ZIV-ZAKAI BOUND FOR TIME DELAY ESTIMATION OF UNKNOWN DETERMINISTIC SIGNALS

Nicolò Decarli and Davide Dardari

DEI, CNIT, University of Bologna, via Venezia 52, 47521 Cesena (FC), Italy
Email: {nicolo.decarli, davide.dardari}@unibo.it

ABSTRACT

This paper investigates theoretical limits on time delay estimation performance when the received waveform is unknown. Considering that the Cramér-Rao bound for the time delay estimation error cannot be derived in this case, the analytical expression of the Ziv-Zakai bound is provided and discussed.

Index Terms— Ziv-Zakai bound, Time delay estimation, Time of arrival, Energy detector, Synchronization

1. INTRODUCTION

Time delay estimation, also referred to as *epoch* estimation or time-of-arrival (TOA) estimation, is a classical signal processing problem with many applications such as synchronization, radar/sonar target distance estimate, and wireless localization [1–5].

Classical TOA estimators rely on coherent correlation, that is, they assume local knowledge of the received signal shape. However their design and implementation is not always practical when the signal shape is unknown [4]. Moreover, correlation-based receivers might result in high complexity implementations, especially when large bandwidth signals are employed. For these reasons, TOA estimators based on energy measurements have been proposed [4, 6], but there is still a no complete understanding of their performance bound. To this purpose, the Cramér-Rao bound (CRB) is the well-known approach adopted in estimation theory to obtain a performance benchmark [3, 7]. Unfortunately, when no prior knowledge on the received signal is available, the CRB cannot be derived because the regularity conditions on the involved signal probability density function (p.d.f.) are not satisfied [7].

The Ziv-Zakai bound (ZZB) [8], with its improved versions such as the Bellini-Tartara bound [9], does not require stringent regularity constraints on the p.d.f. and accounts for both *threshold effect* in the low and moderate signal-to-noise ratio (SNR) regions, and a-priori information of the parameter to be estimated, and hence it can be applied to a wide range of SNRs [2, 10–12]. ZZBs for known signals or assuming statistical knowledge of the channel at the receiver have been widely investigated in the literature (e.g., [9, 12–14]). Differently, at the most of authors' knowledge, no bounds have appeared for the case of unknown deterministic signals.

This paper derives the ZZB on the mean squared error (MSE) associated with the TOA estimation of unknown deterministic signals. Numerical results show that this bound is very tight for all the range of SNR with the actual performance of energy-based estimators, differently from the CRB and ZZB derived in the case of known signal that are quite loose when compared with the estimator performance.

This work has been funded by the Italian ministerial PRIN project GRETA (Grant 2010WHY5PR) and by the European Commission through the FP7 project SELECT (Grant 257544). Authors would like to thank Marco Chiani and Moe Z. Win for the fruitful discussions.

2. SIGNAL MODEL

We consider a received signal given by

$$r(t) = s(t-\tau) + n(t) \tag{1}$$

where $s(t)$ is a time-limited low-pass waveform with duration T_s. We assume $s(t)$ to be *band-limited at level* ϵ with bandwidth W, with W indicating the smallest value for which

$$\int_{|f|>W} |S(f)|^2 \, df < \epsilon \tag{2}$$

where $S(f)$ is the Fourier transform of $s(t)$, and the energy ϵ lying outside the frequency range is less than the smallest amount we are able to detect in the real world [15]. Parameter τ is the unknown TOA of the received signal to be estimated, assumed uniformly distributed in the interval $[0, T_\mathrm{a}]$, and $n(t)$ is additive white Gaussian noise (AWGN) with zero mean and two-sided power spectral density (PSD) $N_0/2$ in the band of interest.[1] The goal is to obtain the estimate $\hat{\tau}$ of τ by observing $r(t)$ in the interval $[0, T_\mathrm{ob}]$, with $T_\mathrm{ob} > T_\mathrm{a} + T_\mathrm{s}$.

We assume $s(t)$ to be an unknown deterministic signal, for which its shape is totally unknown [16]. In the absence of additional hypothesis on $s(t)$ it is not possible to identify

[1] We consider the presence of a zonal filter that removes all the noise components outside W.

