FUNDAMENTAL LOCALIZATION ACCURACY IN NARROWBAND ARRAY-BASED SYSTEMS

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ABSTRACT

Location-awareness is essential for many wireless network applications in both civil and military sectors. In this paper, we determine the localization accuracy of narrowband localization systems in which each mobile agent is equipped with an antenna array. Due to non-coherent estimators, the phases of the received signals can only be exploited for angle-of-arrival (AOA) estimation but not time-of-arrival (TOA). Based on such estimators, we derive the fundamental localization accuracy in terms of the squared position error bound (SPEB) in far-field harsh multipath environments. Moreover, we characterize the effects of the geometry of anchors and array antennas on the localization accuracy, yielding the criteria for optimal array design and network deployment. Our analysis exploits all the TOA and AOA information in the received waveform for localization using narrowband array-based systems, and the resulting SPEB serves as a fundamental limit for such systems.

***Index Terms*—** Antenna arrays, Geometric property, Localization, TOA/AOA, Wireless networks

1. INTRODUCTION

Localization services are essential in many civil and military applications, and currently are mainly provided by the global positioning system (GPS), with effectiveness severely degraded in harsh environments, e.g. in buildings, caves, and urban canyons [1], [2]. The wireless network is a well-performed alternative to handle the degradation, where localization is accomplished by the radio communications between nodes. Specifically, by processing the received signal, some signal metrics can be extracted for localization. Commonly used metrics include time-of-arrival (TOA) [1, 2], time-difference-of-arrival (TDOA) [3, 4], angle-of-arrival (AOA) [2, 5, 6], and received signal strength (RSS) [7].

These localization methodologies all extract some metrics from the received waveforms, i.e. distance (TOA) or direction (AOA), and then use trilateration and triangulation for localization [5, 8]. However, since the extracted metrics may discard useful information for localization, e.g., correlations between metrics, we adopt an alternative methodology which utilizes the received waveforms directly to derive a fundamental bound [9, 10].

In practical systems, the signal propagation will encounter the problem of noise, fading, shadowing, multipath (signal reaches the receiver via multiple paths due to reflection and penetration) and non-line-of-sight (NLOS, i.e. first arriving signal does not travel on a line) [8], which imposes uncertainties on localization. A commonly used lower bound for error estimation is Cramér-Rao bound [5, 8–10], and it is shown in [10] that joint processing between array antennas, i.e. AOA, cannot provide additional information to existing TOA localization information in terms of Cramér-Rao bound for *wideband* systems. However, we will emphasize the contribution of AOA information for narrowband systems in this paper.

The main contributions of this paper are listed as follows. Firstly, we establish a RF signal model to characterize the unavailability for using transmission wave in TOA. Next we derive the equivalent Fisher information matrix (EFIM) for agent's position based on received waveforms and find that it is a SNR-weighed sum of the measuring information from each anchor-antenna pair which contains both distance and direction information. In particular, AOA provides main information for localization in narrowband cases. In addition, we give a tightest SPEB by selecting a proper reference point in the array and then study both impacts of anchors' and array's geometric structure, in which we measure the geometry into some simple metrics, and give criteria for the optimal geometric design for array and anchors.

2. SYSTEM MODEL

Consider a 2-D wireless network with N_{b} anchors and one agent with an array consisting of N_{a} antennas. Anchors have perfect knowledge of their positions, and agent attempts to estimate its position based on received waveforms from neighboring anchors. Let $\boldsymbol{p}_k^{\text{Array}} = (x_k^{\text{Array}}, y_k^{\text{Array}}) \in \mathbb{R}^2$ be the position of the $k(k = 1, 2, \cdots, N_{\mathrm{a}})$-th antenna in the array to be estimated, and $\boldsymbol{p}_j = (x_j, y_j) \in \mathbb{R}^2$ be the position of anchor $j(j \in \mathcal{N}_{\mathrm{b}})$, where $\mathcal{N}_{\mathrm{b}} = \{1, 2, \cdots, N_{\mathrm{b}}\} = \mathcal{N}_{\mathrm{L}} \cup \mathcal{N}_{\mathrm{NL}}$ denotes the set of all anchors, and anchors in $\mathcal{N}_{\mathrm{L}}$ and $\mathcal{N}_{\mathrm{NL}}$ provide line-of-sight (LOS) and NLOS signals respectively. Due to the fixed relative position in the array, the array's whole

