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# POWER ALLOCATION FOR GAUSSIAN MULTIPLE ACCESS CHANNEL WITH NOISY COOPERATIVE LINKS

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## ABSTRACT

In this paper, a new coding scheme for the multiple access channel (MAC) with noisy cooperative links is proposed. The cooperation cost is modelled by powers spent on exchanging common information at transmitters. The optimal power allocation policy is derived to explore the tradeoff between cooperation and transmission. For some important cases, optimal power allocation that maximizes weighted sum rate, is found analytically. The sufficient and necessary condition for which the sum and the individual rates are simultaneously maximized, is identified. Analytical and numerical results suggest that the transmitter, whose power budget is dominated by that of the other, acts purely as a relay. The cooperation gain becomes more significant when the difference between the power budgets is smaller.

***Index Terms***— multiple access channel, cooperation cost, power allocation, convex optimization

## 1. INTRODUCTION

In cellular networks, user cooperation provides spatial diversity against channel fading and can increase the data rate [1, 2]. In this paper, we propose a scheme for two spatially separated transmitters to communicate cooperatively to the common destination. Before channel inputs are generated, two transmitters exchange information through noisy cooperative links which are orthogonal to the MAC, and jointly access the channel with some knowledge of the other's message.

If the links between transmitters are *noise-free* and rate-limited, the cooperation can be realized by holding a conference [3, 4], where two transmitters iteratively communicate words as deterministic functions of respective messages and the words previously heard from the partner. We cannot take this approach here because the words are corrupted by Gaussian noise. We ask each transmitter to encode the message they want to share with the other (common message) **independently** over the noisy cooperative links. The power spent on sending common messages, which helps to enlarge the capacity region, can be viewed as the cooperation cost.

Another transmitter-cooperation scheme is presented in [2], where the cooperation occurs in the same band with the MAC channel. Our scheme is different; by imposing orthogonality, the signals intended exclusively for the receiver will not be overheard by the other transmitter.

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Cooperation over resource (e.g., energy and bandwidth) limited networks has been widely explored in the literature. The survey [5] listed a series of results on the power allocation between source and relay in the relay networks. Power allocation strategy were also designed for wireless sensor networks with cooperative sensors to reduce energy cost and to improve outage performance [6, 7]. The same idea of modelling cooperation cost by the power to exchange common message appeared in [8], where a single user chooses to join the coalition maximizing his utility which is discounted by cooperation cost.

Over the capacity region of the proposed scheme, we find the optimal power allocation to maximize a weighted sum rate. This is a convex problem, and we seek to characterize the optimal policy by analytically deriving the closed-form expressions for the optimal powers and resulting rates. Similar tasks were performed in [9–11] for the capacity region in [2].

Our main contributions are as follows: we derive the capacity region for the proposed scheme with orthogonal cooperation and transmission links. Fixing some power values, we find the optimal power allocation policy for the sum rate and individual rate each. We also identify the condition for which the two optimal policies match and thus we can maximize arbitrarily weighted sum rates. For the non-matching case, we define the optimization problem and derive the associated KKT conditions for global optimality.

## 2. GAUSSIAN MAC WITH ORTHOGONAL COOPERATION AND TRANSMISSION CHANNELS

The system configuration is exhibited in Fig. 1. Two transmitters Tx1 and Tx2, each with power budget $P_1$ and $P_2$, are communicating via a dedicated cooperation channel which is orthogonal to the MAC channel, e.g., parallel frequency bands. The cooperation links are also corrupted by Gaussian noise, i.e., $Z_1^n \sim \mathcal{N}(\mathbf{0}, \sigma_1^2\mathbf{I})$, $Z_2^n \sim \mathcal{N}(\mathbf{0}, \sigma_2^2\mathbf{I})$, $\mathbf{I}$ is the identity matrix.

