due to the power amplifier. For low transmit powers, the EVM at the transmitters is κ_t and, due to the non-linearity, it doubles at a transmit power of $\kappa_t^{(\text{NL}),2}$. Notice that for this choice of impairment functions, $c_{i,m}^{(t),2}$ and $c_{i_k,m}^{(r),2}$ are differentiable w.r.t. $\{\mathbf{V}_{i_k}\}$. When needed, in order to solve to solve (ImpWMMSE-BS), we use the modeling framework Yalmip [19] with the Gurobi solver [20]. In the spirit of reproducible research, the entire simulation source code is available for download at [21].

We employ a $K_t=3$ simulation scenario where each BS has $M_t=4$ antennas and each UE has $M_r=2$ antennas. The BSs are placed 500 m apart, at the corners of an equilateral triangle. The triangle is divided into three cells, each containing $K_c=2$ uniformly dropped users that are served by the closest BS. For a distance d (in meters) between BS and UE, the path loss is described by $\text{PL}_{\text{dB}}=15.3+37.6\log_{10}(d)$. The UEs are never closer than 35 m to the BS. We assume all users to be indoors with a penetration

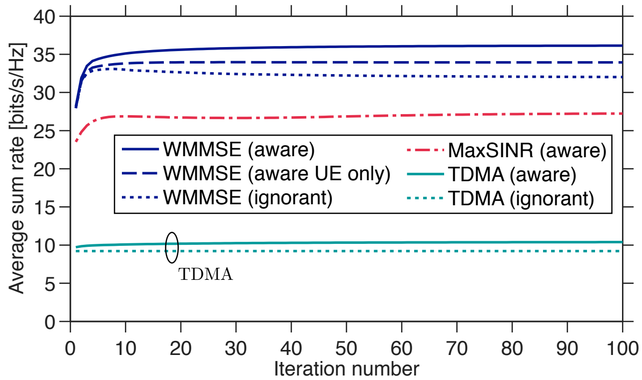

Fig. 1. Sum rate evolution (one realization) for $P_t = 18.2$ dBm, $\kappa_t = \kappa_r = \frac{10}{100}$ and $20\log_{10}(\kappa_t^{(\mathrm{NL})}) = 15.2$ dBm.

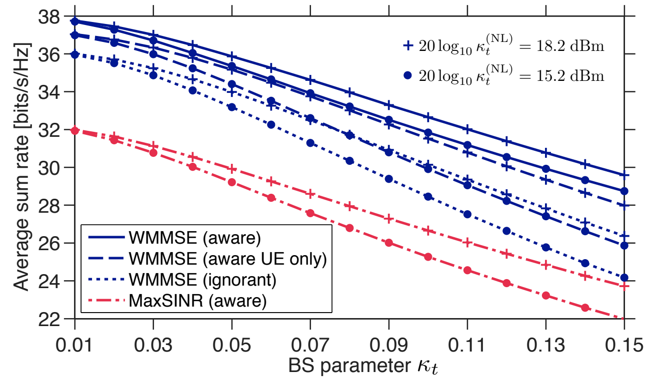

Fig. 2. Sum rate for $P_t = 18.2$ dBm when varying impairment parameters. Note the scale of the vertical axis.

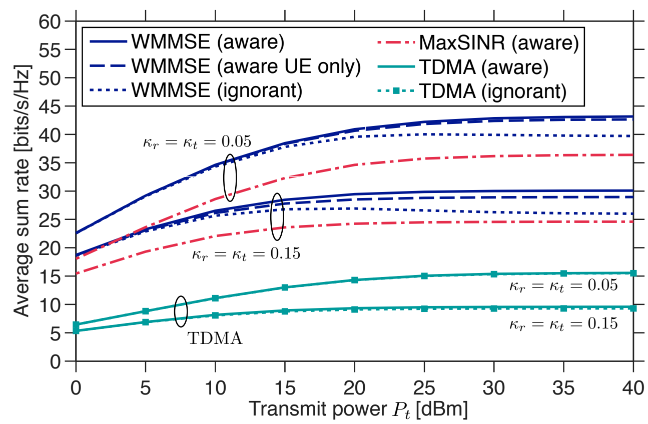

Fig. 3. Sum rate for $\eta(x) = \kappa_t x$ when varying transmit power and impairment parameters.

loss of 20 dB. The BS antenna array boresights are aimed towards the center of the triangle, and the BS antenna gain is $12\left(\frac{\theta}{35^\circ}\right)^2$ dB where θ is the angle from the boresight. The MS antenna gain is 0 dB. The small scale fading is given by i.i.d. $\mathcal{CN}(0,1)$ entries for all antenna-pairs. We study one 15 kHz subcarrier with corresponding noise power $\sigma_r^2 = -127$ dBm, and interpret (2) as a spectral efficiency. The user priorities are $\alpha_{i_k} = 1$ for all i_k.