978-1-4799-2893-4/14/$31.00

uniquely the TOA because the starting and ending instants of $s(t)$ would not be defined. Therefore the only assumption we consider is that $s(t)$ is zero outside the interval $[0, T_s]$.

For further convenience, we make use of orthonormal series representations of signals in $[0, T_{ob}]$ using a suitable complete orthonormal basis $\{\Phi_m(t)\}_{m=1}^M$ [7]. Specifically we can write

$$r(t) = \sum_{m=1}^{M} r_m \, \Phi_m(t), \quad n(t) = \sum_{m=1}^{M} n_m \, \Phi_m(t) \qquad (3)$$

for $0 \le t \le T_{ob}$ with $M = \lfloor 2WT_{ob} \rfloor + 1$ [17]. Adopting the classical Karhunen-Loéve expansion [7] we have that $n_m = c_m \, \sigma_n$, with $\sigma_n^2 = \frac{N_0}{2}$, and where coefficients $\{c_m\}$ are independent zero mean Gaussian random variables (RVs) with unitary variance. According to the previous series expansions, signals $r(t)$ and $n(t)$ are fully represented by the coefficients vectors $\mathbf{r} = [r_1, r_2, \ldots, r_M]^T \in \mathbb{R}^M$, and $\mathbf{n} = [n_1, n_2, \ldots, n_M]^T \in \mathbb{R}^M$, respectively. Signal $s(t)$, instead, is by definition time-limited in $0 \le t \le T_s$, therefore it can be conveniently expanded, using an orthonormal basis $\{\Psi_n(t)\}_{n=1}^N$, as

$$s(t) = \sum_{n=1}^{N} s_n \, \Psi_n(t) \, , \quad 0 \le t \le T_s \qquad (4)$$

where $N = \lfloor 2WT_s \rfloor + 1$. Since $T_s < T_{ob}$, the dimensionality of the vector of coefficients $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T \in \mathbb{R}^N$ is less than that of $\mathbf{r}$, that is, $N < M$. Expression (1) can be written equivalently in the form

$$\mathbf{r} = \mathbf{H}^{(\tau)} \, \mathbf{s} + \mathbf{n} = \mathbf{y} + \mathbf{n} \qquad (5)$$

with $\mathbf{H}^{(\tau)} \in \mathbb{R}^{M \times N}$ being the transform matrix from basis $\{\Psi_n(t)\}$ to basis $\{\Phi_m(t)\}$ depending on τ, and vector $\mathbf{y} \in \mathbb{R}^M$ obeying the linear subspace model $\mathbf{y} = \mathbf{H}^{(\tau)} \, \mathbf{s}$. In particular, vector $\mathbf{y}$ lies in a N-dimensional subspace of $\mathbb{R}^M$ denoted with $\langle \mathbf{H}^{(\tau)} \rangle$,with $N < M$. In the following given the vector $\mathbf{x} \in \mathbb{R}^M$, containing the series expansion coefficients of a generic signal $x(t)$, the orthogonal projection of $\mathbf{x}$ onto $\langle \mathbf{H}^{(\tau)} \rangle$ will be denoted by $\mathbf{P}_{\mathbf{H}^{(\tau)}} \, \mathbf{x}$, where $\mathbf{P}_{\mathbf{H}^{(\tau)}}$ is the orthogonal projection matrix (or projector) $\mathbf{P}_{\mathbf{H}^{(\tau)}} = \mathbf{H}^{(\tau)} \left(\mathbf{H}^{(\tau)T} \mathbf{H}^{(\tau)} \right)^{-1} \mathbf{H}^{(\tau)T}$. It can be shown that

$$\mathbf{x}^T \mathbf{P}_{\mathbf{H}^{(\tau)}} \mathbf{x} = \int_{\tau}^{\tau + T_s} x^2(t) \, dt \qquad (6)$$

which represents the energy of $x(t)$ in the interval $[\tau, \tau + T_s]$.