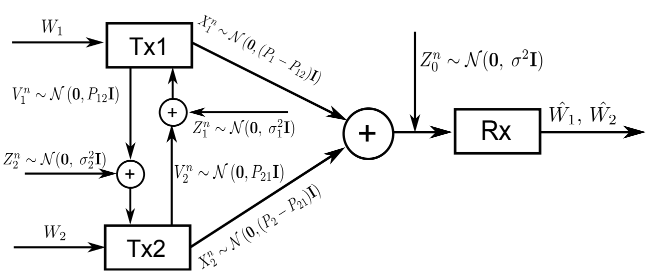


**Fig. 1**. MAC with noisy cooperative links.

Communication is over $T$ blocks of length $n$, where each transmitter has one message to send in each block. In block $t$, message $W_i(t)$ at transmitter $i$, $i \in \{1,2\}$ is split into common and private parts, i.e., $W_i(t) = (W_{ic}(t), W_{ip}(t))$. In contrast to [3,4], we generate the Gaussian codewords $V_1^n\,(W_{1c}(t))$, $V_2^n\,(W_{2c}(t))$ , subject to average power constraints $P_{12}$ and $P_{21}$, to exchange common messages. The rates of $W_{1c}(t)$ $(R_{1c})$, $W_{2c}(t)$ $(R_{2c})$ satisfy:

$$R_{1c} \leqslant \frac{1}{2}\log\left(1+\frac{P_{12}}{\sigma_2^2}\right), \quad R_{2c} \leqslant \frac{1}{2}\log\left(1+\frac{P_{21}}{\sigma_1^2}\right). \tag{1}$$

In the same block $t$, we perform superposition coding for the MAC. We generate $X_{1p}^n\,(W_{1p}(t-1)) \sim \mathcal{N}(\mathbf{0}, P_{1p}\mathbf{I})$ at Tx 1, $X_{2p}^n\,(W_{2p}(t-1)) \sim \mathcal{N}(\mathbf{0}, P_{2p}\mathbf{I})$ at Tx 2, and Gaussian codeword $\widetilde{X}^n\,(W_{1c}(t-1), W_{2c}(t-1))$ at both transmitters subject to unit power constraint. The channel inputs:

$$X_1^n = X_{1p}^n + \sqrt{P_{1c}}\widetilde{X}^n, \tag{2a}$$

$$X_2^n = X_{2p}^n + \sqrt{P_{2c}}\widetilde{X}^n. \tag{2b}$$

The received signals at Tx1 $(Y_1^n)$, Tx2 $(Y_2^n)$, and the receiver $(Y_0^n)$ over the $n$ channel uses are:

$$Y_1^n = V_2^n + Z_1^n, \tag{3}$$

$$Y_2^n = V_1^n + Z_2^n, \tag{4}$$

$$Y_0^n = X_1^n + X_2^n + Z_0^n. \tag{5}$$

where $Z_1$, $Z_2$, and $Z_0$ are real valued white Gaussian noise at respective terminals.

The receiver generates the message estimates $\widehat{W}_1(t-1)$ and $\widehat{W}_2(t-1)$ based on $Y_0^n(t)$:

$$\left(\widehat{W}_1(t-1), \widehat{W}_2(t-1)\right) = \phi(Y_0^n(t)). \tag{6}$$

The rate pair $(R_1, R_2)$ is said to be *achievable* if there exists a sequence of $(n, 2^{nR_1}, 2^{nR_2})$ codes such that

$$\lim_{n\to\infty} \Pr\left[(W_1, W_2) \neq \left(\widehat{W}_1, \widehat{W}_2\right)\right] = 0. \tag{7}$$

Define

$$U_1 \triangleq \frac{1}{2}\log\left(1+\frac{P_{1p}}{\sigma^2}\right) + \frac{1}{2}\log\left(1+\frac{P_{12}}{\sigma_2^2}\right)$$

$$U_2 \triangleq \frac{1}{2}\log\left(1+\frac{P_{2p}}{\sigma^2}\right) + \frac{1}{2}\log\left(1+\frac{P_{21}}{\sigma_1^2}\right)$$

$$U_s^1 \triangleq \frac{1}{2}\log\left(1+\frac{P_{1p}+P_{2p}}{\sigma^2}\right) + \frac{1}{2}\log\left(1+\frac{P_{12}}{\sigma_2^2}\right) + \frac{1}{2}\log\left(1+\frac{P_{21}}{\sigma_1^2}\right)$$

$$U_s^2 \triangleq \frac{1}{2}\log\left(1+\frac{P_{1p}+P_{2p}+P_{1c}+P_{2c}+2\sqrt{P_{1c}P_{2c}}}{\sigma^2}\right)$$