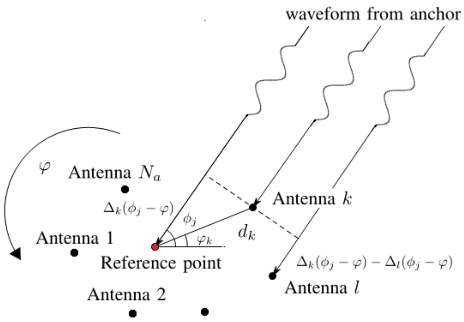


Fig. 1: The array system rotated (with orientation φ) around the predetermined reference point. There are N_a antennas in the array, each of which can be described by the distance d_k and initial angle (with no rotation) φ_k. Waveforms from one anchor are parallel to each other with an identical angle ϕ_j, and $\Delta_k(\phi_j-\varphi)-\Delta_l(\phi_j-\varphi)$ is the wavepath difference between antenna k and antenna l.

status has 3 degrees of freedom, i.e. it can be characterized by $\boldsymbol{p}=(x,y)$, the position of a reference point predetermined relative to the array, and array orientation φ:

$$\boldsymbol{p}_k^{\text{Array}}=\boldsymbol{p}+d_k\begin{bmatrix}\cos(\varphi+\varphi_k)\\ \sin(\varphi+\varphi_k)\end{bmatrix} \tag{1}$$

where d_k is the distance between the reference point and k-th antenna, φ_k is the initial angle when $\varphi=0$ (see Fig.1).

Each anchor provides a known signal to each array antenna, thus antenna k's received waveform from anchor j is [8, 11]

$$r_{jk}(t)=\sum_{l=1}^{L_{jk}}\alpha_{jk}^{(l)}e^{\sqrt{-1}\omega_{jk}^{(l)}(t-\tau_{jk}^{(l)})}s(t-\tau_{jk}^{(l)})+z_{jk}(t) \tag{2}$$

where $s(t)$ is a known complex signal, $\alpha_{jk}^{(l)}$ and $\tau_{jk}^{(l)}$ are the amplitude and delay, respectively, of the l-th path, and L_{jk} is the number of multipath components (MPCs), $z_{jk}(t)$ represents the complex observation noise modeled as additive white Gaussian noise (AWGN) with two-side power spectral density $N_0/2$, $\omega_{jk}^{(l)}$ is the Doppler shift of the specular reflection (zero if $l=1$), and $t\in[0,T_{\text{ob}})$ is the observation interval.

We adopt the far-field assumption in our model, i.e. the distance between the anchor and the agent is far enough to obtain an identical anchor direction to all array antennas:

$$\phi_j\triangleq\phi_{jk}\triangleq\tan^{-1}\frac{y_j-y}{x_j-x},\qquad\forall j\in\mathcal{N}_\text{b} \tag{3}$$

Hence, the time delay can be written as

$$\begin{aligned}\tau_{jk}^{(l)}&=\frac{\|\boldsymbol{p}_j-\boldsymbol{p}\|+d_k\cos(\phi_j-\varphi-\varphi_k)+b_{jk}^{(l)}}{c}\\ &=:\tau_j+\frac{-\Delta_k(\phi_j-\varphi)+b_{jk}^{(l)}}{c}\end{aligned} \tag{4}$$

where c is the propagation speed of the signal, $\|\cdot\|$ is the Euclidean norm, and $b_{jk}^{(l)}$ is the non-negative range bias of l-th path (when $l=1$, it is zero for LOS propagation, and is positive for NLOS). Additionally, we assume that array antennas are quite close to each other to guarantee phase differences between received signals in adjacent antennas less than 2π.

