

# OPTIMAL RESOURCE ALLOCATION FOR TYPE-II HARQ BASED OFDMA AD HOC NETWORKS UNDER INDIVIDUAL RATE AND POWER CONSTRAINTS

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## ABSTRACT

We address multiuser power and bandwidth allocation for Orthogonal Frequency Division Multiple Access (OFDMA) networks employing a Type-II Hybrid Automatic Repeat reQuest (HARQ) mechanism, practical Coding and Modulation Schemes (MCSs) and Bit Interleaved Coded Modulation (BICM). The problem is formulated as minimizing the sum power required to satisfy a goodput constraint for each link while its transmit power does not exceed a certain level. Assuming statistical Channel State Information (CSI), we propose an algorithm to compute the corresponding optimal resource allocation and a practical selection of the MCSs that significantly boosts the proposed algorithm.

## 1. INTRODUCTION

HARQ is considered a powerful link-layer mechanism that allows reliable communications over time-varying channels. Among the different HARQ schemes, the so-called Type-II, which includes Chase Combining (CC-HARQ) and Incremental Redundancy (IR-HARQ) [1], is the most promising in terms of performance. On the physical layer, random Subcarrier Assignment Scheme (SAS) and BICM allow to harvest the inherent diversity in wireless links while OFDMA allows to handle multipath and multiuser interference. It is thus of great interest to address sum transmit power minimization for OFDMA networks that use BICM, MCSs and Type-II HARQ. Although the above problem arises in a wide class of wireless systems, we are mainly interested in this article in ad hoc networks. In such networks, generally a node called “resource manager” is elected to perform the resource allocation. This node typically has only outdated CSI. We therefore assume that resource allocation should be done with only statistical CSI. We also consider individual constraints on the *per-link average transmit powers* as in real-world systems.

In the literature, only few works *e.g.*, [2]-[7], addressed multiuser resource allocation for communication systems utilizing HARQ, and none of them considered individual

power constraints. In [2], the objective is to determine user scheduling and resource allocation that minimize the sum of the information-theoretic data rates in a IR-HARQ based network assuming perfect CSI. The problem is simplified by separately performing power control and bandwidth assignment. The authors of [3] address the problem of sum power minimization for a similar network under average delay constraints with an information-theoretic approach that fails to take practical MCSs into account. In [4, 5], the system goodput is maximized assuming Type-II HARQ and outdated CSI. However, no more than one user can be scheduled at any given time. In the context of cognitive radio, some works have been devoted to resource allocation for secondary users when HARQ is employed [6]. Finally, in [7], transmit power minimization is done in presence of statistical CSIT and practical MCS but only for Type-I HARQ and without individual transmit power constraints. *In this paper, our main contribution is to address the previous problem in the context of Type-II HARQ while also adding per-link transmit power constraints.* The extension from Type-I HARQ to Type-II HARQ is not straightforward since the closed-form expressions for the performance metrics of the latter are much more complicated. The same difficulty arises from adding the individual power constraints.

## 2. SYSTEM MODEL

We focus on network with  $K$  active links. One of its nodes is the resource manager which performs the proposed resource allocation algorithm. Each of the links is considered as a time-varying frequency-selective channel whose  $M$  time-domain taps are Rayleigh distributed. It is assumed that OFDM (with  $N$  subcarriers covering a total bandwidth of  $W$  Hz) is employed and that channels remain constant over one OFDM symbol but change independently between consecutive OFDM symbols. Let  $\mathbf{h}_k(j) = [h_k(j, 0), \dots, h_k(j, M-1)]^T$  be the channel impulse response of link  $k$  during OFDM symbol  $j$  where the superscript  $(\cdot)^T$  stands for the transposition operator. The multivariate complex circular Gaussian distribution with