$$\mathcal{C}_{co} \triangleq \bigcup_{\underline{P}\in\mathcal{P}} \Big\{(R_1,\ R_2) : R_1 \leqslant U_1,\ R_2 \leqslant U_2, \quad R_1 + R_2 \leqslant \min\{U_s^1, U_s^2\}\Big\} \tag{8}$$

where $\mathcal{P} \triangleq \big\{(P_{1p}, P_{2p}, P_{1c}, P_{2c}, P_{12}, P_{21}) : P_{1p} + P_{1c} + P_{12} \leqslant P_1, P_{2p} + P_{2c} + P_{21} \leqslant P_2\big\}$.

**Theorem 1.** *$\mathcal{C}_{co}$ is the capacity region of of the Gaussian MAC with orthogonal noisy cooperative links.*

*Proof.* The achievability part follows by recognizing $W_{1p}, W_{2p}$ and $(W_{1c}, W_{2c})$ are mutually independent, and the achievable rates are the same as those for the three-user Gaussian MAC. The converse and more detailed proofs are in [12]. □

**Remark 1.** *In (8), when $P_{12} = P_{21} = 0$, no information is exchanged between transmitters, and $\mathcal{C}_{co}$ reduces to the capacity region of the Gaussian MAC without cooperation [13]. For noiseless inter-transmitter channels, $(\sigma_1^2 = \sigma_2^2 = 0)$, only $U_s^2$ is finite and limits the achievable rates. Total cooperation is optimal where two transmitters jointly encode $(W_1, W_2)$ with their full transmission powers.*

Because $\mathcal{C}_{co}$ is convex, every boundary point of $\mathcal{C}_{co}$ can be completely specified by some $(U_1, U_2, U_s^1, U_s^2)$. Maximizing $R_\alpha \triangleq \alpha R_1 + R_2$ for arbitrary $\alpha \geqslant 0$ over $\mathcal{C}_{co}$ is a convex program and can be solved by a standard convex optimization solver.

To gain insights about how to allocate power optimally, we consider equal noise variances at all terminals, i.e., $\sigma_1^2 = \sigma_2^2 = \sigma^2$, and define $S_i \triangleq \dfrac{P_i}{\sigma^2}$, $S_{ip} \triangleq \dfrac{P_{ip}}{\sigma^2}$, $S_{ic} \triangleq \dfrac{P_{ic}}{\sigma^2}$, and $S_{ij} \triangleq \dfrac{P_{ij}}{\sigma^2}$ for $i, j \in \{1,2\}$. Then $\underline{P}/\sigma^2$ in (8) can be replaced by $\underline{S}$ for $\underline{S} \in \mathcal{S} \triangleq \big\{(S_{1p}, S_{2p}, S_{1c}, S_{2c}, S_{12}, S_{21}) : S_{1p} + S_{1c} + S_{12} \leqslant S_1, S_{2p} + S_{2c} + S_{21} \leqslant S_2\big\}$.

## 3. POLICY FOR SUM RATE MAXIMIZATION

When $\alpha = 1$, we are maximizing the sum rate. Constraints in (8) on individual rates are not active, and we are maximizing $\min\{U_s^1, U_s^2\}$ over $\mathcal{S}$. We first fix the power pair $(S_{1c}, S_{2c})$, as well as $S_{sp} \triangleq S_{1p} + S_{2p}$ to make $U_s^2$ constant, and maximize $U_s^1$ over the constraint set of $S_{1p}$. For each $(S_{1c}, S_{2c})$, $S_{1r} \triangleq S_1 - S_{1c}$ is the remaining power at transmitter 1, $S_{2r} \triangleq S_2 - S_{2c}$ is the remaining power at transmitter 2. Then $U_s^1$ can be rewritten as $U_s^1(S_{1p}) = \dfrac{1}{2}\log\big((1+S_{sp})f(S_{1p})\big)$ where

$$f(x) = -x^2 + (S_{1r} - S_{2r} + S_{sp})x + (1 + S_{1r})(1 + S_{2r} - S_{sp}) \tag{9}$$

Notice that $S_{1p} \leqslant S_{1r}$, $S_{sp} - S_{1p} = S_{2p} \leqslant S_{2r}$, so $S_{1p} \in [\max\{0, S_{sp} - S_{2r}\}, \min\{S_{1r}, S_{sp}\}]$.