The conditional p.d.f. of $\mathbf{r}$ is given by

$$p\{\mathbf{r}|\tau\} = \frac{1}{\left(\sqrt{\pi N_0}\right)^M} \exp\left\{ -\frac{1}{N_0} \left\| \mathbf{r} - \mathbf{H}^{(\tau)} \, \mathbf{s} \right\|^2 \right\} . \qquad (7)$$

3. THE ZZB FOR UNKNOWN DETERMINISTIC SIGNALS

When τ is uniformly distributed in $[0, T_a]$, the ZZB is given by [9, 18]

$$\mathsf{ZZB} = \frac{1}{T_a} \int_0^{T_a} z \, (T_a - z) \, P_{\min}(z) \, dz \qquad (8)$$

where $P_{\min}(z)$ is the probability of error corresponding to the optimum decision rule based on the likelihood ratio test (LRT)

$$\Lambda(\mathbf{r}) = \frac{p\{\mathbf{r}|\tau\}}{p\{\mathbf{r}|\tau + z\}} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} 1 \, . \qquad (9)$$

for testing the two equally probable hypotheses

$$\mathcal{H}_1 : \; r(t) = s(t - \tau) + n(t) \quad \text{or} \quad \mathbf{r} = \mathbf{H}^{(\tau)} \, \mathbf{s} + \mathbf{n} \qquad (10)$$
$$\mathcal{H}_2 : \; r(t) = s(t - \tau - z) + n(t) \quad \text{or} \quad \mathbf{r} = \mathbf{H}^{(\tau + z)} \, \mathbf{s} + \mathbf{n} \, .$$

In the following the detector and its probability of error will be derived in order to calculate the ZZB in (8).

3.1. Detector Design

Due to the assumption of unknown deterministic signal, the test (9) is composite since unknown series expansion coefficients $\mathbf{s}$ are present. In the absence of any statistical characterization of the unknown coefficients $\mathbf{s}$, a practical and usual approach is to design the detector performing the generalized likelihood ratio test (GLRT) [7]. Unfortunately we cannot claim the optimality of the GLRT, then the corresponding ZZB expression will not result, in general, a lower bound. However it is well known that the GLRT is asymptotically optimum, so the derived expression can be considered a lower bound in the high SNR region [19]. The optimality for certain classes of estimators is under investigation and will be addressed in a following up paper.

The log-GLRT is obtained by replacing the unknown parameters $\mathbf{s}$ by their maximum likelihood (ML) estimates $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$, respectively, under hypothesis $\mathcal{H}_1$ and $\mathcal{H}_2$ [7], that is

$$\ell(\mathbf{r}) = \ln \frac{p\{\mathbf{r}|\tau, \hat{\mathbf{s}}_1\}}{p\{\mathbf{r}|\tau + z, \hat{\mathbf{s}}_2\}} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} 0 \qquad (11)$$

where the ML estimates $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ are given by

$$\hat{\mathbf{s}}_1 = \left(\mathbf{H}^{(\tau)T} \mathbf{H}^{(\tau)} \right)^{-1} \mathbf{H}^{(\tau)T} \, \mathbf{r} \qquad (12a)$$

$$\hat{\mathbf{s}}_2 = \left(\mathbf{H}^{(\tau + z)T} \mathbf{H}^{(\tau + z)} \right)^{-1} \mathbf{H}^{(\tau + z)T} \, \mathbf{r} \, . \qquad (12b)$$

As a consequence, from (7), the statistic $\ell(\mathbf{r})$ in (11) becomes

$$\ell(\mathbf{r}) = \left\| \mathbf{r} - \mathbf{H}^{(\tau + z)} \, \hat{\mathbf{s}}_2 \right\|^2 - \left\| \mathbf{r} - \mathbf{H}^{(\tau)} \, \hat{\mathbf{s}}_1 \right\|^2 = \|\bar{\mathbf{n}}_2\|^2 - \|\bar{\mathbf{n}}_1\|^2$$