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mean  $\mathbf{a}$  and covariance matrix  $\Sigma$  is hereafter denoted by  $\mathcal{CN}(\mathbf{a}, \Sigma)$ . Let  $\mathbf{H}_k(j) = [H_k(j, 0), \dots, H_k(j, N-1)]^T$  be the Fourier Transform of  $\mathbf{h}_k(j)$ . The received signal is thus

$$Y_k(j, n) = H_k(j, n)X_k(j, n) + Z_k(j, n),$$

where  $X_k(j, n)$  is the symbol transmitted at subcarrier  $n$  of the OFDM symbol  $j$ , and where  $Z_k(j, n) \sim \mathcal{CN}(0, N_0 W/N)$ .  $\{h_k(j, m)\}_{j,m}$  are independent random variables with variances  $\varsigma_{k,m}^2$  that are constant w.r.t index  $j$  i.e.,  $\mathbf{h}_k(j) \sim \mathcal{CN}(\mathbf{0}, \Sigma_k)$  with  $\Sigma_k \stackrel{\text{def}}{=} \text{diag}_{M \times M}(\varsigma_{k,0}^2, \dots, \varsigma_{k,M-1}^2)$ . The subcarriers of a single link are thus identically distributed as  $H_k(j, n) \sim \mathcal{CN}(0, \varsigma_k^2)$  where  $\varsigma_k^2 = \text{Tr}(\Sigma_k)$ . Finally, define

$$\text{Gain-to-noise ratio: } G_k \stackrel{\text{def}}{=} \frac{\mathbb{E}[|H_k(j, n)|^2]}{N_0} = \frac{\varsigma_k^2}{N_0}. \quad (1)$$

At the Medium Access Layer (MAC), each link  $k$  receives information packets of  $n_b$  bits each. A Type-II HARQ scheme is then used to transmit each of them in at most  $L$  transmissions. The content of each one of these  $L$  transmissions is called a MAC Packet (MP). It depends on the particular Type-II scheme in use. We examine two of these schemes, namely CC-HARQ and IR-HARQ [1, 8]. In either case, we denote by  $R_k$  the code rate associated with the first transmission. On the physical layer, the symbols are chosen from a  $2^{m_k}$ -QAM constellation. The MCS associated with link  $k$  can thus be represented by the couple  $(m_k, R_k)$ . Let  $\mathcal{E}_{k,l}$  be the event that decoding the information packet based on the first  $l$  MPs results in an error and define  $\pi_{k,l} \stackrel{\text{def}}{=} \mathbb{P}\{\mathcal{E}_{k,l}\}$ . If the BICM and random SAS techniques are tuned to the channel coherence time, the links can be considered as fast fading and the  $l$ th transmission can achieve the maximum diversity gain of  $d_{k,l}$  defined as follows. In the case of IR-HARQ,  $d_{k,1}, \dots, d_{k,L}$  are the minimal Hamming distances associated respectively with transmissions  $l = 1, \dots, L$ . As for CC-HARQ,  $d_{k,l} = l d_{k,1}$ . The results of [9, 10] can thus be applied to show that

$$\pi_{k,l}(\text{SNR}_k) \leq \frac{g_{k,l}}{\text{SNR}_k^{d_{k,l}}}, \quad (2)$$

where  $g_{k,l}$  is a constant designed to upper-bound tightly the simulated  $\mathbb{P}\{\mathcal{E}_{k,l}\}$  curve. The resource manager is assumed to only know the gains  $G_k$  which are subcarrier-independent. It cannot thus decide which subset of subcarriers a link should use, but only how many. Let  $n_k$  designate the number of subcarriers assigned to link  $k$  and define parameter  $\gamma_k \stackrel{\text{def}}{=} \frac{n_k}{N}$  that we allow to take any value in  $(0, 1)$ . It is also natural to use the same power  $P_k \stackrel{\text{def}}{=} \mathbb{E}[|X_k(j, n)|^2]$  on all the  $n_k$  subcarriers. Let  $E_k \stackrel{\text{def}}{=} \frac{P_k}{W/N}$  be the energy consumed to transmit one symbol on one subcarrier and define  $\sigma_k^2 \stackrel{\text{def}}{=} N_0 W/N$  as the noise variance. Note that each subcarrier of  $k$  undergoes an average signal-to-noise ratio (SNR) given by  $\text{SNR}_k \stackrel{\text{def}}{=} \frac{\varsigma_k^2 P_k}{\sigma_k^2} = G_k E_k$ . Finally, we define the goodput  $\eta_k(\gamma_k, E_k)$  as the number of successfully-decoded information bits per channel use.