We obtain the critical point by setting $\dfrac{df(x)}{dx} = 0$, where the optimal value $S_{1p}^{s*}$ for maximizing $U_s^1(S_{1p})$ is given by:

$$S_{1p}^{s*} = \begin{cases} 0, & S_{1r} \leqslant S_{2r} - S_{sp} \\ (S_{1r} - S_{2r} + S_{sp})/2, & S_{2r} - S_{sp} < S_{1r} < S_{2r} + S_{sp} \\ S_{sp}, & S_{1r} \geqslant S_{2r} + S_{sp} \end{cases}. \tag{10}$$

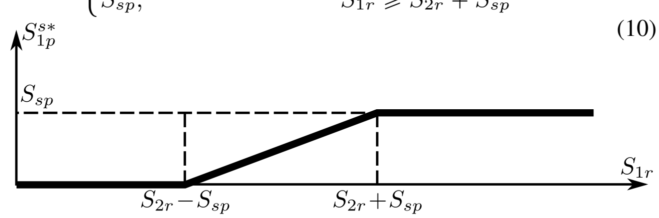


**Fig. 2**. Optimal power allocation for $U_s^1(S_{1p})$.

Once two transmitters agree upon the power each spends transmitting the common messages, we can plot (10) in Fig. 2. Firstly, $S_{1p}^{s*}$ is a non-decreasing function of $S_{1r}$. When transmitter 1 has much less power left compared to transmitter 2, i.e., $S_{1r} \leqslant S_{2r} - S_{sp}$, no private message should be transmitted at transmitter 1, and all of its remaining power should be assigned to the cooperative link and $S_{12} = S_{1r}$. However, when transmitter 1 has much more power left, $S_{1r} \geqslant S_{2r} + S_{sp}$, transmitter 2 should send $W_{2c} = W_2$ to the stronger partner to forward to the receiver ($S_{21} = S_{2r}$). When the remaining powers are comparable, $|S_{1r} - S_{2r}| \leqslant S_{sp}$, $S_{1p}^{s*}$ should increase linearly with respect to $S_{1r}$.

The optimal sum rate is found by realizing $R_s^*(S_{1c}, S_{2c}, S_{sp}) \triangleq \min\{U_s^1(S_{1p}^{s*}), U_s^2(S_{1c}, S_{2c}, S_{sp})\}$, and the overall maximal sum rate is $R_s^* = \max_{S_{1c}, S_{2c}, S_{sp}} \{R_s^*(S_{1c}, S_{2c}, S_{sp})\}$.

## 4. POLICY FOR MAXIMIZING WEIGHTED SUM RATE

Next, without loss of generality, we assume $\alpha > 1$ [1] and rewrite $R_\alpha$ as $(\alpha - 1)R_1 + (R_1 + R_2)$. Have derived the optimal policy to maximize $U_s^1$ in (10), we can determine the scenarios where the optimal policy maximizing $R_1$ coincides with (10), and combine the two policies to construct the overall optimal policy.

Recall in (8), $R_1$ is upper bounded by $U_1$. For fixed ($S_{1c}, S_{2c}, S_{sp}$), we find $S_{1p}^{1*}$ over the constraint set to maximize $U_1(S_{1p}) = \frac{1}{2}\log(1 + S_{1p}) + \frac{1}{2}\log(1 + S_{1r} - S_{1p})$ by taking derivative with respect to $S_{1p}$:

$$S_{1p}^{1*} = \begin{cases} S_{sp} - S_{2r}, & S_{1r} \leqslant 2(S_{sp} - S_{2r}) \\ S_{1r}/2, & 2(S_{sp} - S_{2r}) < S_{1r} < 2S_{sp} \\ S_{sp}, & S_{1r} \geqslant 2S_{sp} \end{cases}. \quad (11)$$

Here, to maximize $R_1$, $S_{1p}$ should never be set to 0.