where $\bar{\mathbf{n}}_1 = \mathbf{r} - \mathbf{H}^{(\tau)}\,\hat{\mathbf{s}}_1 = (\mathbf{I}_M - \mathbf{P}_{\mathbf{H}^{(\tau)}})\,\mathbf{r}$ and $\bar{\mathbf{n}}_2 = \mathbf{r} - \mathbf{H}^{(\tau+z)}\,\hat{\mathbf{s}}_2 = (\mathbf{I}_M - \mathbf{P}_{\mathbf{H}^{(\tau+z)}})\,\mathbf{r}$, with $\mathbf{I}_M$ the Mth order identity matrix. The GLRT results $\ell(\mathbf{r}) = \mathbf{r}^{\mathrm{T}}\,\mathbf{P}_{\mathbf{H}^{(\tau)}}\,\mathbf{r} - \mathbf{r}^{\mathrm{T}}\,\mathbf{P}_{\mathbf{H}^{(\tau+z)}}\,\mathbf{r}$. According to (6) it is

$$\ell(\mathbf{r}) = \int_{\tau}^{\tau+T_\mathrm{s}} r^2(t)\,dt - \int_{\tau+z}^{\tau+z+T_\mathrm{s}} r^2(t)\,dt \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} 0\,. \quad (13)$$

3.2. Detector Performance

The GLRT performance is given by the probability of error

$$P_{\min}(z) = \frac{1}{2}\,\mathbb{P}\left\{\ell(\mathbf{r}|\mathcal{H}_1) < 0\right\} + \frac{1}{2}\,\mathbb{P}\left\{\ell(\mathbf{r}|\mathcal{H}_2) > 0\right\} \quad (14)$$

where $\ell(\mathbf{r}|\mathcal{H})$ denotes the GLRT (13) specified in the case when $\mathcal{H}$ true. Considering that, with $\mathcal{H}_1$ true, $s(t-\tau)$ is by definition zero outside the interval $[\tau, \tau+T_\mathrm{s}]$, it is

$$\ell(\mathbf{r}|\mathcal{H}_1) = \begin{cases} \int_{\tau}^{\tau+T_\mathrm{s}} (s(t-\tau)+n(t))^2 dt - \int_{\tau+z}^{\tau+z+T_\mathrm{s}} n^2(t)dt\,, & z \geq T_\mathrm{s} \\ \int_{\tau}^{\tau+z} (s(t-\tau)+n(t))^2 dt - \int_{\tau+T_\mathrm{s}}^{\tau+z+T_\mathrm{s}} n^2(t)dt, & 0 \leq z < T_\mathrm{s}. \end{cases}$$

An analogue expression can be written when $\mathcal{H}_2$ true. We now proceed with the evaluation of $P_{\min}(z)$ in (14), related to the detector (13). In the following we consider separately the case $z \geq T_\mathrm{s}$ and $0 \leq z < T_\mathrm{s}$. Moreover, as will be detailed in the derivation, it is necessary to further specify the behavior of (13) for small values of z, that is, for $z < \xi = 1/2W$.

1) Evaluation of $P_{\min}(z)$ for $z \geq T_\mathrm{s}$

For further convenience we define the RVs

$$\begin{aligned}
Y_1 &= \frac{2}{N_0}\int_{\tau}^{\tau+T_\mathrm{s}} (s(t-\tau)+n(t))^2\,dt = \sum_{n=1}^{N}\left(\sqrt{\frac{2}{N_0}}s_n + c_{1,n}\right)^2 \\
Y_2 &= \frac{2}{N_0}\int_{\tau+z}^{\tau+z+T_\mathrm{s}} n^2(t)\,dt = \sum_{n=1}^{N} c_{2,n}^2 \\
Y_3 &= \frac{2}{N_0}\int_{\tau+z}^{\tau+z+T_\mathrm{s}} (s(t-\tau-z)+n(t))^2\,dt \\
&= \sum_{n=1}^{N}\left(\sqrt{\frac{2}{N_0}}s_n + c_{2,n}\right)^2 \\
Y_4 &= \frac{2}{N_0}\int_{\tau}^{\tau+T_\mathrm{s}} n^2(t)\,dt = \sum_{n=1}^{N} c_{1,n}^2
\end{aligned} \quad (15)$$

