GLOBALLY OPTIMAL JOINT UPLINK BASE STATION ASSOCIATION AND POWER CONTROL FOR MAX-MIN FAIRNESS

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ABSTRACT

In a heterogeneous network (HetNet) with a large number of low power base stations (BSs), proper user-BS association and power control is crucial to achieving desirable system performance. In this paper, we consider the joint BS association and power allocation problem for an uplink cellular network under the max-min fairness criterion. We first present a binary search method whereby a QoS (Quality of Service) constrained subproblem is solved at each step. Then, we propose a normalized fixed point iterative algorithm to directly solve the original problem and prove its geometric convergence to the global optimal solution, which implies the pseudo-polynomial time solvability of the considered problem. Simulation results show that the proposed normalized fixed point iterative algorithm converges much faster than the binary search method.

1. INTRODUCTION

To meet the surging mobile traffic demand, wireless cellular networks have increasingly relied on low power transmit nodes such as pico base stations (BS) to work in concert with the existing macro BSs. Such a heterogeneous network (HetNet) architecture can provide substantially improved data service to cell edge users.

One crucial problem in the system design of future networks is how to associate mobile users with serving BSs. The conventional greedy scheme that associates receivers with the transmitter providing the strongest signal and its modern variant Range Extension [1] may be suboptimal during periods of congestion. A more systematic approach is to jointly design BS association and other system parameters so as to maximize a network-wide utility. The early work in this direction [2] and [3] proposed a fixed point iteration to jointly adjust BS association and power allocation in the uplink, while subject to QoS (Quality of Service) constraints, with the goal to minimize total transmit power. The global convergence of this algorithm has been established when the problem is feasible. This algorithm has been extended to the uplink transmission with power budget constraints [4] and the downlink transmission [5].

Recently, various approaches have been applied to tackle the BS association problem [6–10]. The work in [6] proposed to solve a utility maximization problem by alternately optimizing over BS association and other system parameters. References [7, 8] considered a partial CoMP (Coordinated Multiple Point) transmission strategy, i.e., allowing one user to be served by multiple BSs. They proposed sparse optimization techniques to compute a desirable BS association that incurs low overhead. References [9, 10] studied the joint design of BS association and frequency resource allocation for a fixed transmission power.

The computational complexity of maximizing a certain utility function by joint BS association and power allocation has been studied in different scenarios [11–13]. For the sum rate utility function, the NP hardness of the joint design problem has been established for both the uplink transmission [11] and the downlink transmission [12]. For the max-min fairness utility, the joint design problem with an equal number of users and BSs was shown to be polynomial time solvable under additional QoS constraints [8]. However, the computational complexity of the general case (possibly unequal number of users and BSs, no QoS constraints) remains unknown.

In this paper, we consider the joint BS association and power allocation problem under the the max-min fairness criterion in an uplink cellular network. We show that this problem can be solved to global optimality via a binary search strategy in which the QoS constrained subproblems are solved by an algorithm in [4]. Also, similar to the work of [14], we propose a normalized fixed point iteration to solve the max-min fairness problem directly. Using results from the concave Perron-Frobenius theory [15, 16], we prove the geometric convergence of the proposed algorithm to the global optimal solution. An immediate consequence is that the considered problem is pseudo-polynomial time solvable. The simulation results show that the normalized fixed point iteration converges much faster than the binary search method.

The normalized fixed point algorithm proposed in this paper extends the previous work of [2–4, 17, 18]. A fixed point iteration was first proposed in [17] to solve the QoS constrained problem for a fixed BS association, and was extended

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to solve the joint BS association and power control problem in [2, 3]. A unified framework that generalizes these algorithms has been presented in [4], but this class of algorithms cannot solve the max-min fairness problem directly (i.e. without binary search). The idea of inserting normalization steps in the fixed point iterations dates back to [18], which solved the max-min fairness problem for a fixed BS association in the noiseless case. Recently, the normalized fixed point iteration approach was applied to solve the max-min fairness problem in the noisy case [14], again assuming a fixed base station association. A main contribution of this paper is to propose a normalized fixed point algorithm for optimal joint BS association and power control which provably achieves the globally maximum of min-rate.