For the WMMSE method of Sec. 3.1, we compare the case of impairments-aware BSs and UEs, with the case of impairments-aware UEs and impairments-ignoring BSs, and with the case of both ignorant UEs and BSs. The case of having aware UEs but ignorant BSs could occur if the UEs estimate their covariances $\mathbf{Q}_{i_k}$ over the air, without having a specific model for the impairments. The impact of the distortions is then picked up by the UEs, and that knowledge is implicitly distributed to the BSs in the WMMSE iterations. The ignorant BSs let $\mathbf{C}_i^{(t)} = \mathbf{0}$ and $\mathbf{C}_{i_k}^{(r)} = \mathbf{0}$ in their optimization.

As a baseline, we apply the popular MaxSINR method [3]. This ad-hoc method iteratively maximizes the SINRs of all the data streams in the network, and although it has not been proven to converge, it often performs excellently in numerical studies without impairments [4, 5, 14]. We modify the method slightly, to account for hardware impairments. In particular, we let $\mathbf{W}_{i_k} = \mathbf{I}$ for all i_k, and optimize the pth column of the precoder for UE i_k w.r.t. the virtual uplink interference, distortions and noise covariance matrix

$$\mathbf{T}_{i_k,p}^{\mathrm{MaxSINR}} = \sum_{(j,l,q)\neq(i,k,p)} \alpha_{j_l}\mathbf{H}_{j_l i}^{\mathsf{H}}\left[\mathbf{U}_{j_l}\right]_{:,q}\left[\mathbf{U}_{j_l}\right]_{:,q}^{\mathsf{H}}\mathbf{H}_{j_l i} + \kappa_t^2 \mathrm{diag}\left(\mathbf{T}_i\right) + \kappa_r^2\widetilde{\mathbf{T}}_i + (\sigma_r^2/P_t)\mathbf{I}.$$

As another baseline, we use TDMA. For impairments-aware UEs and BSs, we use the WMMSE method to find the precoders. For ignorant UEs and BSs, we use eigenbeamforming with water filling.

First, we study the convergence behaviour of the proposed methods and the baselines. We let $N_d = 2$ for WMMSE and $N_d = 1$ for MaxSINR. The power constraint per BS is $P_t = 18.2$ dBm and the impairments parameters are $\kappa_t = \kappa_r = \frac{10}{100}$ and $20\log_{10}(\kappa_t^{(\mathrm{NL})}) = 15.2$ dBm. We generated one user drop, and the corresponding sum rate evolution is shown in Fig. 1. The proposed method converges, and it is clearly important to take hardware impairments into account in order to achieve good performance.

Next, we vary the hardware impairments parameters in (4) for $P_t = 18.2$ dBm. We generated 100 user drops, and 10 small-scale fading realizations per drop. The iterative methods were run until the relative change in achieved sum rate was less than 10^{-3}. The results in Fig. 2, averaged over the Monte Carlo realizations, show that the impairments-ignoring schemes are heavily affected by the non-linearity, but the impairments-aware performance is kept steady.

With the same setup, we study performance as a function of transmit power. We specialize to $\eta(x) = \kappa_t x$ and vary $\kappa_t = \kappa_r$ and the transmit power P_t. The results in Fig. 3 demonstrate that impairments-aware WMMSE outperforms the other methods, but WMMSE with ignorant BSs and aware UEs performs almost equally well. Impairments-ignoring WMMSE performs worse for larger transmit powers, due to the fact that it maximizes an incorrect objective. Both WMMSE and MaxSINR performs around 3 times better than TDMA, and impairments-ignoring TDMA has very similar performance to impairments-aware TDMA. Further, it can be seen that the achieved high-SNR slopes are zero for all schemes, as predicted by theory [10].

5. CONCLUSIONS

The concept of interference alignment has by many been seen as a saviour, due to its ability to achieve the maximum high-SNR scaling of the sum rate in multicell MIMO networks. However, due to the residual hardware impairments, the high-SNR scaling eventually becomes zero, for both coordinated beamforming and traditional TDMA. As shown in this work, applying impairments-aware resource allocation techniques inspired by IA still outperforms impairments-aware TDMA with a large margin.

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