### 3. OPTIMAL POWER AND BANDWIDTH ALLOCATION ASSUMING FIXED MCSS

As the average energy consumed on any link  $k$  to send its part of the OFDM symbol is  $N\gamma_k E_k$ , our goal is to minimize the total average transmit power, proportional to  $\sum_{k=1}^K \gamma_k E_k$ , while a minimum goodput  $\eta_k^{(0)}$  for each link  $k$  is guaranteed

$$\eta_k(\gamma_k, E_k) \geq \eta_k^{(0)}, \quad (3)$$

and while a maximum allowable power is respected:  $\gamma_k E_k \leq Q_k^{(0)}$ . From [8], we know that for any Type-II HARQ:

$$\eta_k(\gamma_k, E_k) = m_k R_k \gamma_k \frac{1 - q_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} q_{k,l}(G_k E_k)}, \quad (4)$$

where  $q_{k,l}(\text{SNR}_k) = \mathbb{P}\{\mathcal{E}_{k,1}, \dots, \mathcal{E}_{k,l}\}$ . It is difficult to get  $q_{k,l}$  in closed-form, but it can be upper-bounded by  $\pi_{k,l}$  expressed analytically by Eq. (2). This bound is relatively tight for all practical values of  $L$  and the SNR [8] leading to

$$\eta_k \geq m_k R_k \gamma_k \frac{1 - \pi_{k,L}(G_k E_k)}{1 + \sum_{l=1}^{L-1} \pi_{k,l}(G_k E_k)}. \quad (5)$$

If we replace the RHS of its goodput constraint (3) with the LHS of Eq. (5) we get the following optimization problem.

**Problem 1.**  $\min_{\gamma_1, \dots, \gamma_K, E_1, \dots, E_K} \sum_{k=1}^K \gamma_k E_k \quad \text{subject to}$

$$\gamma_k \frac{1 - g_{k,L}/(G_k E_k)^{d_{k,L}}}{1 + \sum_{l=1}^{L-1} g_{k,l}/(G_k E_k)^{d_{k,l}}} \geq \frac{\eta_k^{(0)}}{m_k R_k}, \quad (6a)$$

$$\gamma_k E_k \leq Q_k^{(0)}, \forall k \in \{1, \dots, K\}, \quad (6b)$$

$$\sum_{k=1}^K \gamma_k \leq 1, \quad (6c)$$

$$\gamma_k > 0, E_k > 0, \forall k \in \{1, \dots, K\}. \quad (6d)$$

Problem 1 is feasible iff forcing  $\gamma_k E_k = Q_k^{(0)}$  for all  $k$  still leads to a feasible problem. Indeed if Problem 1 is feasible with  $\gamma_{k_0} E_{k_0} < Q_{k_0}^{(0)}$  for  $k_0$ , one can increase  $E_{k_0}$  until equality in Eq. (6b) without violating Eq. (6a). Setting  $E_k = Q_k^{(0)}/\gamma_k$ , constraint (6a) writes as  $\mathcal{G}_k(\gamma_k) \geq 0$ , where

$$\mathcal{G}_k(\gamma) \stackrel{\text{def}}{=} m_k R_k \gamma \frac{1 - g_{k,L}/(G_k Q_k^{(0)}/\gamma)^{d_{k,L}}}{1 + \sum_{l=1}^{L-1} g_{k,l}/(G_k Q_k^{(0)}/\gamma)^{d_{k,l}}} - \eta_k^{(0)}.$$

One can show that  $\lim_{\gamma \searrow 0} \mathcal{G}_k(0) < 0$  and that  $\exists! \gamma_k^{\max} > 0$  s.t.  $\mathcal{G}_k$  is increasing on  $(0, \gamma_k^{\max}]$  and decreasing for  $\gamma > \gamma_k^{\max}$ . Either  $\mathcal{G}_k(\gamma_k^{\max}) < 0$ , then  $\mathcal{G}_k(\gamma) < 0$  for all  $\gamma > 0$  and constraint (6a) cannot be satisfied. Or  $\mathcal{G}_k(\gamma_k^{\max}) \geq 0$ , then function  $\mathcal{G}_k$  has a unique zero on  $(0, \gamma_k^{\max}]$ . We thus define:

$$\gamma_k^{(0)} \stackrel{\text{def}}{=} \begin{cases} +\infty, & \text{if } \mathcal{G}_k(\gamma_k^{\max}) < 0, \\ \text{the zero of } \mathcal{G}_k \text{ on } (0, \gamma_k^{\max}], & \text{if } \mathcal{G}_k(\gamma_k^{\max}) \geq 0. \end{cases} \quad (7)$$