2. SYSTEM MODEL AND PROBLEM FORMUALTION

Consider an uplink cellular network where K mobile users transmit to N base stations (BS). Both the BSs and the users are equipped with a single antenna, and they share the same time/frequency resource for transmission. Each user is to be associated with exactly one BS, but one BS can serve multiple users. Our goal is to maximize the minimum rate by joint BS association and power allocation, subject to the power constraint of each user.

Let $\boldsymbol{a} = (a_1, a_2, \ldots, a_K)$ denote the association profile, i.e., $a_k = i$ if user k is associated with BS i. Denote by $\boldsymbol{p} = [p_1, p_2, \ldots, p_K]$ the transmit power vector, where p_k is the transmit power of user k. Suppose the power budget of user k is $\bar{p}_k$. Denote by g_{ik} the channel gain between user k and BS i.

The problem of maximizing the minimum SINR by joint BS association and power control is formulated as follows:

$$\begin{aligned} \max_{\boldsymbol{p},\boldsymbol{a}} \min_{k=1,\ldots,K} \ & \mathrm{SINR}_k = \frac{g_{a_k k} p_k}{\sigma_{a_k}^2 + \sum_{j\neq k} g_{a_k j} p_j}, \\ s.t. \quad & 0 \le p_k \le \bar{p}_k, \ k = 1, \ldots, K, \\ & a_k \in \{1, 2, \ldots, N\}, \ k = 1, \ldots, K, \end{aligned} \tag{1}$$

where $\sigma_k^2 > 0$ is the receive noise power at user k.

Optimizing $\boldsymbol{p}$ and $\boldsymbol{a}$ separately is easy. Specifically, given a fixed BS association $\boldsymbol{a}$, the formulation (1) is a max-min fairness power control problem for an interfering MAC channel. It can be solved in polynomial time using a binary search strategy whereby a QoS constrained subproblem is solved by LP (Linear Programming) at each step [19]. Moreover, given a power vector $\boldsymbol{p}$, the optimal association of each user k does not depend on the choices of other users and can be easily computed [1]:

$$a_k = \arg \max_{n\in\{1,\ldots,N\}} \left\{ \frac{g_{nk} p_k}{\sigma_n^2 + \sum_{j\neq k} g_{nj} p_j} \right\}. \tag{2}$$

[1]This is because the interference for user k is $\sum_{j\neq k} g_{a_k j} p_j$, which only depends on a_k, and does not depend on $a_j, \forall j \neq k$.

However, it is not straightforward to jointly optimize the continuous variables $\boldsymbol{p}$ and the discrete variables $\boldsymbol{a}$, and this is the focus of our work.

3. A BINARY SEARCH METHOD

The max-min fairness problem is closely related to the QoS (Quality of Service) constrained problem, i.e., minimize the total transmission power subject to the QoS (Quality of Service) constraints. The QoS constrained joint BS association and power allocation problem is given as follows:

$$\begin{aligned} \min_{\mathbf{p},\mathbf{a}} \ & \sum_{k=1}^{K} p_k, \\ s.t. \quad & 0 \le p_k \le \bar{p}_k, \ k = 1, \ldots, K, \\ & a_k \in \{1, 2, \ldots, N\}, \ k = 1, \ldots, K, \\ & \mathrm{SINR}_k = \frac{g_{a_k k} p_k}{\sigma_{a_k}^2 + \sum_{j\neq k} g_{a_k j} p_j} \ge \gamma, k = 1, \ldots, K. \end{aligned} \tag{3}$$

Problem (1) can be solved by a sequence of subproblems of the form (3) and a binary search on γ. Each of these subproblems can be solved by an existing algorithm presented below [4]. Define

$$T_k^{(n)}(\boldsymbol{p}) \triangleq \left\{ \frac{\sigma_n^2 + \sum_{j\neq k} g_{nj} p_j}{g_{nk}} \right\}, \tag{4}$$

$$T_k(\boldsymbol{p}) \triangleq \min_{1\le n\le N} T_k^{(n)}(\boldsymbol{p}), \tag{5}$$

$$A_k(\boldsymbol{p}) \triangleq \arg\min_n T_k^{(n)}(\boldsymbol{p}). \tag{6}$$

Notice that $T_k^{(n)}(\boldsymbol{p})$ represents the minimum power needed by user k to achieve a SINR value of 1 if its associated BS is n and the power of other users are fixed at $p_j, \forall j \neq k$. The minimum power user k needs to achieve a SINR level of 1 among all possible choices of BS association is defined as $T_k(\boldsymbol{p})$, and the corresponding BS association is defined as $A_k(\boldsymbol{p})$. Note that the BS association a_k defined in (2) is precisely $A_k(\boldsymbol{p})$.