where $\{c_{1,n}\}$, $\{c_{2,n}\}$ are related, respectively, to the series expansion coefficients of $n(t+\tau)$ and $n(t+\tau+z)$ in $t \in [0, T_\mathrm{s}]$. According to the signal model considered, $\{c_{1,n}\}$, $\{c_{2,n}\}$ are statistically independent Gaussian RVs with zero mean and unit variance. As a consequence Y_1 and Y_3 are non-central Chi-squared distributed RVs, whereas Y_2 and Y_4 are central Chi-squared distributed RVs, each having N degrees of freedom. The non-centrality parameter μ of Y_1 and Y_3 is given by $\mu = 2\,\mathsf{SNR}$, where $\mathsf{SNR} = \frac{1}{N_0}\sum_{n=1}^{N} s_n^2 = \frac{1}{N_0}\int_0^{T_\mathrm{s}} s^2(t)\,dt$. In addition Y_1, Y_2, Y_3 and Y_4 do not depend on z, then the probability of error $P_{\min}(z)$ results independent on z. Making use of (15) and thanks to the statistical symmetry, we can express $P_{\min}(z)$ in (14) for $z \geq T_\mathrm{s}$ as

$$P_{\min}^{(\mathrm{I})} = \mathbb{P}\{Y_1 < Y_2\} = P_Y(\mu/2, \lceil N/2 \rceil) \quad (16)$$

where

$$P_Y(\gamma, q) = \frac{\exp(-\gamma/2)}{2^q}\sum_{i=0}^{q-1}\frac{(\gamma/2)^i}{i!}\sum_{j=i}^{q-1}\frac{(j+q-1)!}{2^j(j-i)!(q+i-1)!}$$

whose derivation is left to the journal extended version of this work.

2) Evaluation of $P_{\min}(z)$ for $\xi \leq z < T_\mathrm{s}$

We now define the RVs

$$\begin{aligned}
Y_1 &= \frac{2}{N_0}\int_{\tau}^{\tau+z} (s(t-\tau)+n(t))^2\,dt = \sum_{m=1}^{p(z)}\left(\sqrt{\frac{2}{N_0}}\eta_{1,m} + c_{1,m}\right)^2 \\
Y_2 &= \frac{2}{N_0}\int_{\tau+T_\mathrm{s}}^{\tau+z+T_\mathrm{s}} n^2(t)\,dt = \sum_{m=1}^{p(z)} c_{2,m}^2 \\
Y_3 &= \frac{2}{N_0}\int_{\tau+T_\mathrm{s}}^{\tau+z+T_\mathrm{s}} (s(t-\tau-z)+n(t))^2\,dt \\
&= \sum_{m=1}^{p(z)}\left(\sqrt{\frac{2}{N_0}}\eta_{2,m} + c_{2,m}\right)^2 \\
Y_4 &= \frac{2}{N_0}\int_{\tau}^{\tau+z} n^2(t)\,dt = \sum_{m=1}^{p(z)} c_{1,m}^2
\end{aligned} \quad (17)$$

where $p(z) = \lfloor 2\,W\,z \rfloor + 1$, $\{c_{1,m}\}$, $\{c_{2,m}\}$ are related, respectively, to the series expansion coefficients of $n(t+\tau)$ and $n(t+\tau+T_\mathrm{s})$ in $t \in [0, z]$, and $\eta_{1,m}$ and $\eta_{2,m}$ are the series expansion coefficients of $s(t)$ in $t \in [0, z]$ and $t \in [T_\mathrm{s}-z, T_\mathrm{s}]$, respectively. Again, $\{c_{1,m}\}$, $\{c_{2,m}\}$ are statistically independent Gaussian RVs with unit variance. As a consequence Y_1 and Y_3 are non-central Chi-squared distributed RVs, whereas Y_2 and Y_4 are central Chi-squared distributed RVs, each having $p(z)$ degrees of freedom. The non-centrality parameters $\mu_1(z)$ and $\mu_2(z)$ of Y_1 and Y_3 are given by $\mu_1(z) = 2\gamma_1(z)$ and $\mu_2(z) = 2\gamma_2(z)$, where