Setting  $\gamma_k < \gamma_k^{(0)}$  would result in violating constraint (6a) while  $\sum_{k=1}^K \gamma_k > \sum_{k=1}^K \gamma_k^{(0)}$  for any  $\gamma_k > \gamma_k^{(0)}$ . We should thus set  $\gamma_k = \gamma_k^{(0)}$  in order to test the feasibility of Problem 1.

**Lemma 1.** *Problem 1 is feasible iff  $\sum_{k=1}^K \gamma_k^{(0)} \leq 1$ .*

In general, Problem 1 is not convex due to constraint (6a). Nevertheless, by assuming the specific expression (2) for  $\pi_{k,l}$ , we obtain a *geometric program* [11]. Indeed, the LHS of Eq. (6b) and Eq. (6c) are straightforwardly posynomials in  $\{\gamma_k, E_k\}_{k=1, \dots, K}$ . Plugging Eq. (2) into Eq. (6a) leads to the following new expression which is clearly also a posynomial:

$$\frac{\eta_k^{(0)}}{m_k R_k} \gamma_k^{-1} + \sum_{l=1}^{L-1} \frac{\eta_k^{(0)} g_{k,l}}{m_k R_k G_k^{d_{k,l}}} \gamma_k^{-1} E_k^{-d_{k,l}} + \frac{g_{k,L}}{G_k^{d_{k,L}}} E_k^{-d_{k,L}} \leq 1.$$

Problem 1 is thus convex in  $\{x_k, y_k\}_{1 \leq k \leq K}$  where  $\gamma_k = e^{x_k}$  and  $E_k = e^{y_k}$  [11]. Moreover Lemma 1 implies Slater's condition holds for this convex optimization problem. Thus the associated Karush-Kuhn-Tucker (KKT) conditions provide a global solution if the inequality in Lemma 1 is satisfied. Let  $\mu_k, \nu_k, \lambda$  be the Lagrangian multipliers associated with constraints (6a), (6b), (6c) respectively and define function  $x \mapsto f_k(x)$  for any SNR value  $x \in \mathbb{R}_+^*$  as

$$f_k(x) \stackrel{\text{def}}{=} \frac{1 + \sum_{l=1}^{L-1} g_{k,l}/x^{d_{k,l}}}{1 - g_{k,L}/x^{d_{k,L}}}. \quad (8)$$

Note that the LHS of Eq. (6a) is equal to  $\gamma_k/f_k(G_k E_k)$  and that  $f_k$  is decreasing on  $(g_{k,L}^{1/d_{k,L}}, +\infty)$ . The associated KKT conditions should be first derived in variables  $\{x_k, y_k\}_{k=1 \dots K}$ . Rewriting them in  $\{\gamma_k, E_k\}_{k=1 \dots K}$  gives:

$$(1 + \nu_k) \gamma_k E_k - \mu_k \frac{\eta_k^{(0)}}{m_k R_k \gamma_k} \left( 1 - \sum_{l=1}^{L-1} \frac{g_{k,l}}{(G_k E_k)^{d_{k,l}}} \right) + \lambda \gamma_k = 0, \quad (9)$$

$$(1 + \nu_k) \gamma_k E_k - \mu_k \left( \frac{\eta_k^{(0)}}{m_k R_k} \sum_{l=1}^{L-1} \frac{g_{k,l} d_{k,l}}{\gamma_k (G_k E_k)^{d_{k,l}}} + \frac{g_{k,L} d_{k,L}}{(G_k E_k)^{d_{k,L}}} \right) = 0, \quad (10)$$

$$\mu_k \left( \frac{\eta_k^{(0)}}{m_k R_k} - \frac{\gamma_k}{f_k(G_k E_k)} \right) = 0, \quad (11)$$

$$\lambda \left( \sum_{k=1}^K \gamma_k - 1 \right) = 0, \quad \nu_k \left( \gamma_k E_k - Q_k^{(0)} \right) = 0. \quad (12)$$