The algorithm of [4] starts from any positive vector $\boldsymbol{p}(0)$, and updates the power vector by

$$p_k(t+1) = \min\{\gamma T_k(\boldsymbol{p}(t)), \bar{p}_k\}, \ k = 1, \ldots, K, \tag{7}$$

where $\boldsymbol{p}(t) = (p_1(t), \ldots, p_K(t))$ is the power vector at the t-th iteration. It has been shown in [4, Section V.B,Corollary 1] that the above procedure (7) converges to $\boldsymbol{q}$, which is the unique fixed point of the following equation:

$$q_k = \min\{\gamma T_k(\boldsymbol{q}), \bar{p}_k\}, \ k = 1, \ldots, K, \tag{8}$$

Let the corresponding BS association $b_k = A_k(\boldsymbol{q})$, and denote γ_{ach} as the minimum SINR achieved by $(\boldsymbol{q}, \boldsymbol{b})$. Since $q_k \le \gamma T_k(\boldsymbol{q}), \forall k$, we have $\gamma_{\mathrm{ach}} \le \gamma$.

Proposition 1 *If $\gamma_{\text{ach}} = \gamma$, then problem (3) is feasible and $(\boldsymbol{q}, \boldsymbol{b})$ is the optimal solution; if $\gamma_{\text{ach}} < \gamma$, (3) is infeasible.*

Proposition 1 is a corollary of the following fact [4]: if problem (3) is feasible, then the optimal power vector satisfies the fixed point equation (8). Proposition 1 implies that the procedure (7) can be used to check the feasibility of problem (3).

We present a binary search method that solves problem (1) up to a precision of ϵ in Algorithm 1, whereby the subproblem (3) is solved by the procedure (7). The precision error ϵ needs to be determined a priori, and for each SINR target γ, problem (3) needs to be solved up to a precision of ϵ. In practice, it may not easy to check whether problem (1) has been solved to a precision of ϵ; instead, one can set $\|\boldsymbol{p}(t+1)-\boldsymbol{p}(t)\| \le \epsilon_1 \|\bar{\boldsymbol{p}}\|$ as the stopping criterion for the procedure (7), where $\epsilon_1 > 0$ is a small constant.

Algorithm 1: A binary search method
Initialization: pick $0 < \gamma_l < \gamma_h$.
While $\gamma_h - \gamma_l > \epsilon$:
1) $\gamma \leftarrow (\gamma_l + \gamma_h)/2$.
2) Solve problem (3) up to a precision of ϵ to obtain $(\hat{\boldsymbol{q}}, \hat{\boldsymbol{b}})$, and compute the minimum SINR $\hat{\gamma}_{ach}$.
3) if $\hat{\gamma}_{ach} < \gamma - \epsilon$: $\gamma_h \leftarrow \gamma$; otherwise: $\gamma_l \leftarrow \gamma$.

4. A NORMALIZED FIXED POINT ITERATION

In this section, we propose an algorithm that solves problem (1) directly, without resorting to the binary search. Denote

$$\bar{\boldsymbol{p}} \triangleq (\bar{p}_1, \ldots, \bar{p}_K),$$

$$T(\boldsymbol{p}) \triangleq (T_1(\boldsymbol{p}), \ldots, T_K(\boldsymbol{p})),$$

where $\bar{p}_k$ is the power budget of user k, and $T_k(\boldsymbol{p})$ is defined in (5). Define a weighted infinity norm $\|\cdot\|_\infty^{\bar{\boldsymbol{p}}}$ as

$$\|\boldsymbol{x}\|_\infty^{\bar{\boldsymbol{p}}} = \max_{1\le k\le K} \frac{x_k}{\bar{p}_k},\ \forall\, \boldsymbol{x} \in \mathbb{R}^K. \tag{9}$$

If all users have the same power budget $\bar{p}_k = P_{\max}$, the defined norm $\|\boldsymbol{x}\|_\infty^{\bar{\boldsymbol{p}}} = \|x\|_\infty / P_{\max}$.