**Definition** For fixed $(S_{1c}, S_{2c}, S_{sp})$, a **matching** occurs when $S_{1p}^{s*} = S_{1p}^{1*}$.

**Lemma 1.** *There exists triples $(S_{1c}, S_{2c}, S_{sp})$ such that the optimal policy $S_{1p}^{b*}$ to maximize $(\alpha - 1)U_1(S_{1p}) + U_s^1(S_{1p})$, is given by:*

$$S_{1p}^{b*} = \begin{cases} S_{1r}/2, & S_{1r} < S_{2r} + S_{sp}, \text{ and } S_{2r} = S_{sp} \\ S_{sp}, & S_{2r} + S_{sp} \leqslant S_{1r} < 2S_{2r} \\ S_{sp}, & S_{1r} \geqslant 2\max\{S_{2r}, S_{sp}\} \end{cases}. \quad (12)$$

*Proof.* The set of constraints on $(S_{1r}, S_{2r}, S_{sp})$ in (12) is obtained by finding the feasible set which makes a matching possible. Under this condition, $U_1$ and $U_s^1$ are maximized simultaneously, so is $(\alpha - 1)U_1 + U_s^1$. □

Here, the optimal policy $S_{1p}^{b*}$ is independent of $\alpha$ because of the matching. We see from (12) that under the matching condition, similarly as in (10), $S_{1p}^{b*}$ is a non-decreasing function of $S_{1r}$, and $S_{1p}^{b*} = S_{sp}$ when the remaining power at transmitter 1 dominates that at transmitter 2, which implies transmitter 2 should not send any private message. When $S_{1r} < S_{2r} + S_{sp}$, the optimality of $S_{1p}$ can be guaranteed without considering $\alpha$ only when transmitter 1 decides to use same amount of power for its private message ($W_{1p}$) as transmitter 2 uses to communicate its common message ($W_{2c}$), i.e., $S_{1p} = S_{21}$, and the optimal thing to do in this case at transmitter 1 is to equally split the remaining power between $S_{1p}$ and $S_{12}$ so that $S_{1p} = S_{12} = S_{21}$. Notice that different from the optimal rule for the sum rate, when a matching occurs, $S_{1p}^{b*}$ should always be positive, however small $S_{1r}$, this follows by the assumption $\alpha > 1$ and the optimal rule for $U_1(S_{1p})$ in (11).

[1] The case of $\alpha < 1$ can be similarly treated.

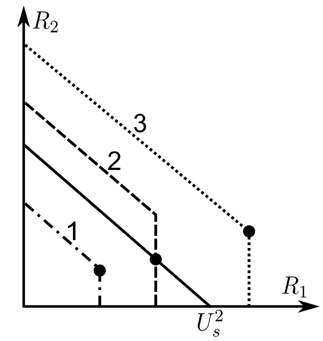


**Fig. 3**. optimal rate pairs

We note that similarly in [9, 10], the authors reduced the number of variables and convexified the optimization problem by first identifying the matching condition.

**Theorem 2.** *For $(S_{1c}, S_{2c}, S_{sp})$ satisfying the matching condition. The overall optimal policy $S_{1p}^* = S_{1p}^{b*}$ and corresponding optimal weighted sum rate $R_\alpha^*$ is given by:*

$$(\alpha - 1)U_1(S_{1p}^{b*}) + U_s^1(S_{1p}^{b*}),\ U_s^2 \geqslant U_s^1(S_{1p}^{b*}) \quad (13a)$$

$$(\alpha - 1)U_1(S_{1p}^{b*}) + U_s^2,\ U_1(S_{1p}^{b*}) \leqslant U_s^2 < U_s^1(S_{1p}^{b*}) \quad (13b)$$

$$\alpha U_s^2,\ U_s^2 < U_1(S_{1p}^{b*}) \quad (13c)$$

*Proof.* After incorporating another sum rate constraint $U_s^2$, which is a constant for given $(S_{1c}, S_{2c}, S_{sp})$, (13a), (13b), (13c) correspond to regions 1, 2, and 3 in Fig. 3. It is obvious that the dark dot is the optimal rate pair for each scenario. □