Our observation $\boldsymbol{r}$ consists of all received waveforms $r_{jk}(t)$, and the parameter $\boldsymbol{\theta}$ to be estimated is given by

$$\boldsymbol{\theta}\triangleq\begin{bmatrix}\boldsymbol{p}^{\boldsymbol{T}} & (\boldsymbol{\kappa}_{11}^{(1)})^{\boldsymbol{T}} & (\boldsymbol{\kappa}_{11}^{(2)})^{\boldsymbol{T}} & \cdots & (\boldsymbol{\kappa}_{N_\text{b}N_\text{a}}^{(L_{N_\text{b}N_\text{a}})})^{\boldsymbol{T}}\end{bmatrix}^{\boldsymbol{T}} \tag{5}$$

$$\boldsymbol{\kappa}_{jk}^{(l)}\triangleq\begin{cases}\begin{bmatrix}\text{Para}(b_{jk}^{(1)}) & \alpha_{jk}^{(1)}\end{bmatrix}^{\boldsymbol{T}} & l=1,\\ \begin{bmatrix}b_{jk}^{(l)} & \omega_{jk}^{(l)} & \alpha_{jk}^{(l)}\end{bmatrix}^{\boldsymbol{T}} & l>1.\end{cases} \tag{6}$$

where $\text{Para}(b_{jk}^{(1)})$ is 0 (empty) if $j\in\mathcal{N}_\text{L}$, and is $b_{jk}^{(1)}$ elsewhere.

The CRB states that for any unbiased estimator $\hat{\boldsymbol{p}}$ for $\boldsymbol{p}$,

$$\mathbb{E}_{\boldsymbol{r}}[\|\hat{\boldsymbol{p}}-\boldsymbol{p}\|^2]\geq\text{tr}\{[\boldsymbol{J}_{\boldsymbol{\theta}}^{-1}]_{2\times2}\}=\text{tr}\{\boldsymbol{J}_\text{e}^{-1}(\boldsymbol{p})\} \tag{7}$$

where $\boldsymbol{J}_{\boldsymbol{\theta}}$ is the Fisher information matrix (FIM) for the parameter vector $\boldsymbol{\theta}$ based on observation $\boldsymbol{r}$, $\boldsymbol{J}_\text{e}(\boldsymbol{p})$ is the 2×2 E-FIM obtained by Schur complement of $\boldsymbol{J}_{\boldsymbol{\theta}}$ [10], and the SPEB is defined as the right-hand side expression in (7).

This methodology can derive SPEB for all signal types, but in this paper, we restrict the signal in a RF signal form and define its effective bandwidth β as follows:

$$s(t)=s_0(t)\exp(j2\pi f_c t) \tag{8}$$

$$\beta\triangleq\left[\frac{\int_{-\infty}^{\infty}f^2|S_0(f)|^2\text{d}f}{\int_{-\infty}^{\infty}|S_0(f)|^2\text{d}f}\right]^{\frac{1}{2}} \tag{9}$$

where $S_0(f)$ is the Fourier transform for $s_0(t)$, and a small β/f_c indicates that the signal is narrowband.

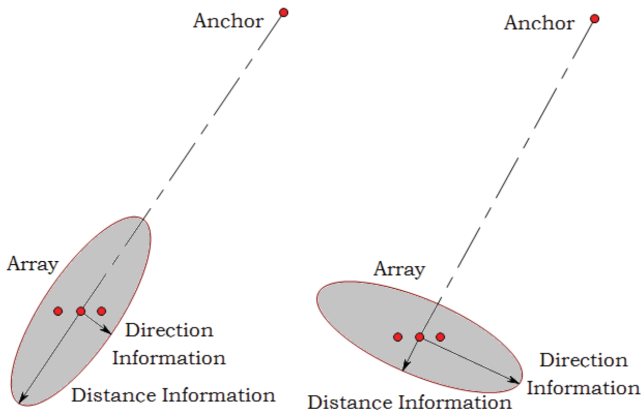
The main purpose for establishing this model is that, in practical TOA systems, the complex envelope $s_0(t)$ rather than the entire signal $s(t)$ is used in matched filtering process [12]. Hence, the differentiation in the CRB for time delay τ_j can only operate on signal envelope, while that for direction ϕ_j can also operate on transmission frequency [13]. This characterizes the fact that AOA can utilize the transmission waveform but TOA cannot, and is the basis for our later conclusion that AOA does improve the localization accuracy.