$$\gamma_1(z) = \frac{1}{N_0}\sum_{m=1}^{p(z)}\eta_{1,m}^2 = \frac{1}{N_0}\int_0^z s^2(t)\,dt \quad (18a)$$

$$\gamma_2(z) = \frac{1}{N_0}\sum_{m=1}^{p(z)}\eta_{2,m}^2 = \frac{1}{N_0}\int_{T_\mathrm{s}-z}^{T_\mathrm{s}} s^2(t)\,dt \quad (18b)$$

now both dependent on z. Note that (18a) and (18b) represent the SNR captured in the intervals $[0, z]$ and $[T_\mathrm{s}-z, T_\mathrm{s}]$, respectively. The probability of error for $\xi \le z < T_{\mathrm{s}}$ results

$$P_{\min}^{(\mathrm{II})}(z) = \frac{1}{2}\,\mathbb{P}\{Y_1 < Y_2\} + \frac{1}{2}\,\mathbb{P}\{Y_3 < Y_4\} \qquad (19)$$
$$= \frac{1}{2}\,P_Y(\gamma_1(z), \lceil p(z)/2 \rceil) + \frac{1}{2}\,P_Y(\gamma_2(z), \lceil p(z)/2 \rceil)\,.$$

3) Evaluation of $P_{\min}(z)$ for $z < \xi$

When $z < \xi$, a low-pass signal is represented with one only coefficient. In this case we exploit approximations reported in [20], leading to $\int_0^z n^2(t)\,dt \approx N_0\,W\,z\,c_1^2$. As a consequence, we define the RVs

$$Y_1 = \frac{2}{N_0}\int_{\tau}^{\tau+z}(s(t-\tau)+n(t))^2 dt = \frac{2}{N_0}\left(\eta_{1,1} + \sqrt{N_0 W z}\,c_{1,1}\right)^2$$
$$Y_2 = \frac{2}{N_0}\int_{\tau+T_{\mathrm{s}}}^{\tau+z+T_{\mathrm{s}}} n^2(t)\,dt = 2\,W\,z\,c_{2,1}^2$$
$$Y_3 = \frac{2}{N_0}\int_{\tau+T_{\mathrm{s}}}^{\tau+z+T_{\mathrm{s}}}(s(t-\tau-z)+n(t))^2\,dt$$
$$= \frac{2}{N_0}\left(\eta_{2,1} + \sqrt{N_0\,W\,z}\,c_{2,1}\right)^2$$
$$Y_4 = \frac{2}{N_0}\int_{\tau}^{\tau+z} n^2(t)\,dt = 2\,W\,z\,c_{1,1}^2 \qquad (20)$$

where $c_{1,1}$ and $c_{2,1}$ are related to the series expansion coefficients of $n(t+\tau)$ and $n(t+\tau+T_{\mathrm{s}})$ in $t \in [0,z]$, and $\eta_{1,1}$ and $\eta_{2,1}$ are the series expansion coefficients of $s(t)$ in $t \in [0,z]$ and $t \in [T_{\mathrm{s}}-z, T_{\mathrm{s}}]$, respectively. Again, $c_{1,1}$ and $c_{2,1}$ are statistically independent Gaussian RVs with unit variance. Now $\sqrt{Y_1}$ and $\sqrt{Y_3}$ are Gaussian RVs, with mean, respectively, $\sqrt{\mu_1(z)} = \sqrt{2\gamma_1(z)}$ and $\sqrt{\mu_2(z)} = \sqrt{2\gamma_2(z)}$, where $\gamma_1(z)$ and $\gamma_2(z)$ are given by (18a) and (18b), and variance $2\,W\,z$. Differently, $\sqrt{Y_2}$ and $\sqrt{Y_4}$ are Gaussian RVs with zero mean and variance $2\,W\,z$. Therefore the probability of error for $z < \xi$ results

$$P_{\min}^{(\mathrm{III})}(z) = \frac{1}{2}\,\mathbb{P}\left\{\sqrt{Y_1} < \sqrt{Y_2}\right\} + \frac{1}{2}\,\mathbb{P}\left\{\sqrt{Y_3} < \sqrt{Y_4}\right\}$$
$$= \frac{1}{2}\,Q\left(\sqrt{\frac{\gamma_1(z)}{2\,W\,z}}\right) + \frac{1}{2}\,Q\left(\sqrt{\frac{\gamma_2(z)}{2\,W\,z}}\right) \qquad (21)$$

where $Q(\cdot)$ is the Gaussian Q-function.