For general non-matching $(S_{1c}, S_{2c}, S_{sp})$, we solve the convex optimization problem to find $S_{1p}^{n*}$ maximizing $(\alpha - 1)U_1(S_{1p}) + U_s^1(S_{1p})$. For $\alpha > 1$:

$$\begin{aligned} \text{minimize} \quad & -(\alpha - 1)\log(1 + S_{1p}) - \alpha\log(1 + S_{12}) \\ & - \log(1 + S_{21}) && (14) \\ \text{subject to} \quad & S_{1p} + S_{12} \leqslant S_{1r}, && (15) \\ & S_{sp} - S_{1p} + S_{21} \leqslant S_{2r}, && (16) \\ & S_{1p}, S_{12}, S_{21} \geqslant 0. && (17) \end{aligned}$$

Associating Lagrangian multipliers $\lambda_1, \lambda_2 \geqslant 0$ to (15) and (16), and also $\epsilon_{1p}, \epsilon_{12}, \epsilon_{21} \geqslant 0$ to (17), we find the Karush-Kuhn-Tucker (KKT) conditions [14] for optimality:

$$\frac{\alpha - 1}{1 + S_{1p}} + \epsilon_{1p} = \lambda_1 - \lambda_2, \quad (18)$$

$$\frac{\alpha}{1 + S_{12}} + \epsilon_{12} = \lambda_1,\ \frac{1}{1 + S_{21}} + \epsilon_{21} = \lambda_2 \quad (19)$$

$$\lambda_1(S_{1p} + S_{12} - S_{1r}) = 0, \quad (20)$$

$$\lambda_2(S_{sp} - S_{1p} + S_{21} - S_{2r}) = 0, \quad (21)$$

$$\epsilon_{1p}S_{1p} = \epsilon_{12}S_{12} = \epsilon_{21}S_{21} = 0. \quad (22)$$

By (19), we have the following inequalities:

$$\lambda_1 - \lambda_2 \geqslant \frac{\alpha - 1}{1 + S_{1p}} \quad (23)$$

$$\lambda_1 \geqslant \frac{\alpha}{1+S_{12}} > 0, \quad \lambda_2 \geqslant \frac{1}{1+S_{21}} > 0 \tag{24}$$

Therefore, by (20) and (21), all remaining power budgets should be used for the optimal allocation:

$$S_{1p} + S_{12} = S_{1r}, \quad S_{1p} - S_{21} = S_{sp} - S_{2r}. \tag{25}$$

If the optimal value for one of $S_{1p}$, $S_{12}$ and $S_{21}$ is 0, the other two can be found by (25). If the optimal values for $S_{1p}$, $S_{12}$ and $S_{21}$ are all strictly positive, the equalities in (23)-(24) will hold and the optimal value $S_{1p}^{n*}$ can be found by solving the equation:

$$\frac{\alpha - 1}{1+S_{1p}} = \frac{\alpha}{1+S_{1r}-S_{1p}} - \frac{1}{1+S_{1p}+S_{2r}-S_{sp}}. \tag{26}$$

## 5. NUMERICAL RESULTS

We use CVX [15] to numerically find and compare the optimal power allocation for different $\alpha$ values in Table 1. $S_1 = S_2 = 30$dB.

| $\alpha$ | $S_{1p}$ | $S_{12}$ | $S_{1c}$ | $S_{2p}$ | $S_{21}$ | $S_{2c}$ |
|---|---|---|---|---|---|---|
| 1 | 10.8 | 10.6 | 29.9 | 10.8 | 10.6 | 29.9 |
| 100 | 18.7 | 16.9 | 29.4 | 20.6 | -55.8 | 29.5 |

**Table 1**. Optimal power (dB) for symmetric and large $\alpha$.

For the symmetric case, $\alpha = 1$, two transmitters have identical optimal power allocations: most part of power is assigned for common information transmission, and the remaining is split approximately equally between cooperation and private message. When $\alpha \gg 1$, two transmitters should jointly transmit only $W_{1c}$, i.e., $S_{21} = 0$. Also at both transmitters, more power should be allocated to private message transmission compared to the symmetric case.