Referring to Eq. (6d), we note that  $\gamma_k E_k > 0$ . We also have  $1 + \nu_k > 0$  because  $\nu_k$  is a Lagrange multiplier. We thus get from Eq. (10) that  $\mu_k \neq 0$ , meaning that the goodput constraint (Eq. 6a) is always active. Eq. (11) thus yields:

$$\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k). \quad (13)$$

**In the following** we write  $E_k$  as function of  $\lambda$  by eliminating  $\mu_k$  and  $\nu_k$ . We thus plug Eqs. (10) and (13) into Eq. (9) to get

$$\lambda = \frac{1}{G_k} F_k(G_k E_k) + \frac{\nu_k}{G_k} (F_k(G_k E_k) + G_k E_k), \quad (14)$$

where we defined for any SNR  $x = G_k E_k \in (g_{k,L}^{1/d_{k,L}}, +\infty)$ ,

$$F_k(x) \stackrel{\text{def}}{=} \frac{x}{\frac{\sum_{l=1}^{L-1} d_{k,l} g_{k,l}/x^{d_{k,l}}}{1 + \sum_{l=1}^{L-1} g_{k,l}/x^{d_{k,l}}} + \frac{d_{k,L} g_{k,L}/x^{d_{k,L}}}{1 - g_{k,L}/x^{d_{k,L}}}} - x. \quad (15)$$

The following lemma states some properties of function  $F_k$ .

**Lemma 2.**  $\forall k, \exists! s_k > g_{k,L}^{1/d_{k,L}} > 0$  s.t. i)  $F_k(s_k) = 0$ , ii)  $F_k(x) < 0 \forall x < s_k$ , iii)  $F_k^{-1}$  is increasing on  $[0, +\infty)$ .

**For the moment, assume that a genie tells us the value of the Lagrange multiplier  $\lambda$ .** In this case, due to Eq. (14):

$$\nu_k = \frac{G_k \lambda - F_k(G_k E_k)}{G_k E_k + F_k(G_k E_k)}. \quad (16)$$

We thus need to write  $E_k$  as function of  $\lambda$ . For *links with active transmit power constraint*, the bandwidth parameter  $\gamma_k$  is equal to  $\gamma_k^{(0)}$  defined by Eq. (7) and  $E_k = Q_k^{(0)}/\gamma_k^{(0)}$ . As for *links with inactive transmit power constraint*, we plug  $\nu_k = 0$  into Eq. (16) and we refer to Eq. (13) to obtain:

$$E_k = \frac{1}{G_k} F_k^{-1}(G_k \lambda), \quad \gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \lambda)). \quad (17)$$

We now can determine the subset of links, denoted as  $\mathcal{B}(\lambda)$ , for which  $E_k = Q_k^{(0)}/\gamma_k^{(0)}$ .  $\mathcal{B}(\lambda)$  is simply composed of links  $k$  whose associated multiplier  $\nu_k$ , as given by Eq. (16), is strictly positive i.e.,  $G_k \lambda > F_k(G_k E_k)$ . We thus have

$$\mathcal{B}(\lambda) = \left\{ k \mid G_k \lambda > F_k \left( G_k Q_k^{(0)} / \gamma_k^{(0)} \right) \right\}. \quad (18)$$

**We now turn our attention to the determination of  $\lambda$ .** We should thus extend the definition of  $\mathcal{B}(\lambda)$  to any value  $\Lambda \geq 0$ :  $\mathcal{B}(\Lambda) \stackrel{\text{def}}{=} \left\{ k \mid G_k \Lambda > F_k \left( G_k Q_k^{(0)} / \gamma_k^{(0)} \right) \right\}$ . Note that  $\mathcal{B}(\Lambda)$  has a “physical” meaning *only when  $\Lambda = \lambda$*  and that  $\mathcal{B}(\Lambda) = \{1, \dots, K\}$  for  $\Lambda$  large enough. Now define  $\forall \Lambda \geq 0$

$$\Gamma(\Lambda) \stackrel{\text{def}}{=} \sum_{k \in \mathcal{B}(\Lambda)} \gamma_k^{(0)} + \sum_{k \in \overline{\mathcal{B}(\Lambda)}} \frac{\eta_k^{(0)}}{m_k R_k} f_k(F_k^{-1}(G_k \Lambda)) \quad (19)$$

which is continuous and decreasing. We finally obtain the following theorem and optimal allocation algorithm.