3. DERIVATION FOR EFIM

We call a RF signal relatively narrowband iff the spectrum of baseband signal gathers around a certain frequency. Under this assumption, the expression of EFIM is shown in Theorem 1, where all proofs are omitted for space limitations.

Theorem 1. *When the orientation φ is known, the EFIM for position is*

$$\boldsymbol{J}_\text{e}(\boldsymbol{p})=\sum_{j\in\mathcal{N}_\text{L}}\sum_{k=1}^{N_\text{a}}\lambda_{jk}\boldsymbol{q}_{jk}\boldsymbol{q}_{jk}^{\boldsymbol{T}} \tag{10}$$



(a) High β/f_c: distance provides main measuring information. (b) Low β/f_c: direction provides main measuring information.

Fig. 2: Each anchor-antenna pair forms ranging information in two orthogonal directions, where the major part differs with β/f_c.

where

$$\lambda_{jk} \triangleq \frac{8\pi^2 \mathrm{SNR}_{jk}^{(1)}(1-\chi_{jk})}{c^2} \tag{11}$$

$$\boldsymbol{q}_{jk} \triangleq \beta \boldsymbol{q}_j^{(\mathrm{TOA})} + \frac{f_c}{D_j}\left(\frac{\partial}{\partial \phi_j}\Delta_k(\phi_j - \varphi)\right)\boldsymbol{q}_j^{(\mathrm{AOA})} \tag{12}$$

$$\boldsymbol{q}_j^{(\mathrm{TOA})} \triangleq \begin{bmatrix} \cos\phi_j \\ \sin\phi_j \end{bmatrix}, \boldsymbol{q}_j^{(\mathrm{AOA})} \triangleq \begin{bmatrix} -\sin\phi_j \\ \cos\phi_j \end{bmatrix} \tag{13}$$

$$\mathrm{SNR}_{jk}^{(l)} \triangleq \frac{|\alpha_{jk}^{(l)}|^2}{N_0}\int_{-\infty}^{\infty}|S_0(f)|^2 df \tag{14}$$

and $\mathrm{SNR}_{jk}^{(l)}$ *is the signal-to-noise ratio,* $\chi_{jk} \in [0,1]$ *is the Path-Overlap Coefficient (POC) defined in [10],* D_j *is the distance from anchor* j *to the reference point.*

Theorem 1 indicates that the EFIM for position is a weighed sum of measuring information (i.e. $\boldsymbol{q}_{jk}\boldsymbol{q}_{jk}^{\boldsymbol{T}}$) from each anchor-antenna pair with intensity λ_{jk}, where high intensity requires large SNR of first arriving signal, little severity in path-overlap and LOS signals. In particular, NLOS signals provide no information for localization, and $\chi_{jk} = 0$ if the first arriving signal does not overlap with the others.

Note that $\boldsymbol{q}_{jk}$ is a weighed sum of $\boldsymbol{q}_j^{(\mathrm{TOA})}$, the distance (TOA) information with direction towards the anchor, and $\boldsymbol{q}_j^{(\mathrm{AOA})}$, the direction (AOA) information with direction perpendicular to the anchor, with weights proportional to β and f_c, respectively. Hence, each anchor-antenna pair provides measuring information in two orthogonal directions (see Fig.2). Obviously, in narrowband cases (i.e. $\beta \ll f_c$), AOA provides the main information for localization.

4. GEOMETRIC PROPERTIES FOR SPEB

Since EFIM in (10) is affected by the selection of reference point, a tighter SPEB can be obtained via a better reference point selection. Generally, when localization is possible, we can prove that there must exist a reference point yielding a tightest SPEB. This point can be found explicitly based on the far-field assumption that $\lambda_j \triangleq \lambda_{jk}, \forall k$ in certain cases, such as the TOA-only (AOA-only) cases where direction (distance) information is small enough to be safely neglected.