The proposed algorithm is based on the following lemma, which states that the optimal power vector satisfies a fixed point equation.

Lemma 2 *Suppose $(\boldsymbol{p}^*, \boldsymbol{a}^*)$ is the optimal solution to problem (1), then $\boldsymbol{p}^*$ satisfies the following equation:*

$$\boldsymbol{p}^* = \frac{T(\boldsymbol{p}^*)}{\|\boldsymbol{T}(\boldsymbol{p}^*)\|_\infty^{\bar{\boldsymbol{p}}}}. \tag{10}$$

Proof of Lemma 2: For a given power allocation $\boldsymbol{p}^*$, the optimal BS association is $a_k^* = A_k(\boldsymbol{p}^*) = \arg\min_n T_k^{(n)}(\boldsymbol{p}^*)$. Therefore, the SINR of user k at optimality is

$$\text{SINR}_k^* = \frac{p_k^*}{T_k^{(a_k^*)}(\boldsymbol{p}^*)} = \frac{p_k^*}{\min_n T_k^{(n)}(\boldsymbol{p}^*)} = \frac{p_k^*}{T_k(\boldsymbol{p}^*)}. \tag{11}$$

Let γ^* denote the optimal value $\min_k \text{SINR}_k^*$, then we have

$$\text{SINR}_k^* = \gamma^*, \quad \forall\, k. \tag{12}$$

In fact, if $\text{SINR}_j^* > \gamma^*$ for some j, then we can reduce the power of user j so that SINR_j decreases and all other SINR_k increases, yielding a minimum SINR that is higher than γ^*. This contradicts the optimality of γ^*, thus (12) is proved.

According to (11) and (12), we have

$$\gamma^* T_k(\boldsymbol{p}^*) = p_k^*, \quad \forall\, k. \tag{13}$$

Next, we show that at least one user transmits at full power, i.e.

$$\max_k \frac{p_k^*}{\bar{p}_k} = 1. \tag{14}$$

Assume $\mu = \max_k \frac{p_k^*}{\bar{p}_k} < 1$. Define a new power vector $\boldsymbol{p} = \boldsymbol{p}^*/\mu$, then $\boldsymbol{p}$ satisfies the power constraints $p_k \le \bar{p}_k, \forall k$. The SINR of user k achieved by $(\boldsymbol{p}, \boldsymbol{a}^*)$ is $\text{SINR}_k = p_k / T_k^{(a_k^*)}(\boldsymbol{p}) = p_k^* / (\mu T_k^{(a_k^*)}(\boldsymbol{p}^*/\mu)) > p_k^* / (T_k^{(a_k^*)}(\boldsymbol{p}^*)) = \text{SINR}_k^*$, which contradicts the optimality of $(\boldsymbol{p}^*, \boldsymbol{a}^*)$.

Plugging (13) into (14), we obtain

$$\frac{1}{\gamma^*} = \max_k \frac{T_k(\boldsymbol{p}^*)}{\bar{p}_k} = \|T(\boldsymbol{p}^*)\|_\infty^{\bar{p}}. \tag{15}$$

Combining (13) and (15), we obtain (10). Q.E.D.

Based on the fixed point equation (10), we propose the following algorithm to solve problem (1).

Algorithm 2: A normalized fixed point iteration
Initialization: pick random positive power vector $\boldsymbol{p}(0)$.
Loop t:
1) Compute BS association: $a_k(t) \leftarrow A_k(\boldsymbol{p}(t))$, $\forall\, k$.
2) Update power: $\boldsymbol{p}(t+1) \leftarrow T(\boldsymbol{p}(t))$;
3) Normalize: $\boldsymbol{p}(t+1) \leftarrow \frac{\boldsymbol{p}(t+1)}{\|\boldsymbol{p}(t+1)\|_\infty^{\bar{\boldsymbol{p}}}}$, where $\|\boldsymbol{p}(t+1)\|_\infty^{\bar{p}} = \max_k \frac{p_k(t+1)}{\bar{p}_k}$.
Iterate until $\|\boldsymbol{p}(t) - \boldsymbol{p}(t+1)\| \le \epsilon_2 \|\bar{p}\|$.