**Theorem 1.** *Let the feasibility condition in Lemma 1 hold. The optimal solution to Problem 1 is as follows.*

*If  $\tilde{\Gamma}(0) \leq 1$ , then  $\forall k \in \mathcal{B}(0)$ :  $\gamma_k = \gamma_k^{(0)}$ ,  $E_k = Q_k^{(0)}/\gamma_k^{(0)}$ , and  $\forall k \in \overline{\mathcal{B}(0)}$ :  $E_k = \frac{1}{G_k} F_k^{-1}(0)$ ,  $\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k)$ . Else,  $\forall k \in \mathcal{B}(\lambda)$ :  $\gamma_k = \gamma_k^{(0)}$ ,  $E_k = Q_k^{(0)}/\gamma_k^{(0)}$ , and  $\forall k \in \overline{\mathcal{B}(\lambda)}$ :  $E_k = \frac{1}{G_k} F_k^{-1}(G_k \lambda)$ ,  $\gamma_k = \frac{\eta_k^{(0)}}{m_k R_k} f_k(G_k E_k)$ , with  $\lambda$  the unique solution in  $\mathbb{R}_+^*$  to  $\Gamma(\Lambda) = 1$ .*

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**Algorithm 1** Optimal resource allocation for Problem 1

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 $\Lambda \leftarrow 0$ 
for all  $k \in \{1, \dots, K\}$  do  $\gamma_k^{(0)} \leftarrow$  RHS of Eq. (7) end for
repeat
   $\mathcal{B}(\Lambda) \leftarrow \left\{ k \mid G_k \Lambda > F_k \left( G_k Q_k^{(0)} / \gamma_k^{(0)} \right) \right\}$ 
  for all  $k \in \mathcal{B}(\Lambda)$ :  $\gamma_k \leftarrow \gamma_k^{(0)}$  do  $E_k \leftarrow \frac{Q_k^{(0)}}{\gamma_k^{(0)}}$  end for
  for all  $k \in \overline{\mathcal{B}(\Lambda)}$  do  $\gamma_k, E_k \leftarrow$  RHS of Eq. (17) end for
  increment  $\Lambda$ 
until  $\sum_{k=1}^K \gamma_k \leq 1$ 
return  $\{\gamma_k, E_k\}_{k=1 \dots K}$ 

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#### 4. MCS SELECTION

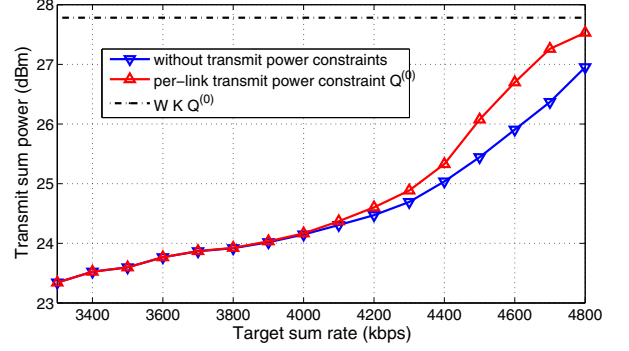
Let  $\mathcal{M}$  designate the set of available modulation schemes and  $\mathcal{R}$  the set of available codes. Fixing  $\mathbf{m} \stackrel{\text{def}}{=} [m_1, \dots, m_K]^T$  and  $\mathbf{R} \stackrel{\text{def}}{=} [R_1, \dots, R_K]^T$ , Algorithm 1 returns the optimal parameters  $\{\gamma_k, E_k\}_k$ . Define  $Q_T^*(\mathbf{m}, \mathbf{R}) \stackrel{\text{def}}{=} \sum_{k=1}^K \gamma_k E_k$  as the minimal total transmit power. The optimal MCS selection is  $(\mathbf{m}^*, \mathbf{R}^*) = \arg \min_{(\mathbf{m}, \mathbf{R}) \in \mathcal{M}^K \times \mathcal{R}^K} Q_T^*(\mathbf{m}, \mathbf{R})$ . It can be found by an exhaustive search that is prohibitively costly in computations. Instead, we resort to the suboptimal greedy algorithm used in [7] and inspired by [12]. Let  $\mathcal{M}$  and  $\mathcal{R}$  be sorted s.t.  $\mathcal{M} = \{m_1, \dots, m_{|\mathcal{M}|}\}$  and  $\mathcal{R} = \{R_1, \dots, R_{|\mathcal{R}|}\}$  with  $m_1 \leq \dots \leq m_{|\mathcal{M}|}$  and  $R_1 \leq \dots \leq R_{|\mathcal{R}|}$ . The idea behind the algorithm is to change the MCS of only one link per iteration. This is done by assigning to each link the next MCS in the ordered set  $\mathcal{M} \times \mathcal{R}$  and by selecting the link whose MC modification results in the lowest total power.