Next, we explore the impact of power ratio $\eta \triangleq S_2/S_1$ on the optimal policy to maximize the sum rate, ($\alpha = 1$). $S_2 = 10$dB.

| $\eta$ | $S_{1p}$ | $S_{12}$ | $S_{1c}$ | $S_{2p}$ | $S_{21}$ | $S_{2c}$ |
|---|---|---|---|---|---|---|
| 1 | 3.0 | 1.7 | 8.1 | 3.0 | 1.7 | 8.1 |
| 0.1 | 19.5 | -3.5 | 10.0 | -66.4 | -68.4 | 10.0 |
| 0.01 | 30.0 | -13.9 | 10.0 | -62.6 | -66.0 | 10.0 |

**Table 2**. Optimal power (dB) for different $\eta$.

When two transmitters have same power budget, most of the power should be given to the common information transmission. When the power budget at one transmitter starts to dominate the other, the transmitter with less power budget should spend all of its power to relay the common message of its stronger partner, which implies a unidirectional cooperation.

To understand how much performance gain is achieved by the proposed cooperative scheme, we plot the prelog factor, which is half for non-cooperative Gaussian MAC [13]: $R_1+R_2 \leqslant \frac{1}{2}\log(1+S_1+S_2)$, for various $\eta$ values across different SNR regimes in Fig. 4. We find that for all $\eta$ and SNR values, the prelog factors of this scheme are at least 0.5. The performance gain resulting from the cooperation is more significant at the medium SNR. Also transmitters with more balanced power budgets benefit more from cooperation, this property can be used to guide the selection of cooperation partners.

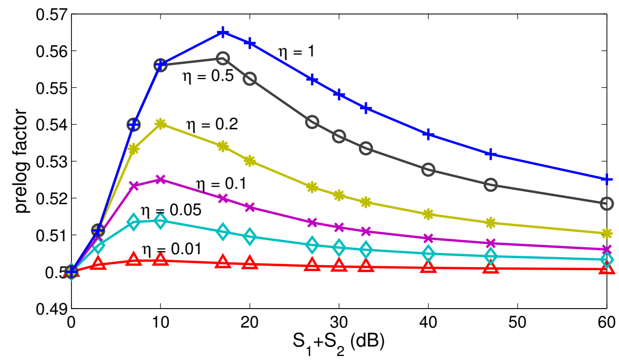


**Fig. 4**. prelog factor of the proposed cooperative scheme.

Next, we plot the sum rates in Fig. 5 when using the optimized policy and a simple policy which divides $P_i$ equally among $P_{ip}$, $P_{ic}$ and $P_{ij}$ at each transmitter.

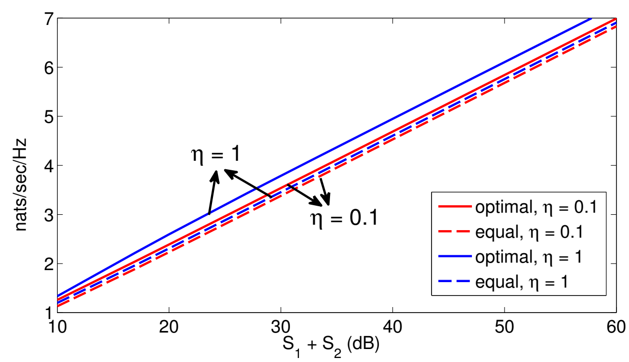


**Fig. 5**. Sum rates resulting from the optimal policy and an equal allocation policy.

We observe for both policies, the sum rates increase approximately linearly with respect to the total power budget. The optimal power allocation achieves constantly higher sum rate than the equal allocation scheme. The performance gap between the two policies is more noticeable for a more balanced power budget across transmitters, which implies that the optimal policy presented in this paper provides an easy design rule to guide the power allocation for cooperative partner with comparable power budgets.

## 6. CONCLUSIONS

We propose a new cooperative scheme for accessing the Gaussian MAC and derive the corresponding capacity region. As the cooperation cost, a part of transmission power is spent to exchange common messages on noisy links. Fixing a part of the power allocation policy, we identify the existence of a matching and give closed form expressions of optimal policy to maximize $\alpha R_1 + R_2$, $\alpha \geqslant 1$. Also, we solve a constrained convex optimization problem to tackle the more general non-matching situations. Numerical results demonstrate the performance gain by allocating power optimally and indicate that it is more beneficial to implement this scheme on transmitters with more balanced power budgets.

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