Theorem 2. *In both TOA-only and AOA-only cases, setting the reference point to array coordinate center yields a tightest SPEB, where the EFIM is* $\boldsymbol{J}_{\mathrm{e}}(\boldsymbol{p}) = N_{\mathrm{a}}\sum_{j\in\mathcal{N}_{\mathrm{L}}}\lambda_j \boldsymbol{I}_j$*, and*

$$\boldsymbol{I}_j \triangleq \begin{cases} \beta^2 \boldsymbol{q}_j^{(\mathrm{TOA})}(\boldsymbol{q}_j^{(\mathrm{TOA})})^{\boldsymbol{T}} & \textit{TOA-only} \\ (f_c/D_j)^2 G(\phi_j - \varphi)\boldsymbol{q}_j^{(\mathrm{AOA})}(\boldsymbol{q}_j^{(\mathrm{AOA})})^{\boldsymbol{T}} & \textit{AOA-only} \end{cases} \tag{15}$$

and we call $G(\theta)$ *as the array geometric factor (AGF):*

$$G(\theta) \triangleq \frac{1}{N_{\mathrm{a}}^2}\sum_{1\le k<l\le N_{\mathrm{a}}}\left[\frac{\mathrm{d}}{\mathrm{d}\theta}(\Delta_k(\theta) - \Delta_l(\theta))\right]^2 \tag{16}$$

To gain insights into array geometry, (15) indicates that a larger AGF yields to a larger EFIM. In particular, $G(\theta) = 0$ when $N_{\mathrm{a}} = 1$, thus direction information needs joint processing among array antennas. Furthermore, the name AGF is originated from the fact that $G(\theta)$ fully characterizes array geometry, i.e. arrays with identical AGF perform identically for localization.

We define the array diameter as the diameter of a smallest circle which can cover the array. In addition, we call the array with $G(\theta)$ invariant with θ and all antennas placed on a circle centered at its coordinate center as uniformly circular oriented array (UCOA). The following theorem indicates that, UCOA has the best performance in terms of expected SPEB given that the orientation φ is uniformly distributed in $[0, 2\pi)$. This is a guideline for designing the optimal array geometry when *expected* SPEB is of our concern, e.g., we cannot adjust the array orientation freely because it is fixed on the car roof.

Theorem 3. *For fixed anchor locations and array diameter,*

$$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{SPEB}(\boldsymbol{p},\varphi)d\varphi \ge \mathrm{SPEB}^{\mathrm{UCOA}}(\boldsymbol{p}) \tag{17}$$

where $(\boldsymbol{p},\varphi)$ *indicates array position* $\boldsymbol{p}$ *and orientation* φ*.*

As for the assessment of the choice for anchor directions, Theorem 4 below provides a useful measure.

Theorem 4. *Denote* $u_i \triangleq \lambda_i/D_i^2$ *in AOA-only case provided that the array is UCOA, or* $u_i \triangleq \lambda_i$ *in TOA-only case, then*

$$\mathrm{SPEB} \propto \left[\left|\sum_{i\in\mathcal{N}_{\mathrm{L}}} u_i\right|^2 - \left|\sum_{i\in\mathcal{N}_{\mathrm{L}}} u_i \exp(j2\phi_i)\right|^2\right]^{-1} \tag{18}$$

Theorem 4 indicates that $|\sum_{i\in\mathcal{N}_{\mathrm{L}}} u_i \exp(j2\phi_i)|$ is a good measure for anchor geometry, and its small value is preferred to enhance accuracy. Note that this measure is small when anchor directions are weighed uniformly distributed, implying that it's better to place anchors on diversified directions.

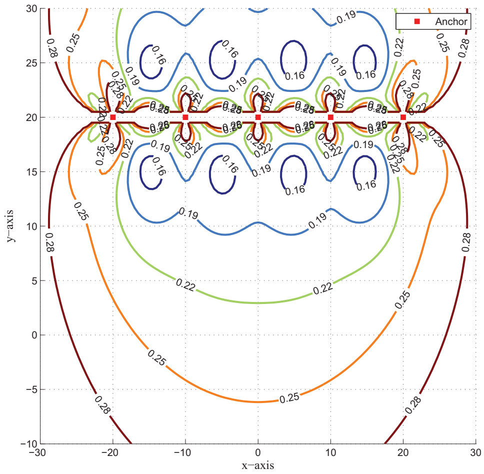


Fig. 3: PEB contours with 5 anchors placed on a line and ULA with $N_{\mathrm{a}} = 6$. $\beta/f_c = 0.01$ and the array is parallel to anchors.