By substituting (16), (19), and (21) in (8), we obtain the ZZB on TOA estimation MSE of unknown deterministic signals, that is

$$\mathsf{ZZB} = \frac{1}{T_{\mathrm{a}}}\int_0^{1/2W} z(T_{\mathrm{a}}-z)P_{\min}^{(\mathrm{III})}(z)\;dz \qquad (22)$$
$$+ \frac{1}{T_{\mathrm{a}}}\int_{1/2W}^{T_{\mathrm{s}}} z(T_{\mathrm{a}}-z)P_{\min}^{(\mathrm{II})}(z)\,dz + \left(\frac{T_{\mathrm{a}}^2}{6} - \frac{T_{\mathrm{s}}^2}{2} + \frac{T_{\mathrm{s}}^3}{3T_{\mathrm{a}}}\right)P_{\min}^{(\mathrm{I})}\,.$$

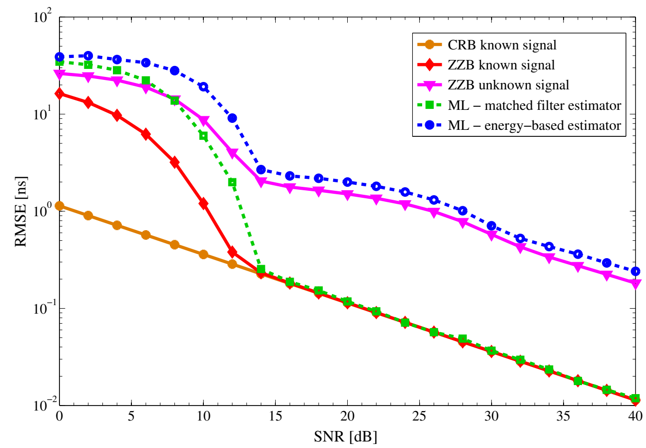

Fig. 1. CRB and ZZB for known and unknown signals. Dashed lines denote the performance of the MF estimator (known signal), and the energy-based estimator (unknown signal).

4. NUMERICAL RESULTS

We consider a root raised cosine (RRC) received pulse with pulse width parameter $T_{\mathrm{w}} = 3.2$ ns and roll-off $\nu = 0.6$. This signal is exactly band-limited, so its time-limited version, obtained by cutting its main two lobes, is considered. Figure 1 shows the root-mean-squared error (RMSE) predicted by the CRB [7] and the ZZB [13] in the case of known signals, and the derived ZZB (22) valid for unknown received signal. A receiver bandwidth $W = 8/T_{\mathrm{w}}$ is considered. For comparison, the performance of the ML estimator, that is the matched filter (MF) estimator in the case of known signals, and the performance of a classical energy-based TOA estimator [6]

$$\hat{\tau} = \underset{\tau}{\operatorname{argmax}} \ln p\{\mathbf{r}|\tau, \mathbf{s}=\hat{\mathbf{s}}\} = \underset{\tau}{\operatorname{argmax}} \int_{\tau}^{\tau+T_{\mathrm{s}}} r^2(t)\,dt \quad (23)$$

in case of unknown signals, are depicted in the figure.

The presence of the threshold effect is evident from the ZZB. On the other hand this behavior cannot be observed in the CRB. For high SNR the estimation error of the MF approaches that predicted by the CRB and the ZZB for known signals [13]. The CRB and ZZB obtained considering the signals as they were known are very loose, also in the high SNR region, in comparison with the performance of energy-based estimator (23), which does not require any knowledge on the signal. In this case the new expression (22) provides a very tight and more realistic bound for all the range of SNR.