The following theorem shows that the above algorithm converges to the optimal solution to (1) at a geometric rate.

Theorem 3 *Suppose $(\boldsymbol{p}^*, \boldsymbol{a}^*)$ is the optimal solution to problem (1). Then the sequence $\{\boldsymbol{p}(t)\}$ generated by Algorithm 2 converges geometrically to $\boldsymbol{p}^*$, i.e.,*

$$\|\boldsymbol{p}(t) - \boldsymbol{p}^*\|_\infty^{\bar{p}} \le C\kappa^t, \tag{16}$$

where $C > 0$, $0 < \kappa < 1$ are constants that depend only on the problem data.

Proof of Theorem 3: By definition (4), the mapping $T(\boldsymbol{p}) = (T_1(\boldsymbol{p}), \ldots, T_K(\boldsymbol{p})) : \mathbb{R}_+^K \to \mathbb{R}_+^K$ is the pointwise minimum of affine linear mappings $T^{(n)}(\boldsymbol{p}) = (T_1^{(n)}(\boldsymbol{p}), \ldots, T_K^{(n)}(\boldsymbol{p}))$, for $n = 1, \ldots, N$. It follows that T is a concave mapping. According to Lemma 2, $\boldsymbol{p}^*$ is a fixed point of (10). According to the concave Perron-Frobenius theory [15, Theorem 1], (10) has a unique fixed point, and Algorithm 2 converges to this fixed point. Therefore, Algorithm 2 converges to $\boldsymbol{p}^*$.

To show the geometric convergence, we define U as the set of power vectors $\boldsymbol{p}$ with $\|\boldsymbol{p}\|_\infty^{\bar{p}} = 1$ (i.e. $\max_k \frac{p_k}{\bar{p}_k} = 1$). It can be easily verified that

$$A_k \le T_k(\boldsymbol{p}) \le B_k, \quad \forall\, \boldsymbol{p} \in U, \tag{17}$$

where $A_k = \min_n \min_l \frac{\sigma_n^2 + g_{nl}\bar{p}_l}{g_{nk}}$ and $B_k = T_k(\bar{\boldsymbol{p}}) = \min_n \frac{\sigma_n^2 + \sum_{j \neq k} g_{nj}\bar{p}_j}{g_{nk}}$ are both constants that only depend on the problem data. For two vectors x, y, we denote $x \ge y$ if $x_k \ge y_k,\ \forall\ k$. Define $\kappa = 1 - \min_k \frac{A_k}{B_k} \in (0, 1)$ and $\boldsymbol{e} = (B_1, \ldots, B_K) > 0$. Then (17) implies

$$(1 - \kappa)e \le T(\boldsymbol{p}) \le e, \quad \forall\, \boldsymbol{p} \in U. \tag{18}$$

According to the concave Perron-Frobenius theory [16, Lemma 3, Theorem], if T is a concave mapping and satisfies (18), then Algorithm 2 converges geometrically at the rate κ. Q.E.D.

Remark: Theorem 3 implies the pseudo-polynomial time solvability of problem (1). Without loss of generality, we can assume $\sigma_n^2 = 1$; in fact, replacing g_{nk}^2 by g_{nk}^2/σ_n^2 and σ_n^2 by 1 for all n, k does not change problem (1) and Algorithm 2. It is easy to verify that $\kappa \le 1 - 1/(KG \cdot \mathrm{SNR})$, where $\mathrm{SNR} = \max_k \bar{p}_k$ and $G = \max_{n,k}\{g_{nk}\}$. To achieve an ϵ-optimal solution, Algorithm 2 takes $T \le \frac{\log(1/\epsilon)}{\log(1/\kappa)} \le \log(1/\epsilon) KG \cdot \mathrm{SNR}$ iterations. Since $KG \cdot \mathrm{SNR}$ is polynomial in the input parameters $K, \{\bar{p}_k\}$ and $\{g_{nk}\}$, we obtain the pseudo-polynomial time solvability of problem (1).