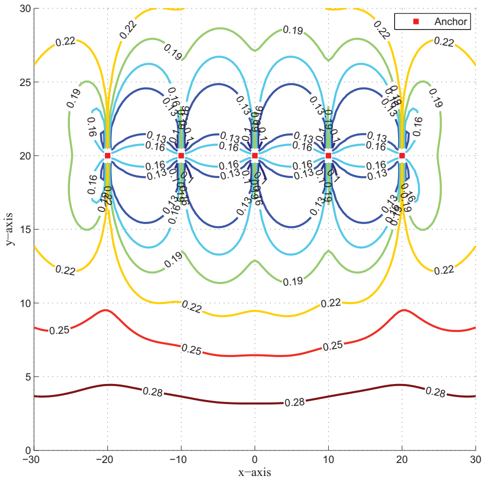


Fig. 4: PEB contours with 5 anchors placed on a line and ULA with $N_{\mathrm{a}} = 6$. $\beta/f_c = 0.01$ and the array is perpendicular to anchors.

5. NUMERICAL RESULTS

We place five identical anchors on $(-10k, 20)(-2 \leq k \leq 2)$ and a uniformly linear array (ULA) with diameter 1 and six antennas. All intensities in (11) are set to be unit for simplicity, and the relative bandwidth is $\beta/f_c = 0.01$. Then Fig.3 and Fig.4 show the PEB (defined as root SPEB) contours with arrays parallel and perpendicular to anchors respectively.

When array is parallel to anchors, the line on which anchors are placed yields the lowest accuracy since AGF is zero on it. Moreover, there are valleys on contour map, where anchor geometry and distance are balanced for localization. However, when array is perpendicular to anchors, there are peaks on $x = 10k(-2 \leq k \leq 2)$ for similar reasons on AGF, but the line $y = 20$ no longer yields lowest accuracy.

Then we fix the array position on $(0, 0)$ and change its orientation from 0 (parallel to anchors) to $\pi/2$ (perpendicular to anchors) with different relative bandwidths (f_c remains constant). Fig.5 illustrates the relationship between PEB and orientation. When β/f_c is large, PEB is almost invariant with orientation, which does not hold with small β/f_c. In addition, $\varphi = 0$ yields the lowest PEB, which can be theoretically explained by largest AGF obtained in this orientation. Moreover, when β decreases with f_c fixed, the PEB rises but tends to converge. This means AOA can provide information alone when there is no available TOA information.

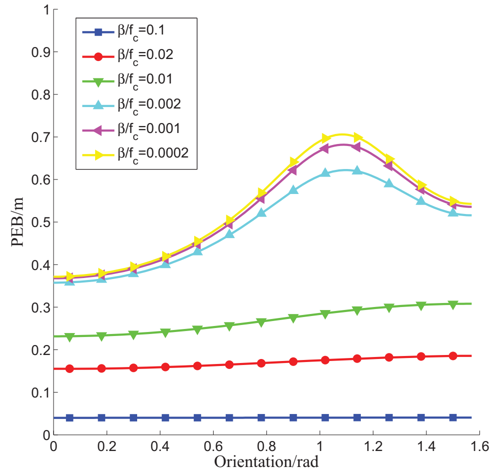


Fig. 5: PEB(φ) with fixed f_c and $\boldsymbol{p} = (0, 0)$. Six curves correspond to $\beta/f_c = 0.1, 0.02, 0.01, 0.002, 0.001, 0.0002$, respectively.

6. CONCLUSION

In this paper, we determined the localization accuracy of narrowband array-based systems, where the TOA and AOA are obtained by the received signal envelope and phases, respectively. Under far-field conditions, we showed that the EFIM for position is a weighed sum of measuring information from each anchor-antenna pair, which can be decomposed into distance and direction parts with orthogonal information direction. Unlike localization systems using wideband antenna arrays, the AOA provides the main information for localization for the narrowband case. In addition, we derived the tightest SPEB by choosing an optimal reference point and characterized the effects of array and anchor geometry. We found that UCOA and anchors with weighed uniform direction distributions have the best performance in localization. These results can be used as design guidelines for localization systems, and serve as a basis for further research to relax the restrictions in our work such as TOA(AOA)-only cases and UCOA.

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