5. CONCLUSION

We have presented the ZZB on time delay estimation error of unknown deterministic signals. The derived ZZB foresees the presence of different regions, clearly showing the SNRs threshold values of the ambiguity region. The proposed new bound results very tight with the actual performance of energy-based TOA estimators, usually adopted in the presence of unknown signals.

6. REFERENCES

[1] G. C. Carter, "Coherence and time delay estimation," *Proc. IEEE*, vol. 75, no. 2, pp. 236–255, Feb. 1987.

[2] A. J. Weiss and E. Weinstein, "Fundamental limitations in passive time delay estimation - Part I: Narrow-band systems," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 31, no. 2, pp. 472–486, Apr. 1983.

[3] B. Sadler and R. Kozick, "A survey of time delay estimation performance bounds," in *IEEE Work. on Sensor Array and Multichannel Process.*, Waltham, MA, Jul. 2006, pp. 282–288.

[4] D. Dardari, A. Conti, U. Ferner, A. Giorgetti, and M. Win, "Ranging with ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009.

[5] D. Dardari, E. Falletti, and M. Luise, *Satellite and Terrestrial Radio Positioning Techniques - A signal processing perspective*. Elsevier Ltd, London, 2011.

[6] C. Carbonelli and U. Mengali, "Synchronization algorithms for UWB signals," *IEEE Trans. Commun.*, vol. 54, no. 2, pp. 329–338, Feb. 2006.

[7] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*, 1st ed. New York, NY 10158-0012: John Wiley & Sons, Inc., 1968.

[8] J. Ziv and M. Zakai, "Some lower bounds on signal parameter estimation," *IEEE Trans. Inform. Theory*, vol. 15, no. 3, pp. 386–391, May 1969.

[9] S. Bellini and G. Tartara, "Bounds on error in signal parameter estimation," *IEEE Trans. Commun.*, vol. 22, no. 3, pp. 340–342, Mar. 1974.

[10] E. Weinstein and A. J. Weiss, "Fundamental limitations in passive time delay estimation - Part II: Wide-band systems," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 32, no. 5, pp. 1064–1078, Oct. 1984.

[11] ——, "A general class of lower bounds in parameter estimation," *IEEE Trans. Inform. Theory*, vol. 34, no. 2, pp. 338–342, Mar. 1988.

[12] D. Dardari and M. Z. Win, "Ziv-Zakai bound of time-of-arrival estimation with statistical channel knowledge at the receiver," in *Proc. IEEE Int. Conf. on Ultra-Wideband (ICUWB)*, Vancouver, BC, Canada, Sep. 2009, pp. 624–629.

[13] D. Dardari, C.-C. Chong, and M. Z. Win, "Improved lower bounds on time-of-arrival estimation error in realistic UWB channels," in *Proc. IEEE Int. Conf. on Ultra-Wideband (ICUWB)*, Waltham, MA, Sep. 2006, pp. 531–537.

[14] B. M. Sadler, L. Huang, and Z. Xu, "Ziv-Zakai time delay estimation bound for ultra-wideband signals," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Process.*, vol. 3, Honolulu, HI, Apr. 2007, pp. 549–552.

[15] D. Slepian, "On bandwidth," *Proc. IEEE*, vol. 64, no. 3, pp. 292–300, Mar. 1976.

[16] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, no. 4, pp. 523–531, Apr. 1967.

[17] H. Landau and H. Pollak, "Prolate spheroidal wave functions, Fourier analysis and uncertainty: Part III," *Bell Sys. Tech. Journal*, vol. 41, pp. 1295–1336, 1962.

[18] D. Chazan, M. Zakai, and J. Ziv, "Improved lower bounds on signal parameter estimation," *IEEE Trans. Inform. Theory*, vol. 21, no. 1, pp. 90–93, Jan. 1975.

[19] J. Gabriel and S. Kay, "On the relationship between the GLRT and UMPI tests for the detection of signals with unknown parameters," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4194–4203, Nov. 2005.

[20] D. Slepian and E. Sonnenblick, "Eigenvalues associated with prolate spheroidal wave functions of zero order," *Bell Sys. Tech. Journal*, vol. 44, 1965.