#### 5. NUMERICAL RESULTS

We consider a network with  $K = 10$  links with a bandwidth  $W = 5$  MHz centered around  $f_0 = 2400$  MHz. Each information packet is  $n_b = 128$  bits long. The distance  $D_k$  associated with any link  $k$  is randomly drawn from a uniform distribution on  $[0.1, 1]$  km. The path-loss parameter  $\zeta_k^2$  follows a free-space model. For the sake of simplicity, each link has the same target goodput  $\eta^{(0)}$  so that the required sum rate is equal to  $K\eta^{(0)}$ . Finally, we fix  $N_0 = -170$  dBm/Hz.

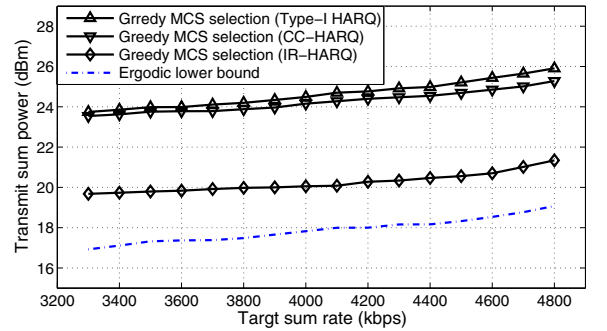
Assume that each link uses QPSK and a CC-HARQ based on the 1/2-rate convolutional code from [13]. In Figure 1, we plot  $W \sum_{k=1}^K \gamma_k E_k$  obtained using 200 Monte-Carlo runs of Algorithm 1 with  $WQ^{(0)} = 18$  dBm. The sum  $W \sum_{k=1}^K Q_k^{(0)} = WKQ^{(0)}$  of the maximum allowable levels is also shown. Note that the power constraints have negligible effect for target sum rates smaller than 4 Mbps while they lead to an increase in the total transmit power in the range 4-5 Mbps. For target sum rates larger than 5 Mbps, neither the constrained nor the unconstrained problems can be feasible.

Now assume that  $\mathcal{M} = \{1, 2, 4, 6\}$  and  $\mathcal{R} = \{1/2, 1\}$ . In Figure 2, we plot the total power resulting from applying



**Fig. 1.** Sum transmit power of Algorithm 1 (CC-HARQ)

the proposed MCS selection with the following schemes: IR-HARQ based on the two nested convolutional codes from [14] with an initial rate equal respectively to 1/2 and 1, CC-HARQ and Type-I HARQ based either on the 1/2-rate convolutional code from [13] or on uncoded transmission. In the same figure, we compare the sum transmit power obtained by the proposed MCS selection to the ideal lower bound reached by endowing the links with the possibility of achieving their *ergodic capacity*. Figure 2 shows that the proposed greedy MCS selection removes up to 2 dBm from the optimality gap with the ergodic lower bound as compared to Figure 1 where the MCS is fixed trivially.



**Fig. 2.** Sum transmit power of the proposed MCS selection

#### 6. CONCLUSION

We proposed an optimal algorithm to minimize sum power consumption in OFDMA wireless networks which use Type-II HARQ subject to per-link goodput and transmit power constraints. Under the assumption of statistical CSI and practical MCSs, our algorithm returns the optimal power and bandwidth parameters. Finally, a computationally-efficient algorithm for MCS selection was provided that, when coupled with the proposed resource allocation, yields significant reductions in the total power emitted by the network.

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