5. NUMERICAL RESULTS

Consider a HetNet (heterogeneous network) that consists of 25 hexagon macro cells, each containing one macro BS in the center. The distance between adjacent macro BSs is 1000m. There are 3 pico BSs randomly placed in each macro cell, thus in total there are $N = 100$ BSs. The channel gain from user k to BS n at a distance d_{nk} is $g_{nk} = S_{nk}(200/d_{ik})^{3.7}$, where $10\log_{10} S_{i,k} \sim \mathcal{N}(0, 64)$ models the shadowing effect. In Algorithm 1, we set $\epsilon_1 = 10^{-6}$ and $\epsilon = 10^{-3}$; in Algorithm 2, we set $\epsilon_2 = 10^{-6}$. There are $K = 160$ users with the same power budget $\bar{p}_k = P_{\max}$ in the network, and we consider two user distributions: in "Uniform", users are uniformly distributed in the network area; in "Congested", $K/4 = 40$ user are placed randomly in one macro cell, while other users are uniformly distributed in the network area. Suppose the noise power is $\sigma = 1$, and define the signal to noise ratio as $\mathrm{SNR} = 10\log_{10}(P_{\max})$.

Fig. 1 depicts the CDF (Cumulative Distribution Function) of the number of iterations needed for the following three algorithms to converge: Algorithm 1, 2 and the algorithm "Oracle". In the algorithm "Oracle", we fix the BS association to be the optimal one $\boldsymbol{a}$, and compute the optimal power allocation by the following procedure (proposed in [14])

$$p_k(t+1) \leftarrow \frac{T_k^{(a_k)}(\boldsymbol{p})}{\|T_k^{(a_k)}(\boldsymbol{p})\|_\infty^{\bar{p}}}. \tag{19}$$

A little surprisingly, Algorithm 2 and the algorithm 'Oracle' converge equally fast: they usually converge in 10∼30 iterations. Due to the binary search step, Algorithm 1 takes more than 200 iterations in total to converge (each subproblem takes 10∼30 iterations to converge).

Fig. 2 compares the minimum rate achieved by Algorithm 1, Algorithm 2 and the "max-SNR" algorithm. The "max-SNR" algorithm computes the BS association based on the maximum receive SNR, i.e. $a_k = \arg\max_n\{g_{nk}\bar{p}_k\}$. For a fair comparison, the optimal power allocation corresponding to "max-SNR" algorithm is then computed by (19). Each point in the figure is obtained by averaging over 500 monte carlo runs. Algorithm 1 and Algorithm 2 have similar performance in terms of the minimum rate. For the setting "Uniform", Algorithm 2 outperforms "max-SNR" by approximately 70% (when SNR= 35db); for "Congested", Algorithm 2 outperforms "max-SNR" by 400% (when SNR= 35db).

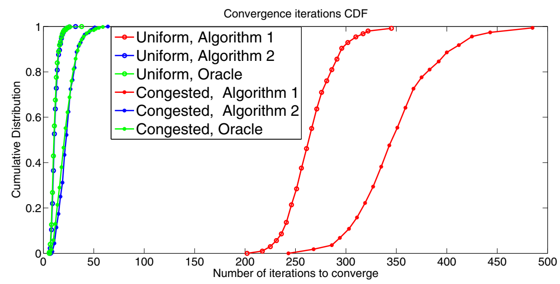

Fig. 1. Distribution of the number of iterations required to converge. $N = 100$ BSs, $K = 160$ users, $\mathrm{SNR} = 25dB$.

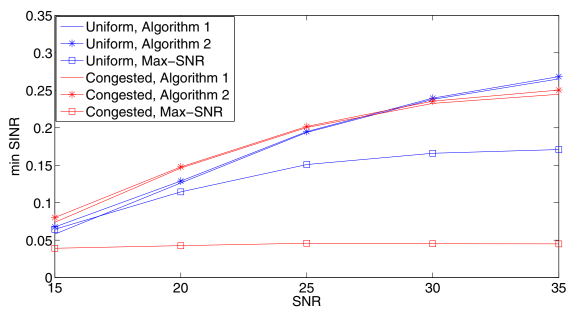

Fig. 2. Comparison of the minimum SINR achieved. $N = 100$ BSs, $K = 160$ users.

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